THE RATIONAL ARITHMETIC

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GRAMMAR SCHOOL

MYERS AND BROOKS

RATIONAL GRAMMAR SCHOOL

ARITHMETIC

BY

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PREFACE

The present book is an outgrowth of the notion that arithmetic as a science of pure number, and arithmetic as a school science, must be treated from two essentially different standpoints. Viewed as a finished mental product, arithmetic is an *abstract* science, taking its bearings solely from the needs of the subject; but viewed as a school subject, arithmetic should be an *abstracted* science, taking its bearings mainly from the needs of the learner. The former calls only for logical treatment, while the latter calls for psychological treatment as well. In other words, to be of high educational value the school science of arithmetic must take into full account the particular stage of the pupil's development. The abstract stage must be approached by steps which begin with the learner, rise with his unfolding powers, and end leaving him in possession of the outlines of the science of arithmetic. To break vital contact with the learner at any stage of the unfolding process is fatal. A controlling principle in the development of the various topics of this book is that any phase of arithmetical work, to be of value, must make an appeal to the life of the pupil.

But the social and industrial factors in American communities enter largely into the pupil's life. This renders material drawn from industrial sources and from everyday affairs of high pedagogical value for arithmetic. The recent infusion of new life into the curricula of elementary schools through the wide introduction into them of nature study, manual training, and geometrical drawing furnishes a basis for a closer unifying of the pupil's work in arithmetic with his work in the other school subjects. Wide use has been made of all these sources of arithmetical material.

A rational presentation of the processes and principles of arithmetic can be secured as well through material representing real conditions as through material representing artificial conditions. Not only have most of the problems been drawn from real sources

but a very earnest effort has also been made to have all data of problems absolutely correct and consistent, to the end that inferences from them may be relied upon. With so rich a store as the book contains, however, it is perhaps too much to hope that no errors remain. The authors will deem it a favor to be notified of any errors that may be detected.

It is well known that the majority of the pupils of the elementary school never reach the high school. Even these pupils, whose circumstances cut them off from advanced mathematical study, have a right to claim of the elementary school some useful knowledge of the more powerful instruments of algebra and geometry. For those who will continue their studies into the high school it is important that the roots of the later mathematical subjects be well covered in the soil of the earlier. The present book seeks to meet the needs of both classes of pupils through the organic correlation of the elements of geometry and algebra with the arithmetic proper. Treated thus, the geometry serves to illustrate the work of arithmetic and algebra, and the algebra emerges from the arithmetic as generalized number.

In particular, this text aims to accomplish four main purposes, viz.:

(1) To present a pedagogical development of elementary mathematics, both as a tool for use and as an elementary science;

(2) To base this development on subject-matter representing real conditions;

(3) To open to the pupil a wide range and variety of uses for elementary mathematics in common affairs—to aid him to get a working hold of his number sense; and

(4) To give the pupil some training in ways of attacking common problems arithmetically and some power to analyze quantitative problems.

It is not intended that teachers should have their classes solve all the problems; but rather that each teacher should select such topics and problems as have a particular interest for his school. We trust that the suggestiveness of this book as to sources of problems and ways of handling them in the arithmetic class will be appreciated by many teachers. Such teachers will doubtless

prefer to work out some topics more fully than is done in the text. It is thought that what is given in the text will make the fuller working out of special topics practicable and not difficult.

The work makes continual call for estimating magnitudes and for actual measurement by the pupils. Let the children be supplied with measures, foot-rules, yardsticks, meter-sticks, etc., and encourage their constant use. Make so regular a feature of this work that pupils form the habit of estimating distances, areas, volumes, weights, etc., always correcting their estimates by actual measurement.

All models, scales, or standards of measure made by pupils should be carefully kept and used in the later work.

Pupils should also be supplied with instruments with which the exercises in constructive geometry may be actually done. A cheap pair of compasses is the only necessary purchase. It is better that the pupil make the rest of the apparatus needed. If for any reason the pupil cannot actually do the constructive work it should be omitted altogether.

The Introduction which precedes the work in the formal operations is a departure from current text-book procedure believed to be worthy of special remark. It consists of fifteen pages of simple, practical problems, relating to matters with which the pupil's experiences, in school and out of school, have familiarized him in an indefinite way, for the right understanding of which the use of numbers and of the arithmetical processes is necessary. This chapter, by furnishing to the pupil a gradual transition from his vacation experiences to the rather severe study of the formal processes of arithmetic, by impressing the pupil with a sense of the real need and purpose of such study, thus preparing his interest for it, will be recognized as a deviation from common practice resting upon sound pedagogical grounds.

Especial attention is invited to this introductory chapter; to the chapter on measurement, §§71-81, preceding and laying the foundation for common fractions; to the treatment of proportion, §§85-97, 115-118, 190, 191; to the lists of data for original problems, §§29, 36, 44, 132; to the work in geography, §§23, 27, 54, 157; in farm account keeping, §35; on farm products, §62; in

commerce, §28; in nature study, §§45, 122, 124, 155, 156, and elsewhere; in physical measurements, §§6-8, 125, 133; in paper-folding, §174; on the locomotive engine, §166; in constructive geometry, §§70, 168-180, 187; and in applied algebra, §206. Most teachers will approve dropping many of the time-honored but antiquated subjects and the curtailing of other topics of slight utility, and the introduction in their places of more timely and more real subjects. The many lists of data for original problems, §§29, 36, 44, 132; the treatment of longitude and time, §§181, 182, the descriptive work leading up to this topic; the extensive use of graphs to put meaning into arithmetical measures and to bring out the laws involved in numerical data, will commend themselves to teachers as useful and instructive means of keeping the number faculty employed on practical material.

Another special feature is the numerous lists of problems that bear on the development of some important idea or law, having an interest on its own account. In these lists each problem is a step in a connected line of thought culminating in an important truth. This plan furnishes numerous problems, miscellaneous as to process, thereby requiring original mathematical thought, and still organically related to a central idea, thereby calling for the constant exercise of judgment. Examples of this may be seen in any section of the introduction; in the problems on geography, commerce, nature study, and elementary science, and also in the following sections: 35, 67, 68, 82, 83, 110, 115, 116, 118, 133, 134, 136, 155, 156, 157, 160, and in practically all the matter from p. 264 to the end of the book. For the maturity of pupils of the later grades this is believed to be an important feature. It avoids the danger, always present with lists of promiscuous problems when classified under the arithmetical processes to be exemplified, of reducing to the mechanical what should never be allowed to become mechanical, viz.: the analysis of relations. Throughout the book the instruction is addressed to the pupil's understanding rather than to his memory.

But while accomplishing this, due regard has been had to the necessity of sufficient drill in pure number to enable the pupil to obtain both a conscious recognition of processes and considerable

facility in their automatic use. This is done out of the conviction that the fundamental arithmetical operations should be reduced to the automatic stage as early as possible, consistently with a clear understanding of them.

The attention of teachers is called to the section on Short Methods and Checking at the close of the book. After pupils have clearly grasped the meaning of the arithmetical processes and have acquired some mastery of their uses, a relief from the tedium of long arithmetical calculations becomes a matter of great importance. No one can become a rapid computer without short methods and, considering that the problems of daily life that call for arithmetical treatment do not have answers with them, no one can be certain of his results without means of checking calculations. Every expert accountant uses them and the more expert he is the more does he use them. In fact, expertness consists very largely in the ability to shorten calculations and to apply rapid checks. Training in the use of short cuts and checks should constitute a much more important part of the pupil's work in arithmetic than is common. Constant use of these sections should be made through the seventh and eighth grades.

Definitions, processes, rules, and even special subjects are worked out under the guidance of the principle: "First its informal, though rational, use, and afterwards conscious recognition of its formal use." Definitions of merely technical terms are given when called for by the development and where the need for them arises.

Unusual attention has been paid to the numerous illustrations. The publishers have spared no pains to make them an important aid to the teacher in the development of the subject. None have been inserted for mere adornment and none are mere pictures of things or relations that are perfectly obvious to the pupil. Their aim throughout is to secure clearness, precision, and certainty of thought. It is thought the book may lay rightful claim to an innovation in this particular.

For convenience in reviews and for reference, a synopsis of definitions and a full index are given.

The authors' acknowledgments for suggestions and ideas are

due to numerous arithmetical writers and teachers. They desire to express their obligations in particular to Miss Katherine M. Stilwell of the School of Education, University of Chicago, for valuable help throughout the book; to Mr. J. B. Russell, Superintendent of Schools, Wheaton, Ill., and to Miss Ada Van Stone Harris, Supervisor of Primary Schools, Rochester, N. Y., both of whom read much of the proof. They desire most heartily to thank Mr. Stephen Emery of Lewis Institute, whose unceasing diligence and pains with the proofs have wrought distinct improvements on nearly every page of the book.

If the book shall in some measure aid in putting the teaching of arithmetic on a more rational basis, thereby bringing about results more nearly commensurate with the time and energy put upon the subject in the elementary schools, the authors will deem their efforts repaid.

THE AUTHORS.

CHICAGO, August, 1905.

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INTRODUCTION

§1. Scale Drawing. ORAL WORK

1. The scale of the drawing of the picture frame is 1″: 4″, or $\frac{1}{4}$. How high is the frame? how long?

2. How long and how high is the place for the picture? Answer question 1 for a scale of $\frac{1}{12}$; of $\frac{1}{10}$.

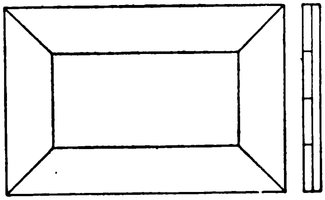


FIGURE 1

3. What is the scale of a drawing in which a line 1 in. long represents 1 ft.? in which 1 in. represents 40 ft.? 100 ft.?

4. In Fig. 2, which is a scale drawing of a schoolroom, one-sixteenth of an inch represents 1 ft. How long is the room? the teacher's desk? How long and how wide are the pupils' desks?

5. In what direction do the pupils face, when seated at their desks?

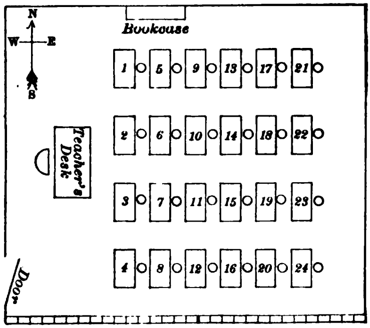


FIGURE 2

6. Find by measurement how far it is from the northwest corner of desk 1 to the southwest corner of desk 4; from the west edge of desk 2 to the east wall of the room.

7. Make and answer other questions on this plan.

8. Make a similar (like) scale drawing from measurements of your schoolroom and the fixed objects within it.

9. Make a scale drawing from measurements of your schoolhouse and grounds.

§2. Reading Scale Drawings.

Using a foot rule marked to 8ths and 16ths of an inch and noticing the scale, answer by measurement the problems on the drawing of figure 3.

1. How wide is the actual block that is shown by the drawing *M N O P*? How long is the block?

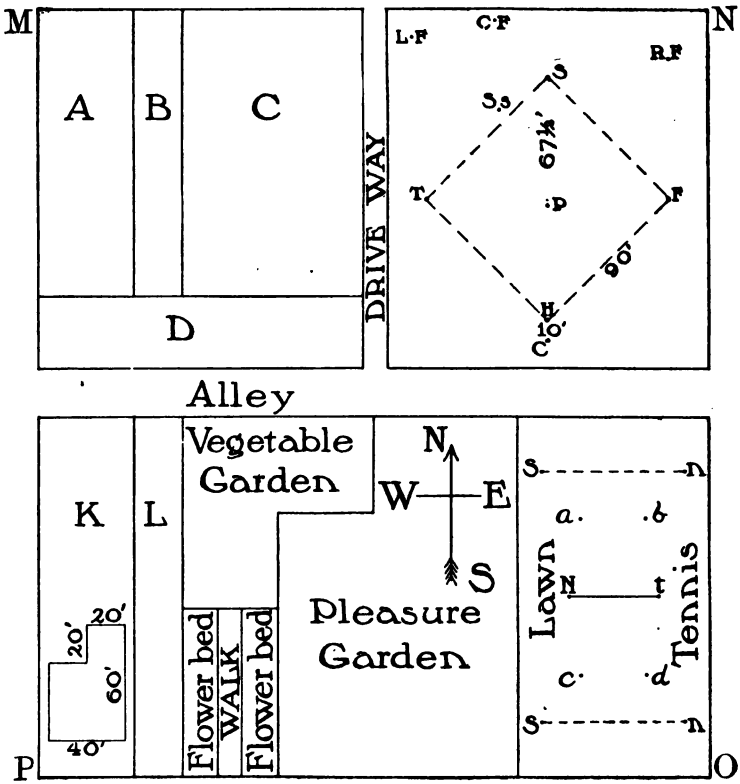


FIGURE 3 SCALE 1″ = 100′.

2. How wide is lot *A*? *B*? *K*? *L*?

3. How long is the vegetable garden? How wide?

4. How wide is the pleasure garden? The southeast corner lot? The northeast corner lot?

5. The alley (25 ft. wide) is taken equally from the north and south halves of the block. How long is lot *K*? The northeast lot? The southeast lot?

6. How long are the flower beds? How wide is the walk?

7. If a man cuts a strip along the west side of lot *A* with a lawn mower that cuts a swath 1 ft. wide, how many square feet of the lawn does he cut in passing along the side once? Twice? Ten times? How many strips must he cut to mow the entire lot?

8. How many square feet are there in lot *A*? *B*? In the northeast corner lot? In the southeast corner lot? In lot *K*?

9. The baseball "diamond" on the northeast lot is a 90-ft. square. How many square feet are there within the lines connecting the bases?

10. If a player strikes the ball and runs to third along the dotted lines, how far does he run? How far must a player run to make a "home run"?

11. The left fielder stands at *L F* and throws the ball to the pitcher at *P*. How far does he throw?

12. The center fielder at *C F* catches a ball which was struck at the home plate, throws it to third at *T*, where it is caught and thrown to the home plate. How far has the ball gone?

13. How much will it cost to enclose the northeast quarter of the block with a fence costing 25 cents a foot?

14. Make and solve other problems on the drawing.

15. Notice the scale of any good maps in your geography and find by measurement about how far it is from Chicago to New York; to Washington, D. C.; to New Orleans; from New York to St. Louis; to Pittsburg; to Boston; to San Francisco.

§3. Cost of Living. ORAL WORK

1. If the living expenses of a family are $70 per mo., what is the expense per year?

2. If house rent is $25 per mo. and other expenses are $50, what part of the total expense is house rent?

3. How many work days are there in a month? How many weeks in a year?

WRITTEN WORK

1. Find out at home or from your neighbor how much hard coal is required to supply some furnace or stove 1 mo. Supposing hard coal costs \$7.50 per T., how much will the coal cost for the 6 winter months from November to May?

2. If soft coal costs \$3.75 a T., and $1\frac{3}{4}$ T. goes about as far as 1 T. of hard coal, about how much would it cost to supply this furnace with soft coal for the 6 winter months?

The prices given below are fair averages for a large city. They would be less in smaller places, and prices current in your community are to be preferred to these.

GROCER'S PRICES

Apples	pk.	\$.35
Apricots	can	.25
Baking powder	½ lb. can	.20
Bread	loaf	.05
Butter	lb.	.28
Cabbage	lb. 2c., head	.10
Cauliflower	head	.15
Celery	bunch	.10
Cheese	lb.	.15
Coffee	lb.	.30
Crackers	lb.	.10
Eggs	doz.	.25
Flour	25 lb. sack	.50
Lemons	doz.	.25
Lettuce	bunch	.05
Lima beans	can	.10
Milk	qt.	.06¼
Oatmeal	2 lb. pkg.	.10
Olives	pt.	.30
Onions	pk.	.20
Oranges	doz.	.30
Peaches	can	.25
Pears	can	.25
Peas	can	.12½
Pickles	doz.	.10
Potatoes	pk.	.25
Prunes	lb.	.10
Rice	lb.	.08⅓

GROCER'S PRICES—*Continued*

Rolls	doz.	\$.10
Salt	lb.	.05
Soap	bar	.05
Starch	lb.	.10
Sugar	lb.	.05
Tea	lb.	.60
Tomatoes	can	.15

BUTCHER'S PRICES

Bacon	lb.	\$.20
Chicken	lb.	.14
Fish, fresh	lb.	.12½
Fish, salt	lb.	.10
Ham	lb.	.18
Lard	lb.	.13
Lobsters, shrimps, etc.	lb.	.25
Mutton	lb.	.14
Oysters	qt.	.30
Pork	lb.	.10
Pork tenderloin	lb.	.20
Porterhouse steak	lb.	.18
Round steak	lb.	.10
Salmon	lb.	.20
Sausage meat	lb.	.10
Sirloin steak	lb.	.16
Spare ribs	lb.	.08⅓
Turkey, duck, etc.	lb.	.12½
Veal	lb.	.14

NOTE.—It will be assumed that rates are the same for either large or small quantities.

Make such problems as:

3. If you purchase of a grocer 2 loaves of bread, 1 lb. of butter, 1 lb. of coffee, and 1 pk. of potatoes, and give him $1, how much change should you receive, if prices are as quoted (named) in the table on the opposite page?

4. If you buy a 2-lb. sirloin steak and give the butcher 50¢, what change should you receive, prices being as in the table?

5. Pupils may make out lists in the form of a statement and find the change due if the account is paid with $2, $5, or $10, thus:

Bought				Paid		
Item				Item		
1.	2 doz. rolls	@ 10c.	$0.20	1.	Cash	$5.00
2.	1 doz. eggs	25c.	.25			
3.	1 doz. oranges	30c.	.30			
4.	2 bunches celery	10c.	.20			
5.	1 lb. tea	60c.	.60			
6.	2 cans peas	12½c.	.25			
7.	1 lb. starch	10c.	.10			
8.	5 bars soap	5c.	.25			
9.	1 pkg. oatmeal	10c.	.10			
		Total cost				
	Balance due in change					

6. Foot the statement above.

7. Make out a bill of supplies such as a hotel keeper might purchase at the grocer's or the butcher's.

8. Pupils may make and solve problems similar to the above, using local prices when convenient.

Jan., 1902.	
Meat......	$18.50
Groceries .	27.75
Milk	1.88
Butter	2.60
Soft coal..	9.65
Clothing ..	15.00

9. The actual living expense account of a family for January is as shown in the table. If this is a fair average monthly expense account for the family, what will be its living expense for one year?

10. The family pays $3000 for a home and is thereby saved $25 per

mo. in rent. The yearly rent amounts to what part of the cost of the home?

11. If the home is in the suburbs of a city and the man must pay 10¢ morning and evening 6 d. per wk. for car fare, how much does this add to his yearly expense account?

The table given here shows the average yearly cost to one person for food, clothing, and household utensils from 1897 to 1901 inclusive:

For Year Ending	Bread-stuffs	Meat	Dairy & Garden	Other Food	Clothing	Metals	Miscel-laneous	Totals
Dec. 31, 1897	$13.51	$7.34	$12.37	$8.31	$14.65	$11.57	$12.11	
Dec. 31, 1898	13.82	7.52	11.46	9.07	14.15	11.84	12.54	
Dec. 31, 1899	13.25	7.25	13.70	9.20	17.48	18.09	16.31	
Dec. 31, 1900	14.49	8.41	15.56	9.50	16.02	15.81	15.88	
Dec. 31, 1901	20.00	9.67	15.25	8.95	15.56	15.38	16.79	
Sums.......								
Averages...								

Note.—In the above table breadstuffs include wheat, corn, oats, rye, barley, beans, and peas; meat includes lard and tallow; dairy and garden products include vegetables, milk, butter, eggs, and fruit; the miscellaneous articles include a variety of things which make a part of the cost of living for the average family.

12. Fill out a column like the "totals" column, thus finding the total cost of living for 1 person for each of the 5 years.

13. What was the increase in cost of living from 1897 to 1901?

14. Has the cost of any of the items decreased during this period? How much has been the change in cost in each case?

15. What fractional part (what part expressed as a fraction) of the whole cost of living for the year is the cost of breadstuffs in 1897? in 1899? in 1901? 1st *Ans.* $\frac{13.51}{79.86} = \frac{1351}{7986}$.

Note.—The average of any five numbers is $\frac{1}{5}$ of their sum.

16. Find the average yearly cost of living for the 5 years.

17. Find the average cost of each item (breadstuffs, meat, etc.).

18. Fill out a column like the "totals" column, with the sums of the numbers in the second, third, fourth and fifth lines. What does each of the totals mean?

19. Give the change, from year to year, in the total cost of foods for one person by finding the difference between each total and the one next after it.

§4. **Fencing a Farm.**—A farm was divided into fields and seeded as shown in the drawing (Fig. 4). The scale of the drawing is 1 in. to 80 rd. This means that 1 in. in the drawing represents 80 rd. in the farm, and that all other lines longer or shorter than 1 in. represent distances in the farm proportionately longer or shorter than 80 rd.

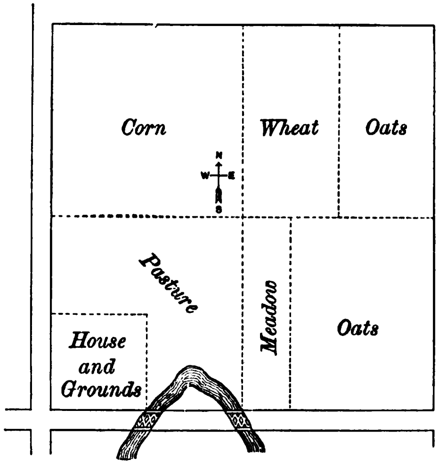


FIGURE 4

ORAL WORK

1. How long is the cornfield north and south? how wide?

2. How long and how wide is the wheatfield? the north oatfield?

3. How long and how wide is the meadow? the south oatfield?

4. The drawing of the road is $\frac{1}{10}$ in. wide; how wide is the road?

5. How long and how wide is the farm?

6. How long must a wire be to reach entirely round the farm?

7. How many rods of barbed wire will be needed to inclose the farm with a 3-wire fence? how many miles (320 rd. = 1 mile)?

WRITTEN WORK

1. How many rods long is the partition (division) fence across the farm from east to west? what part of a mile is its length? How many rods of wire will make it a 4-wire fence? 1st *Ans.* 160.

2. How many rods of wire will run a 4-wire fence across the farm from the middle of the north side to the middle of the south side? *Ans.* 640.

3. How many rods of wire will run a 4-wire fence between the wheatfield and the north oatfield? between the meadow and the south oatfield?

4. How many bales of wire of 100 rd. each will be needed for these cross fences and division fences? *Ans.* 19.2.

5. How much will the wire cost for the 3-wire inclosing fence @ $3.50 a bale of 100 rods? *Ans.* $67.20.

6. The posts for all the barbed wire fences are set 1 rd. apart. How many posts will be needed to fence around the farm?

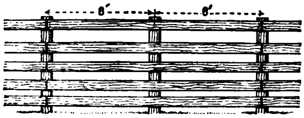
7. How much will these posts cost @ 25¢ apiece?

8. How much will the wire cost @ $2.60 a bale for the two 4-wire cross fences, and the two division fences between the wheat and the north oatfield and between the meadow and south oatfield?

9. How much will the posts for these fences cost @ 15¢ apiece?

10. Find the total cost of wire and posts for these fences.

11. The house and barn lot is separated from the pasture by a 5-board fence. The boards used are 1 in. thick, 6 in. wide, and 12 ft. long. How many boards will reach once along the north side of the house and barn lot? *Ans.* 55.



TWO FENCE PANELS
FIGURE 5

12. How many boards will reach once along the east side of the lot?

13. How many boards will be needed for the 5-board fence along both sides?

14. The posts are set 6 ft. between centers, that is, a post is set every 6 ft. (Fig. 5). How many posts will be needed for the north side? for the east side? for both?

1st *Ans.* 110; 2d *Ans.* 109.

15. How much will these posts cost @ 23¢ apiece?

Lumber is sold by the *board foot.* A board foot is a board 1 ft. long, 1 ft. wide, and 1 in., or less, thick.

16. How many board feet of lumber in a board 1 in. thick, 1 ft. wide, and 5 ft. long? the same thickness and width and 10 ft. long? 12 ft. long?

17. How many board feet in a board 1 in. thick, 10 ft. long, and 2 ft. wide? the same thickness and length and 6 in. wide? 9 in. wide? 18 in. wide? 1 in. wide? 8 in. wide?

18. How many board feet in a board 12 ft. long, 4 in. wide, and 1 in. thick? 2 in. thick?

19. How many board feet in a "two by four" scantling 10 ft. long (2″ × 4″ × 10′)* ? 16 feet long?

20. How many board feet in a "four by four" scantling 10 ft. long? 12 ft. long? 18 ft. long?

21. How many board feet in a fencing plank 1″ × 6″ × 12′? in 5 such planks?

22. How much will the fencing lumber of problem 12 cost @ $18 per M board feet?

23. What will be the total cost of the lumber and posts for the fence on the north and east sides of the lot?

24. It cost 8¢ apiece to have the post holes bored, and it took 4 days' work by 3 men @ $1.50 per day to build the fence. Three patent gates costing $12 apiece were put in the fence. Find the total cost of the lot fence, including gates.

25. What was the total cost of all the fencing done on the farm?

§5. Areas of Fields. ORAL WORK

1. The whole farm contains 160 acres; how many acres are there in the cornfield? in the wheatfield? in the meadow? in the south oatfield? in the pasture? in the lot around the house and barn?

2. What is the width of each of these fields in rods?

3. If the meadow were divided by a north and south central line, how many rods wide would each half be? How many acres would there be in each half?

4. How many square rods in a strip 1 rd. wide and 80 rd. long?

5. How many square rods in an acre?

6. How wide must a strip of land be to contain an acre if it is 80 rd. long? 40 rd. long? ½ mile long?

7. Compare the sizes of the wheatfield and the north oatfield; of the cornfield and the wheatfield; of the meadow and the wheatfield; of the meadow and the south oatfield; of the meadow and the house and barn lots; of the lots and the pasture; of the pasture and the south oatfield.

* The mark (′) means *foot* or *feet*; (″) means *inch* or *inches*.

WRITTEN WORK

1. What would be the income per acre from the farm planted in corn, if the average yield were 47 bu. per acre and were sold at 28¢ per bu.? What would be the gross (total) income from the whole farm for this year?

2. If the farm were rented @ $5 per acre, how much would remain for the tenant * from each acre? from the entire farm?

3. What would be the owner's income from the whole farm, not allowing for expenses?

4. Which is the more profitable way for the owner to rent his farm, @ $6 per acre cash, or for $\frac{1}{2}$ of all the crop delivered to market, supposing that the whole farm is planted to corn and that a yield of 48 bu. per acre and a price of 30¢ per bushel can be obtained every year?

NOTE.—Rent of the first sort is called "cash rent," of the second sort, "grain rent."

5. If this same farm will produce 20 bu. of wheat per acre and a price of 60¢ per bushel can be obtained for it, which is the more profitable crop to the owner, wheat or corn? how much more profitable?

6. Compare the profits to the owner of a grain-rented farm from an oats crop of 40 bu. per acre and a price of 22¢ per bushel with the profits of the wheat crop of problem 5; also with corn crop of problem 4.

7. In problem 5 which would bring the larger income to the owner, and how much, cash rent @ $5 per A., or grain rent @ $\frac{1}{2}$ delivered?

8. Answer the same question for the oats crop mentioned in problem 6.

9. A cornfield of 68 acres produced an average yield of 48 bu. per acre. How many bushels did the farm yield?

10. It cost $5.85 per acre to raise the crop. At 46¢ per bu. what was the net value of the crop?

11. If the tenant paid cash rent @ $5.50 per acre, what was his net profit from the crop? How much did the owner receive?

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* The tenant is the farmer who raises the crop on another man's farm.

§6. **Physical Measurements (a).**—A tin can, or pail, 18″ high and 10″ in diameter is filled nearly full of water. A second can, or pail, 9″ to 12″ high and 9″ in diameter is inserted, mouth downward, within the water. A smooth hole should be punched in the bottom of the smaller can.

A rubber tube passed under the edge of the inverted can and stretched over the end of a piece of glass tubing, which is held in an upright position inside the inverted can by light cross-braces, may be used to get air into the can.

If a cork, which fits into the hole at A, is drawn, the inner can sinks readily in the water; after which the cork is inserted air-tight.

FIGURE 6

A foot rule, marked with eighths or sixteenths of an inch, stuck with putty to the side of the inside can, permits the readings to be taken.

Such an apparatus, when used as in the problems below, is called a *spirometer*. A more elaborate instrument is shown in Fig. 6.

Common spools may be used for the pulleys that the cords pass over, and sand may be pressed into the cans W, W, until they just balance the weight of the inverted can or pail.

1. If the inside diameter of the inverted can is 8.8 in., for each inch the inner can sinks into the water, there will be 60.85 cu. in. of water inside the can, the stopper at A being removed. How many cubic inches (capacity) of water will there be in the can when it has sunk 2 inches? 5 inches? 8 inches?

NOTE.—If a pail is used, the corresponding number of cubic inches is found by measuring the distance in inches across the mouth of the pail, multiplying one-half this distance by itself, and the product by $3\frac{1}{7}$.

2. How many cubic inches of water will there be in the can when it is sunk in the water only $1\frac{1}{2}$ inches? $1\frac{3}{4}$ inches? $2\frac{1}{8}$ inches? $2\frac{5}{16}$ inches?

3. Let the can now be pushed down as far as it will go and the stopper at A be pushed in tightly. Read the scale. When a pupil blows through the tube until the inside can rises 1 inch, how

many cubic inches of air has he expelled from his lungs into the can? How many when the scale indicates that the can has risen $\frac{1}{8}$ inch? $1\frac{1}{16}$ inches? $1\frac{1}{4}$ inches? $1\frac{5}{16}$ inches? $1\frac{3}{8}$ inches?

4. Fill your lungs full and blow into the tube as long as you can without danger. Suppose you raise the can 2″, what is the capacity of your lungs in cubic inches?

5. The average lung capacity for a man 5 ft. 8 in. tall is 204 cu. in. This average capacity may be called the *normal* lung capacity. How much does your lung capacity fall short of the normal value for a man?

6. The normal lung capacity in cubic inches for a man is 3 times his height in inches. For a woman, it is 2.6 times the height. Divide your lung capacity by your height in inches, and compare your quotient with these numbers.

7. The chest measure of a man should be not less than half his height. How much greater or less is your chest measure than it should be according to this law?

§ 7. Physical Measurements (b).

1. Fill out in your notebook a record like the one below, which is a copy from a student's notebook:

Name, *John Morrow;* age, *10 yr.;* height, *52 in.;* weight, *61 lb.*

Chest measures,	on Inspiration (breathing in) *28 in.*	Lung capacity,
	on Expiration (breathing out) *24½ in.*	*104 cu. in.*
	Mean,* *26¼ in.*	

Height in inches by 3 (if a boy). Result *52 × 3 = 156 cu. in.*
Height in inches by 2.6 (if a girl).
Average to each inch of height, *2 cu. in.*, for *104 ÷ 52 = 2.*
Chest average = *26¼ in.* Half height = *26 in.*

2. How much do your measures exceed or fall short of the normal values?

3. Do your measures show that you need chest exercise?

4. How many in the class or room are above or below the normal?

* The *mean* of 2 numbers is half their sum.

5. Compare class or room averages* to see whether the average for class or room is about normal.

6. Make the comparison of problem 5 after a course of exercises in physical training, and record the changes due to the exercises.

7. Every month or two during the year find results like those called for in problems 2 and 5; keep your results, and show what changes are taking place in your measures.

§8. **Physical Measurements** (c).—The following table gives the average height in inches and the average weight in pounds for boys and for girls year by year from 4 to 15 years of age. The numbers are the averages of careful measures of heights and weights of hundreds of boys and girls in the schools of Chicago, Boston, Cincinnati, and St. Louis. These averages may be called *normal values* for boys and girls of school age:

AGE	HEIGHT		YEARLY GROWTH		WEIGHT		YEARLY GROWTH	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
4 yr.	39.2 in.	39.0 in.			37.3 lb.	35.3 lb.		
5 "	41.6 "	41.4 "			40.6 "	39.7 "		
6 "	43.7 "	43.3 "			44.7 "	43.0 "		
7 "	45.8 "	45.5 "			48.7 "	47.0 "		
8 "	47.9 "	47.6 "			53.8 "	52.0 "		
9 "	49.7 "	49.5 "			58.7 "	57.1 "		
10 "	51.6 "	51.3 "			64.8 "	62.2 "		
11 "	53.5 "	53.4 "			70.1 "	68.1 "		
12 "	55.2 "	55.8 "			76.7 "	77.4 "		
13 "	57.3 "	58.5 "			84.9 "	88.4 "		
14 "	60.0 "	60.2 "			95.0 "	98.3 "		
15 "	62.4 "	61.3 "			106.5 "	105.0 "		

1. Subtract each number in the column of heights from the number next *below* it and find the growth in height for boys and for girls from year to year. Do the same for the weights.

2. When is the yearly growth in height greatest for boys? for girls? What is the yearly growth in each case?

3. Make and solve similar problems for weights.

4. How much do your height and weight exceed, or fall short of, the normal values for your age?

* The class average is the sum of the measures for all pupils in the class divided by the number of pupils.

5. How do the averages for your room compare with the values in the table for the same years?

6. How much do your height and weight exceed or fall short of the averages for pupils of your own age in your room?

§9. Wind Pressure.

ORAL WORK

1. Which is easier, to walk against or with the wind?

2. If you are walking against the wind with a raised umbrella, why do you have to push forward harder than if you are walking with the wind?

3. If you hold a board 1′ wide and 10′ long flatwise against the wind, how hard does the board push against you if the wind pushes against each square foot of the board with a force of 2 pounds? 3 pounds? 5 pounds? 2½ pounds? 20 ounces? 32 ounces?

WRITTEN WORK

The wind is a stream of air moving along with a certain speed, or *velocity*. The greater the speed, or *velocity*, the stronger does it push against objects it strikes

The velocity of wind in miles per hour and the pressure per square foot are as given here:

Light breeze......	3¼ mi.	.75 oz.	Strong gale......	56¼ mi.	15 lb. 9 oz.
Moderate breeze ..	6½ "	3.33 "	Hurricane.......	79½ "	31 " 4 "
Fresh breeze......	16¼ "	1 lb. 5 "	Violent hurricane	97½ "	46 " 12 "
Stiff breeze.......	32½ "	5 " 3 "			

NOTE.—Use measures from your own schoolhouse or from other buildings in your neighborhood in preference to the numbers in the problems.

1. What is the total pressure in pounds on the west side of a house 30 ft. by 25 ft., due to a light breeze from the west? to a stiff breeze? to a strong gale?

2. What is the pressure on the side of a tall building 265 ft. by 80 ft., due to a hurricane blowing squarely against it?

3. Find the wind pressure against the side of a load of hay 25 ft. long and 12 ft. high, due to a strong gale blowing squarely against it.

4. Find the number of pounds pressure against a signboard fence 100 ft. by 14 ft., due to a strong gale blowing squarely against it.

5. What is the wind pressure tending to overturn a square chimney 30 ft. wide at the base, 15 ft. wide at the top, and 175 ft. high, due to a violent hurricane blowing squarely against one of the flat faces?

NOTE.—To obtain the area of the effective surface against which the wind is blowing, multiply the height of the chimney by its breadth halfway up. This breadth is the half sum of the widths at the top and bottom (Fig. 7).

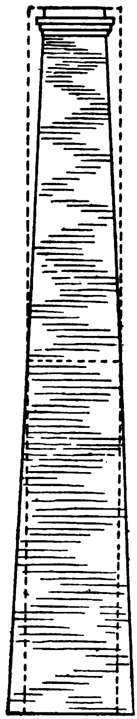

FIGURE 7

6. Find the wind pressure on the side of a passenger coach 70 ft. long and 15 ft. high, due to a stiff breeze blowing squarely against it:

(*a*) If the wind blows against the whole rectangle $15' \times 70'$;

(*b*) If the wind is not obstructed by the strip $4\frac{1}{2}' \times 70'$, from the track to the bottom of the coach box and running the length of the coach.

7. Find the wind pressure on the side of a train of 10 such cars as that of problem 6 (*b*), due to a strong gale, blowing squarely against it.

8. Find the wind pressure on a diamond-shaped signboard, due to a violent hurricane blowing squarely against it, if the diamond is 12′ long (AB = 12′) and 6′ broad at the middle (CD = 6′). (See Fig. 8.)

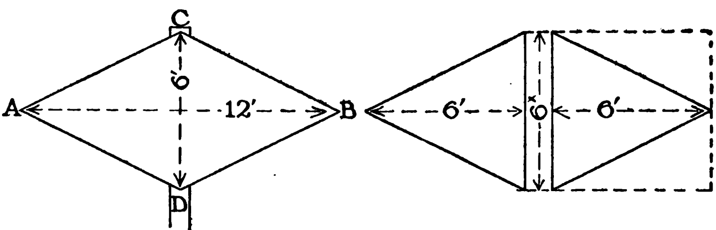

FIGURE 8

§10. Dairying.—The following table is an extract from the milk record of the University of Illinois herd of milk-cows:

WEIGHT OF MORNING (A.M.) AND EVENING (P.M.) MILKINGS IN POUNDS

1901 Nov.	Queen		Beauty		Beech-wood		Myrtle		Spot		Rose		Total
	a.m.	p.m.	a.m.	p.m.	a.m.	p.m.	a.m.	p.m.	a.m.	p.m.	a.m.	p.m.	
1	6.5	6.5	11.0	10.4	12.5	10.2	7.3	5.7	12.0	11.5	11.0	9.6	
2	5.6	6.0	11.0	8.4	11.0	11.0	7.2	5.6	11.5	10.5	11.3	8.8	
3	6.4	6.0	10.8	9.5	11.7	11.5	7.3	5.7	12.0	10.4	11.2	9.8	
4	6.0	6.5	12.5	10.2	12.6	13.0	7.2	6.0	11.5	11.5	11.0	9.8	
5	6.0	7.5	11.4	10.2	12.0	11.4	6.7	5.8	11.6	11.3	11.0	10.2	
6	6.6	6.8	12.0	10.0	8.0	11.6	8.0	5.0	12.5	11.8	11.7	10.0	
7	7.4	8.5	10.8	12.4	12.0	11.7	7.0	7.2	12.0	12.5	11.6	12.0	
8	7.8	7.3	11.2	11.6	13.2	12.2	7.2	5.7	11.5	11.3	11.5	10.1	
9	7.5	6.4	11.6	10.4	12.0	11.6	6.6	5.4	12.0	12.2	10.7	10.0	
10	7.8	7.2	11.0	10.4	12.0	12.0	6.7	4.7	11.5	8.5	11.8	10.6	
11	6.8	7.3	12.2	11.0	13.6	11.5	7.7	5.3	9.0	10.0	12.3	10.0	
12	7.8	7.7	12.5	12.0	13.0	11.6	8.0	5.5	12.0	12.6	12.2	10.6	
13	7.6	8.0	13.4	11.5	13.5	13.0	7.2	5.8	11.1	13.0	11.8	11.2	
14	7.0	8.0	12.0	11.8	14.0	12.7	7.7	5.7	12.5	12.7	11.5	10.3	
15	8.8	7.5	13.0	10.7	15.0	12.0	7.6	5.0	14.0	12.4	12.3	9.0	
16	9.0	7.0	12.5	11.0	15.2	13.7	7.0	5.5	12.7	12.3	14.1	8.5	
17	8.0	7.0	12.5	10.5	14.0	13.0	6.1	6.4	10.0	9.0	12.0	10.5	
18	8.1	7.7	13.4	10.9	14.0	11.5	6.6	5.0	12.5	11.5	11.0	9.1	
19	7.0	7.0	11.5	11.4	13.6	11.0	6.2	5.3	12.2	11.7	11.1	10.0	
20	7.5	6.5	12.6	10.5	12.7	12.0	6.5	4.9	11.8	11.4	11.5	9.5	
21	7.0	7.6	12.0	10.2	13.7	11.3	6.1	5.2	13.0	11.0	10.6	9.6	
22	8.0	7.5	12.2	11.9	14.7	11.2	6.7	6.0	12.4	10.7	11.3	9.5	
23	7.5	9.0	12.2	11.0	14.4	12.0	7.0	5.5	12.8	10.8	11.3	10.4	
24	7.0	7.1	13.5	10.6	13.8	11.5	7.2	5.1	13.0	10.7	12.0	10.4	
25	7.4	8.0	12.5	11.2	14.0	10.5	7.4	5.7	12.7	11.7	12.0	11.0	
26	6.5	8.4	12.0	11.8	12.0	12.0	6.1	5.8	13.2	11.5	11.5	10.5	
27	7.5	8.5	13.7	11.8	14.0	13.0	6.1	5.8	13.0	12.7	11.4	10.5	
28	7.2	9.2	12.0	12.0	13.6	13.8	6.6	4.9	13.1	12.5	11.7	10.5	
29	6.5	9.0	13.5	12.6	15.0	13.5	6.7	5.0	10.2	12.6	12.5	12.6	
30	7.2	8.0	11.7	12.7	14.2	12.2	6.0	4.3	12.4	11.5	12.7	10.3	

1. How many pounds of milk did Queen give the first week of Nov., 1901, at the morning milkings? at the evening milkings? at both milkings?

2. At which milking did Queen give most milk and how much more than at the smallest milking?

3. Make and answer similar questions for the remaining weeks and for any or all of the 6 cows.

NOTATION AND NUMERATION

§11. Digits.—0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

1. How many characters are given above? By what names do you know the first character? It is called *zero*, or *naught.*

These ten characters are called *digits* or *figures;* and by means of them all numbers, are expressed.

§12. Place and Name Values of Digits.

1. Read the following numbers:

$30	3	30	30,000
300 mi.	.3	300	300,000
3 yd.	.03	3000	$\frac{3}{15}$

What is the same in all these numbers? What is different in the numbers?

2. Read:

7 feet 7 days 7 hundreds 7 millions 7 tenths

How many feet? 7 what? How many hundreds? 7 what? How many millions? 7 what? In all these cases, what is the same? What is different?

3. In the number 666, what does the first 6 on the right mean? the second 6? the third? *1st Ans.* 6 ones *or* 6 units.

4. If the value which depends on the name of a digit is called its *name value*, what shall we call the value which depends on its *place?*

5. How many values has a digit? What gives the first value? the second? Which is the permanent (fixed) value? Which is the changeable value?

§13. ORAL WORK

1. Give the place value of 2 in each of the following numbers:

20 2 200 .2 20,000 2000 2222

2. In 286, for what number does the 2 stand? the 8? the 6?

We may read the number 2 hundreds, 8 tens, 6 units, or two hundred eighty-six, meaning two hundred eighty-six units.

Writing the names of the three places, they appear:

Hundreds	Tens	Units
2	8	6

meaning $200 + 80 + 6$.

3. Read the following numbers, first as hundreds, tens, and units, and then as units:

729	916	335	494
845	642	538	739

§14. Periods.—For convenience in reading, the digits of large numbers are grouped into periods of three digits each. These periods have names depending upon their position; but the three places in each period are always hundreds, tens, and units.*

The first seven periods are units, thousands, millions, billions, trillions, quadrillions, and quintillions. The first three periods are sufficient for common purposes.

QUINTILLIONS			QUADRILLIONS			TRILLIONS			BILLIONS			MILLIONS			THOUSANDS			UNITS		
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units
2	5	3	4	6	2	5	7	1	3	2	4	9	1	8	6	4	5	2	8	4

WRITTEN WORK

Write the following in figures:

1. Forty-five thousand, two hundred eighty-four; six thousand, nine hundred thirty-four; three hundred twenty-one thousand, one hundred thirteen; five thousand, five.

NOTE.—Fill all vacant places with zeros.

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*In Great Britain a notation different from that explained in §14 is used. The method of §14 is called the French notation and is used altogether in the United States, France, and Germany. The English notation groups the digits into periods of six figures each:

Billions	Millions	Units
865324,	786593,	279368

In the English notation a million millions make a billion, a million billions a trillion, etc. In the French notation a thousand millions make a billion, a thousand billions a trillion, etc.

2. Nine thousand, ten; ten thousand, nine; six hundred thousand, six hundred; five million, fifty thousand, three; eighty thousand, eighty.

§15. Decimal Notation. ORAL WORK

1. 10 equals how many units? 1 equals what part of 10?

2. 1 hundred equals how many tens? how many units? 1 ten equals what part of 1 hundred? 1 unit equals what part of 1 hundred?

3. 1 thousand equals how many hundreds? 1 hundred equals what part of 1 thousand?

4. The number denoted by each digit in 1111 is how many times as great as the number denoted by the digit to its right? The number denoted by each digit equals what part of the number denoted by the digit to its left?

5. How many tenths in 1 unit? 1 unit equals how many times 1 tenth (.1)?

6. 1 hundredth (.01) equals what part of 1 tenth? 1 tenth equals how many times 1 hundredth?

7. How does the place value of the unit change from right to left? from left to right?

§16. Reading Numbers.

3,007	.6
365,834	.27
4,004,246	1.07

2. Read and tell the meaning of numbers of the tables on pp. 30, 39, or 40.

3. Read aloud the following:

Mean radius of the earth	=	636,739,510 centimeters
Mean distance to moon	=	76,429,120 rods
Mean radius of sun	=	138,560,000 rods
Mean distance to sun	=	92,897,500 miles
Mean distance to sun	=	149,500,000,000 meters
Light travels in one minute	=	11,199,000 miles

4. The table gives the number of vessels engaged in the trade of the chief countries of the world for 1901, and the number of tons they could carry. Read some of the numbers:

Country	Number		Tonnage	
	Steamers	Sailing Vessels	Steamers	Sailing Vessels
Greece	150	1,078	139,187	320,797
Austria-Hungary	243	366	294,960	322,897
Denmark	377	1,200	256,631	387,727
The Netherlands	318	1,004	330,132	451,949
Japan	511	2,035	338,354	510,175
Spain	489	1,112	460,193	562,068
Sweden	672	2,328	314,572	607,868
Russia	659	3,372	347,375	850,695
Italy	381	1,931	435,426	947,079
France	850	2,595	547,895	961,259
Norway	866	2,913	507,158	1,393,096
Germany	1,350	2,560	1,561,078	2,106,895
The United States	794	4,614	916,153	2,318,276
The United Kingdom	7,906	15,219	7,944,095	10,304,338

§17. Writing Numbers. WRITTEN WORK

Write in numerals the following:

1. Four billion, two hundred thirteen million, one hundred twenty-two thousand, six hundred seven; forty-five billion, four hundred fifty million, three hundred twenty-seven thousand, one.

2. Seven hundred eighteen billion, two hundred million; one hundred ninety thousand, four hundred two; three million, one hundred twenty-four; fifteen thousand, nine; ninety; one hundred eighty-six billion; one hundred forty-seven million, one hundred thousand, one hundred.

The digits, excepting zero, were first used by the Hindus, but they were introduced into Europe by the Arabs, and we call the numbers written with these digits *Arabic numerals.*

§18. Roman Notation.—The Romans used letters instead of digits to represent numbers. This notation is still used upon monuments, and in numbering chapters and volumes of books.

In the Roman notation,

I = 1	L = 50	D = 500
V = 5	C = 100	M = 1000
X = 10		

When a letter of less value, as I, is placed before a letter of greater value, as V, the value of the less is to be taken from that of the greater; thus, IV means 4 and XC means 90.

When a letter of less value follows one of greater value, its value is added to that of the greater; as, VI for 6, XV for 15, and CX for 110.

Repeating a letter repeats its value; as, XX for 20, CC for 200.

A horizontal line over a letter increases its value a thousand-fold; as, $\overline{\text{C}}$ meaning 100,000, $\overline{\text{D}}$ meaning 500,000.

To interpret (change to Arabic notation) numbers written in the Roman notation, always begin on the right.

§19. Change of Notation. WRITTEN WORK

1. Change from Roman to Arabic:

XCV	CMXIX	CDLXXXIV
DXXV	MLXXX	MDCCLXXVI
DCIV	MCCLXX	LXIX
XCIX	DLVI	CCXI

2. Change from Arabic to Roman:

10	65	100	572	619	1902
30	77	500	78	584	1774
60	80	140	50	400	1565
49	95	46	246	2000	2590

ADDITION

§20. Definitions. ORAL WORK

1. A bicyclist rides 19 mi. Monday and 10 mi. Tuesday; how far does he ride both days?

2. I paid $28 for a suit of clothes and $20 for a bicycle; how much did I pay for both?

3. A four-sided lot has sides of the lengths: 25 yd., 4 yd., 11 yd., and 28 yd.; how far is it around the lot?

4. A man works for me 7 hr. one day, 6 hr. the next day, 7 hr. the next, and 8 hr. the next; how many hours does he work for me in all?

5. A newsboy sold 18 papers Wednesday, 12 papers Thursday, and 16 papers Friday; how many papers did he sell in all?

6. The number of hours the sun shone on each of the days of a certain week was: 8 hr., 6 hr., 3 hr., 7 hr., 5 hr., 9 hr., 2 hr. How many hours of sunshine were there during the week?

7. A farmer's fields are of the following sizes: 40 acres, 80 acres, 16 acres, and 34 acres. How many acres in all?

8. How many dollars are \$25 and \$35 and \$20?

9. How many feet are 12 ft., 8 ft., 6 ft., 12 ft., and 4 feet?

The answer to problem 1 is the same as the answer to the question: "What single distance is just as long as the distances 19 mi. and 10 mi. combined?" Answering problem 2 answers the question: "What single amount equals \$28 and \$20 combined?"

Combining numbers into a single number is *addition.*

The result of addition is called the *sum* or *amount.*

The numbers to be combined or added are the *addends.*

Thus in the problem

$$\begin{array}{ll} \left.\begin{array}{l} 19 \text{ mi.} \\ 10 \text{ mi.} \end{array}\right\} & \text{Addends} \\ \hline 29 \text{ mi.} & \text{Sum} \end{array}$$

19 mi. and 10 mi. are the addends and 29 mi. is the sum.

The *sign* of addition is +. It is read "plus." In some cases it may be read "and," though it is better to use the correct reading, "plus," at all times.

The sign = when written between two numbers means that they are equal. It should not be read "is" or "are," but always "equal" or "equals."

10. Read and answer these questions:

(1) $3+8+4=?$	(4) $12+7+8=?$	(7) $3+15+4+9=?$
(2) $7+6+8=?$	(5) $6+13+7=?$	(8) $6+16+8+9=?$
(3) $9+8+5=?$	(6) $5+18+9=?$	(9) $5+14+9+7=?$

11. In these questions, what numbers should stand in place of the question mark?

(1) 2 8's + 5 8's = ? 8's.	(5) 9 y's + 8 y's = ? y's.
(2) 6 9's + 7 9's = ? 9's.	(6) 15 z's + 10 z's = ? z's.
(3) 12 13's + 7 13's = ? 13's.	(7) 8 a's + 17 a's = ? a's.
(4)* 7 x's + 5 x's = ? x's.	(8) 16 b's + 20 b's = ? b's.

* Read 4 thus, "7 x's plus 5 x's equal how many x's?"

§21. **Exercises.** WRITTEN WORK

1. In five trips a street-car carried the following number of passengers: 30, 42, 45, 60, 65. Find the whole number of passengers.

When the sums can not be held in memory, it is convenient to arrange the addends in this form:

SOLUTION.—Writing the addends so that units, tens, hundreds, etc., shall stand in the same vertical column, we add the units first; thus, 5, 10, *12* units = *1* ten and *2* units. Write the 2 units in units column under the line and the 1 ten in tens column. Then, 6, 12, 16, 20, *23* tens = *2* hundreds and *3* tens. Write the 3 in tens column and the 2 in hundreds column. Then add the partial sums as shown.

CONVENIENT FORM

$$\begin{array}{r} 30 \\ 42 \\ 45 \\ 60 \\ 65 \\ \hline 12 \\ 23 \\ \hline 242 \end{array}$$

2. On these trips the conductor collected $1.50, $2.10, $2.25, $3.00, $3.25. The fares for the five trips amounted to what?

3. In one week my street-car fare was 35¢, 25¢, 40¢, 25¢, 30¢, 30¢. Find the whole amount.

4. In one evening a telegraph operator sent messages containing the following numbers of words: 2, 6, 3, 12, 6, 3, 3. What was the total number of words dispatched?

5. Fred bought a note book for 10¢, a lead pencil for 5¢, a foot rule for 10¢, a compass for 25¢, and a bottle of ink for 10¢. What was the amount of his bill?

6. A train moves the following distances in five successive hours: 40 mi., 45 mi., 50 mi., 50 mi., 48 mi. What distance is traveled in the given time?

Find the total sales in each of the following problems:

7. Silk sales: $2\frac{1}{2}$ yd., $3\frac{1}{2}$ yd., $\frac{3}{4}$ yd., $2\frac{1}{4}$ yd., 3 yards.

8. Linen sales: $2\frac{1}{2}$ yd., 5 yd., 6 yd., $2\frac{1}{2}$ yd., 5 yards.

9. Ribbon sales: $1\frac{1}{2}$ yd., $\frac{1}{2}$ yd., 3 yd., 2 yd., 12 yards.

10. Thread sales: 25 spools white cotton, 6 spools black cotton, 3 spools black linen, 4 spools A silk, 6 spools twist.

11. Handkerchief sales: 6, 12, 3, 6, 12, 6, 3, 2. How many dozen were sold?

12. Notion sales: 6 papers pins, 2 papers safety pins, 6 papers needles, 2 cards darning wool, 2 combs, 12 bunches tape, 4 bunches braid. How many articles were sold?

13. Coffee sales: $\frac{1}{2}$ lb., 2 lb., $2\frac{1}{2}$ lb., 3 lb., $\frac{3}{4}$ lb., $1\frac{1}{4}$ lb., 5 lb., 4 pounds.

14. Sugar sales: 18 lb., 12 lb., 12 lb., 20 lb., 5 lb., 6 pounds.

15. Flour sales: 50 lb., 150 lb., 25 lb., 15 lb., 25 pounds.

16. Potato sales: 2 bu., $1\frac{1}{2}$ bu., 4 bu., 6 bu., 2 bu., $1\frac{1}{2}$ bu., 3 bushels.

17. Apple sales: 2 bu., 1 bu., 3 pk., 2 pk., 1 pk., 2 bu., $1\frac{1}{2}$ bu., 4 bushels. (Answer in bushels.)

18. Egg sales: 7 doz., 3 doz., 6 doz., 4 doz., 8 doz., 5 doz., 2 dozen.

19. Butter sales: 5 lb., 3 lb., 3 lb., $2\frac{1}{2}$ lb., $1\frac{1}{2}$ lb., 3 lb., 5 pounds.

§22. Problems.

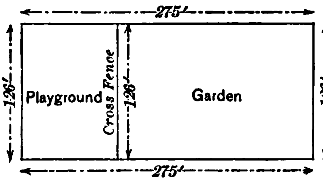


FIGURE 9

1. A fifth grade staked out a rectangular plot of ground to be used for a playground and school garden. The eighth grade pupils fenced the plot in and ran a cross fence separating the parts. The entire plot was 126 ft. wide by 275 ft. long (126′ x 275′). How many feet of fence were needed?

SOLUTION.—Arrange the addends in convenient form. Beginning at the bottom of units column, think the sums thus, 12, 17, 23, *28* equals *2* tens and *8* units. Write the 8 in units column. Add the 2 tens to the tens digits, thinking (not speaking) 4, 6, 13, 15, *22* tens, equals *2* hundreds and *2* tens. Write the 2 tens in tens place and add the hundreds to the hundreds in the third column, thus 3, 4, 6, 7, *9* hundreds. Write the 9 in hundreds place, and since we are adding feet the sum is 928 ft. Test the correctness of the sum by beginning at the top of each column and adding downward. If the sum is the same the work is probably correct. This is called *checking* the work.

CONVENIENT FORM	
275 ft.	Addends
126 ft.	
275 ft.	
126 ft.	
126 ft.	
928 ft.	Sum

2. Add 187, 892, and 478.

3. In a certain school there were enrolled one month:

96 children in the kindergarten
182 in the first grade
143 in the second grade
133 in the third grade
123 in the fourth grade
100 in the fifth grade
95 in the sixth grade
83 in the seventh grade
76 in the eighth grade

How many pupils were in the whole school?

4. During the same month, the numbers of cases of tardiness were as follows:

Kindergarten...	10	Third grade.......	8	Sixth grade	12
First grade......	12	Fourth grade.....	5	Seventh grade..	4
Second grade...	4	Fifth grade........	9	Eighth grade ...	2

What was the total number of cases of tardiness?

5. The numbers of cases of absence for the same month were as follows, beginning with the kindergarten: 87, 60, 46, 21, 15, 20, 15, 14, 9. Find the total number of cases.

Teachers may take reports of their own or of other schools and make similar problems.

6. Fill out for each of these cities the total number of rains during May and June, 1902, and the total rainfall in inches. Do not rewrite the numbers.

City	Number of Rains			Rainfall in Inches		
	May	June	Total	May	June	Total
Chicago.............	11	15		4.46	6.00	
Des Moines...........	13	16		4.57	6.76	
Detroit..............	14	18		3.46	6.16	
Kansas City.........	13	15		5.02	4.19	
Omaha	13	16		2.83	7.29	
St. Louis............	10	13		2.21	9.08	

The length in feet, the number of officers and men, the displacement in tons, and the indicated horse power of eight of the first-class battleships of the United States navy are as given in the table on the following page.

NAME	LENGTH	MEN	DISPLACEMENT	HORSE POWER
Alabama	368	585	11,525	11,366
Wisconsin	368	585	11,525	10,000
Kearsarge	368	520	11,525	11,954
Kentucky	368	520	11,525	12,318
Iowa	360	444	11,340	12,105
Indiana	348	465	10,288	9,738
Massachusetts	348	424	10,288	10,403
Oregon	348	424	10,288	11,111
Total				

7. If these 8 vessels stood in a straight line with their ends touching, how far would they reach?

8. How many officers and men are needed to man the 8 ships?

9. The displacement is the number of tons of water the vessel pushes aside as it floats. What is the combined displacement?

10. What is the combined horse power of the engines of the 8 ships?

11. At the close of 1900 the numbers of teachers, schools, and pupils in the 6 provinces of Cuba were as follows:

PROVINCE	TEACHERS	SCHOOLS	PUPILS
Havana	904	904	41,383
Puerta Principe	246	246	9,355
Santa Clara	879	879	44,872
Santiago	645	645	33,983
Pinar del Rio	274	274	13,282
Matanzas	619	619	29,398
Total			

Find the totals and tell what they mean.

12. The 6 European countries from which most immigrants came to the United States in 1901 were as follows:

Country	Immigrants in 1901			Immigrants in 1900		
	Male	Female	Total	Male	Female	Total
Italy	106,306	29,690		76,088	24,047	
Austria-Hungary	78,725	34,665		79,978	34,499	
Russia	54,070	31,187		60,091	31,066	
Ireland	12,894	17,667		16,672	19,058	
Sweden	12,875	10,456		10,262	8,388	
Total						

Fill out all the totals and tell what they mean.

13. The published daily circulation of a city newspaper from week to week is here given:

First Week	Second Week	Third Week	Fourth Week	Fifth Week
Day Copies	Day Copies	Day Copies	Day Copies	Day Copies
1...255,572	8...305,725	15...304,636	22...304,836	29...303,383
2...303,062	9...303,489	16...302,173	23...304,698	30...302,005
3...323,513	10...304,991	17...302,650	24...310,870	
4...312,724	11...304,746	18...304,255	25...299,780	
5...306,009	12...305,515	19...302,942	26...301,455	
6...297,918	13...299,095	20...297,684	27...298,446	Total
Total	Total	Total	Total	Total for mo.

Find the weekly totals and the total for the whole month.

14. In the year 1891 the United States imported 94,628,119 lb. of coffee, which was 54,262,757 lb. less than was imported during the two previous years. How many pounds were imported during the two previous years?

15. The United States imported 79,192,253 lb. of tea in 1889; 83,494,956 lb. in 1890; and 82,395,924 lb. in 1891. How many pounds of tea were imported in the three years?

16. In 1889 $27,024,551 worth of molasses was imported into the United States; in 1890 $31,497,243 worth; and in 1891 $2,659,172 worth. How many dollars' worth was imported during the three years?

17. Of the yearly trade within New York state, \$1,650,000,000 worth of the freight passes over the railroads; \$150,000,000 over the canals; and \$250,000,000 over Long Island Sound and the lakes. What is the total value of this trade?

THE TEN LARGEST CITIES OF THE UNITED STATES

	Population
New York	3,437,202
Chicago	1,698,575
Philadelphia	1,293,697
St. Louis	575,236
Boston	560,892
Baltimore	508,957
Cleveland	381,768
Buffalo	352,387
San Francisco	342,782
Cincinnati	325,902

THE TEN LARGEST FOREIGN CITIES

	Population
London	4,433,018
Paris	2,536,834
Canton	2,000,000
Berlin	1,677,304
Vienna	1,364,548
Tokyo	1,268,930
St. Petersburg	1,267,023
Pekin	1,000,000
Moscow	988,610
Constantinople	900,000

18. Find the total population of the ten largest cities of the United States.

Test the correctness of your addition by adding the columns from top downward.

Teachers may omit the following method of checking if thought too difficult.

A very useful method of checking long problems in addition is known as *casting out the nines.*

To cast out the nines from a number add its digits and whenever the sum equals or exceeds 9, drop 9 and continue adding the next digits to what remains, dropping 9 whenever the sum equals or exceeds 9. The last remainder is called the *excess.*

ILLUSTRATION.—Cast the 9's out of 647,255.

Beginning on the left, $6+4=10$; drop 9, giving the remainder 1. $1+7+2=10$, drop 9. $1+5+5=11$, drop 9. The last remainder is 2 and this is the excess of 647,255.

To check addition by casting out nines, cast the nines out of the addends and the sum. Then cast out the nines from the excesses of the addends. If this last excess of the excesses equals the excess of the sum, the work is probably correct.

ILLUSTRATION	EXCESS
8,465	5
3,282	6
4,497	6
2,957	5
7,642	1
26,843	$5=5$

$5=$ excess of the sum.
$5=$ excess of excesses of addends.

To obtain the excess of the excesses of the addends:

$5+6=11$, drop 9, giving 2. $2+6+5=13$, drop 9, giving 4. $4+1=5$, the excess of the excesses of the addends. Since this equals the excess of the sum, the work is checked.

19. Find the population of the ten largest cities of Europe and Asia. (See table, p. 28.)

20. Find the total population of these twenty cities.

21. Find the population of those cities in both lists having more than 900,000 inhabitants.

22. Make and solve similar problems based on the table.

§23. Distribution of Population in the United States in 1900.

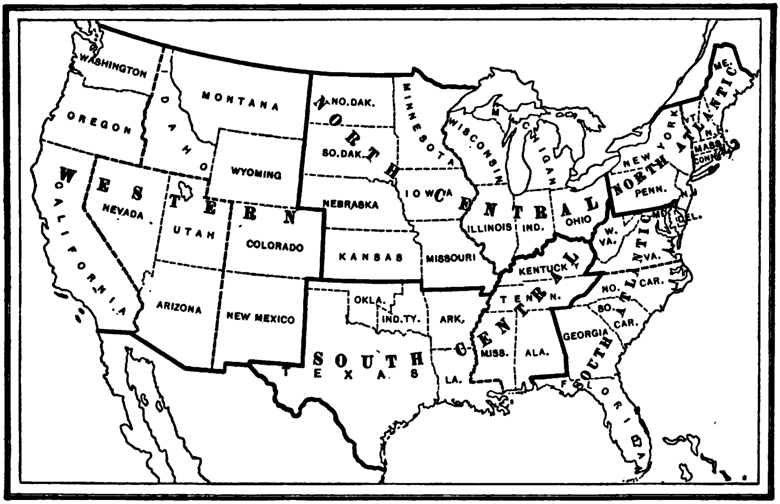


FIGURE 10

On the map the main geographical divisions are surrounded by heavy full lines: ▬. The divisions are separated into sections by heavy dotted lines: ----. Thus the New England states are the eastern section of the North Atlantic division; and New York, Pennsylvania, and New Jersey are the western.

Teachers may select such problems from this list as have a special geographical interest to their school.

1. From the table of numbers given on the following page, find the total population of the New England states for 1900.

2. Of the North Central division.

3. Of the Western division.

4. Of the United States with outlying territory.

5. Find the total area in square miles of the United States with outlying territory.

AREA AND POPULATION OF STATES AND TERRITORIES

STATE OR TERRITORY	AREA IN SQ. MILES	POPULATION 1900	PUPILS IN ELEM. & SEC. SCHOOLS
Me.	33,040	694,666	130,918
N. H.	9,305	411,588	65,193
Vt.	9,565	343,641	65,964
Mass.	8,315	2,805,346	474,891
R. I.	1,250	428,556	64,537
Conn	4,990	908,420	155,228
N. Y.	49,170	7,268,894	1,209,574
N. J.	7,815	1,883,669	315,055
Penn.	45,215	6,302,115	1,151,880
North Atlantic division			
Del	2,050	184,735	33,174
Md.	12,210	1,188,044	229,332
D. C.	70	278,718	46,519
Va.	42,450	1,854,184	358,825
W.Va.	24,780	958,800	232,343
N. C.	52,250	1,893,810	400,452
S. C.	30,570	1,340,316	281,891
Ga	59,475	2,216,331	482,673
Fla.	58,680	528,542	108,874
South Atlantic division			
Ohio	41,060	4,157,545	829,160
Ind.	36,350	2,516,462	564,807
Ill.	56,650	4,821,550	958,911
Mich.	58,915	2,420,982	498,665
Wis.	56,040	2,069,042	445,142
Minn.	83,365	1,751,394	399,207
Iowa	56,025	2,231,853	554,992
Mo.	69,415	3,106,665	719,817
N. D.	70,795	319,146	77,686
S. D.	77,650	401,570	96,822
Neb.	77,510	1,066,300	288,227
Kan.	82,080	1,470,495	389,583
North Central division			

STATE OR TERRITORY	AREA IN SQ. MILES	POPULATION 1900	PUPILS IN ELEM. & SEC. SCHOOLS
Ky.	40,400	2,147,174	501,893
Tenn.	42,050	2,020,616	485,354
Ala.	52,250	1,828,607	376,423
Miss.	46,810	1,551,270	360,177
La.	48,720	1,381,625	196,169
Tex.	265,780	3,048,710	578,418
Okl.	39,030	398,331	....
Ark.	53,850	1,311,564	314,662
Ind. T.	31,400	392,060	99,602
South Central division			
Mont.	146,080	243,329	39,430
Wy.	97,890	92,531	14,512
Col.	103,925	539,700	117,555
N. M.	122,580	195,310	36,735
Ariz.	113,020	122,931	16,504
Utah	84,970	276,749	73,042
Nev.	110,700	42,335	6,676
Idaho	84,800	161,772	36,669
Wash.	69,180	518,103	97,916
Ore.	96,030	413,536	89,405
Cal.	158,360	1,485,053	269,736
Western division			
U. S. without outlying territory			
Alaska	590,884	63,592	
Hawaii	6,449	154,001	
Phil. Is.	114,410	6,961,339	
Tutuila	77	6,100	
Guam	150	9,000	
Porto Rico	3,531	953,243	
Total U. S. with outlying territory			

6. Find the area in square miles of the North Central division.

7. Of the South Central division. Which has the greater territory?

8. What state in the Union contains the smallest number of square miles? the greatest? What state supports (contains) the greatest population? the smallest? How do the areas of these two states compare?

9. Find the number of pupils attending school in the New England states; in the North Central division.

10. Find the total number of children attending school in the United States.

11. Make such original problems as these: Find the total population in 1900 of the 13 original states; the total area of the 13 original states; of the states east of the Mississippi river, etc.

12. Check the correctness of your work in problems 2 and 7 by adding the columns first as a whole and then adding the footings of the separate sections and comparing the two sums.

13. Find the population and the area of the eastern North Atlantic states; of the western.

14. Find the number of pupils attending the elementary and secondary schools in both the sections mentioned in problem 13.

15. Check the work of problems 1 and 4 by casting out the nines.

§24. Measurements.

A square unit, or a unit square, is a square each of whose sides is 1 unit long.

The area of a surface is the number of square units in it.

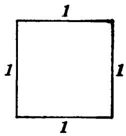


FIGURE 11

1. What is a square inch? a square foot? a square yard? a square mile? a square rod?

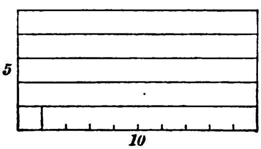


FIGURE 12

2. If a rectangle is made up of 5 rows of 10 square units each, what is its area? How many tens of square units are in the rectangle?

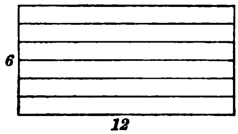


FIGURE 13

3. If a rectangle is 12 units long and 6 units wide, how many twelves of unit squares are there in the area?

4. If a rectangle is x units long and 9 units wide, how many x's of unit squares are in its area?

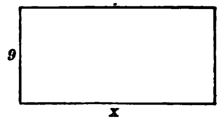


FIGURE 14

NOTE.—Seven x's is written $7x$ and is read "seven x."

5. How wide must a rectangle z inches long be to contain $6z$ sq. in.? $15z$ square inches?

6. How long must a rectangle 15 in. wide be to contain $15a$ square inches?

7. The area of one rectangle is $8z$ sq. in. and of another $7z$ sq. in.; how many square inches in their sum?

8. How many 9's are 8 9's and 7 9's? How many 12's are 8 12's and 7 12's? How many x's are $8x$ and $7x$? $8z + 7z = ?$

9. Add these numbers:

(1) $9a$	(2) $8a$	(3) $26x$	(4) $48z$	(5) $76y$	(6) $69b$
$8a$	$6a$	$47x$	$98z$	$49y$	$79b$
(7) $6m$	(8) $68n$	(9) $84c$	(10) $73d$	(11) $786x$	(12) $960x$
$8m$	$75n$	$76c$	$39d$	$319x$	$869x$
$9m$	$98n$	$37c$	$79d$	$876x$	$48x$

SUBTRACTION

§25. Definitions.

ORAL WORK

1. A man earns \$180 a month and spends \$140. What is the difference between his earnings and expenditures?

2. One farmer owns 320 A. of land, another 80 A. Find the difference in size of the two farms?

3. From a cheese weighing 43 lb., 23 lb. were sold; how many pounds remained?

4. From a bin containing 72 bu. of oats, I use 32 bu. How many bushels remain?

5. The sum of two numbers is 48, and one of the numbers is 28; what is the other?

6. What is required in the first two problems of this section? in the second two? in the fifth problem?

7. What is subtraction? What is the result in subtraction called?

Subtraction is the way of finding:

(1) The difference of two numbers expressed in the same unit.

(2) The remainder after taking a part of a number away from the whole number.

The number from which we subtract is the *minuend*. The number to be subtracted is the *subtrahend*. The result is the *difference*, or *remainder*.

Subtraction is also the way of finding either one of two addends when their sum and the other addend are known. The known sum is the *minuend*, the known addend is the *subtrahend*, and the unknown addend is the *difference*, or *remainder*.

The sign of subtraction $-$ is read "minus," and when placed between two numbers means that their difference is to be found. The minuend is always written before the sign.

8. Tell what number the letter stands for in these problems:

(1) $8-5=x$	(3) $16-7=y$	(5) $15-8=z$
(2) $17-9=x$	(4) $25-y=8$	(6) $16-z=8$

9. What digit should take the place of the question mark in these problems?

(1) 7 8's $-$ 5 8's = ? 8's	(4) $9\ x-6\ x=?\ x$
(2) 8 12's $-$ 3 12's = ? 12's	(5) $15\ x-7\ x=?\ x$
(3) $4\ x-2\ x=?\ x$	(6) $15\ x-?\ x=8\ x$

WRITTEN WORK

1. A traveler has a journey of 437 mi. to make and he has already traveled 199 mi. of it; how much farther must he travel?

SOLUTION.—437 means $400+30+7$, and 199 means $100+90+9$. We arrange the numbers conveniently, thus:

$$\begin{array}{lrl} \text{Minuend} & 437 \text{ mi.} & = 300 \text{ mi.} + 120 \text{ mi.} + 17 \text{ mi.} \\ \text{Subtrahend} & 199 \text{ mi.} & = 100 \text{ mi.} + 90 \text{ mi.} + 9 \text{ mi.} \\ \hline \text{Remainder} & 238 \text{ mi.} & = 200 \text{ mi.} + 30 \text{ mi.} + 8 \text{ mi.} \end{array}$$

Beginning on the right, 9 units can not be taken from 7 units. We take one of the 3 tens and add it to the 7 units, giving 17 units. Then 17 units — 9 units = *8* units. Write *8* in units column. Passing to tens

column, we can not take 9 tens from the 2 tens remaining, so we take one of the 4 hundreds and add it to 2 tens, making 12 tens. Then 12 tens — 9 tens = *3* tens. Write *3* in tens column. Finally, 3 hundreds — 1 hundred = *2* hundreds. Write the *2* in the hundreds column. The remainder is 238 miles.

2. How can you prove 238 to be the correct remainder or difference in problem 1?

3. The minuend is 1284, and the subtrahend is 876. What is the difference, or remainder? Prove.

4. When the water was dried out of 38 oz. of natural soil, the dry soil weighed 29 oz. Make a problem from these facts, solve it, and prove your solution correct.

5. A subtrahend is 99, and the difference is 338. What is the minuend? How is it found?

6. From 72 hr. subtract 1 da. and 4 hours.

NOTE.—A correct answer to this problem would be 72 hr.—1 da. and 4 hr. But the answer is to be expressed in a single unit, or name. To do this, first change 1 da. and 4 hr. to hours. 1 da. = 24 hr.; 24 hr. + 4 hr. = 28 hr. 72 hr. — 28 hr. = 44 hours.

7. From 36 ft. take 5 yd. What must first be done to obtain the result in a single unit? Make complete statements.

8. 20 qt. – 2 pk. = x qt. What is the first thing to be done? Find the value of x.

9. From $\frac{3}{4}$ yd. take $\frac{5}{8}$ yd. What is the unit of $\frac{3}{4}$ yd.? of $\frac{5}{8}$ yd.? What then must be done before subtracting? Why?

NOTE.—Only numbers expressed in the same unit can be subtracted if the result is to be expressed in a single unit.

10. Write the last problem, using the sign of subtraction.

§26. Exercises.

1. From a barrel of vinegar containing $31\frac{1}{2}$ gal., suppose the following quantities to be removed, and give successive remainders: 6 gal.; $2\frac{1}{2}$ gal.; 5 gal.; 3 gal.; $2\frac{1}{2}$ gal.; $2\frac{1}{2}$ gallons.

2. From a wagon box containing 42 bu. of potatoes, a farmer sold the following quantities: 2 bu., 4 bu., 3 bu., 2 bu., 5 bu., 6 bu., 9 bu., 4 bu. How many bushels remained?

3. On March 31 the gas meter read 30,000 cu. ft., and on April 30, 45,000 cu. ft. How many cubic feet of gas had been used during the month?

4. A horse and his rider weigh 1375 lb. The man weighs 160 lb. What is the weight of the horse?

5. Find and read the differences in these exercises:

(1)	(2)	(3)
$9274.75	$4246.28	1,024,637 yd.
1846.19	1571.16	725,448 "

6. Find the differences:

(1)	(2)	(3)
6,420,014 rd.	$3472.06	$1228\frac{3}{4}$
1,382,741 "	2189.24	$936\frac{1}{4}$

7. During one week a merchant's transactions (or business dealings) at his bank were as follows: deposits (moneys put in), $217.40, $343.27, $290.00, $365.18, $380.24, $415.17; withdrawals (moneys taken out), $200, $320.25, $195.75, $286, $240.15, $395.25. Which item was the greater at the close of the week, and how much?

8. During one week of November, 1901, 63,188 cattle were received at the Chicago stock yards; during the corresponding week of 1899, 32,722 were received. What was the increase?

9. Great Salt Lake is 4200 ft. above sea level, and Lake Superior 602 ft. x is the difference in elevation. Find value of x.

10. Cuba contains 45,884 sq. mi., and the Philippines, 114,410 sq. mi. Find the difference in areas.

For the numbers for problems 11-17, see the tables, p. 36.

11. Find the total receipts at Chicago for the week Nov. 25-30, 1901, of cattle; of calves; of hogs; of sheep.

12. Find the total shipments.

13. Find the difference between receipts and shipments.

14. Compare the receipts with those of the previous week; with those of the corresponding week of 1900; of 1899.

15. Compare shipments in the same manner.

16. Find total receipts for one week in the four markets, Chicago, Kansas City, Omaha, and St. Louis, of cattle; of hogs; of sheep.

17. Compare Chicago receipts with those of the other cities.

A Week's Receipts and Shipments of Live Stock. Chicago, Nov. 30, 1901

RECEIPTS	Cattle	Calves	Hogs	Sheep
Monday, Nov. 25	16,078	537	39,630	26,170
Tuesday, " 26	6,941	735	41,654	18,297
Wednesday, " 27	17,583	481	51,275	12,393
Thursday, " 28 (Holiday)				
Friday, " 29	5,857	162	44,090	12,462
Saturday, " 30	800	40	25,000	2,000
Total				
Previous week	63,188	3,814	273,426	104,528
Corresponding week 1900	58,965	1,822	191,104	55,046
Corresponding week 1899	32,722	1,335	159,724	61,718

SHIPMENTS	Cattle	Calves	Hogs	Sheep
Monday, Nov. 25	2,578	2	7,543	2,852
Tuesday, " 26	1,904	1	1,495	4,518
Wednesday, " 27	5,647	16	5,712	4,501
Thursday, " 28 (Holiday)				
Friday, " 29	1,875	205	4,308	2,127
Saturday, " 30	400	30	5,000	1,000
Total				
Previous week	18,699	324	29,486	24,429
Corresponding week 1900	18,438	352	23,575	14,832
Corresponding week 1899	9,556	210	16,637	3,182

Live Stock Receipts for One Week at Four Markets

	Cattle	Hogs	Sheep
Chicago	47,300	201,600	71,300
Kansas City	29,300	89,000	14,900
Omaha	16,800	65,200	12,800
St. Louis	10,700	38,100	5,800
Total			
Previous week	144,600	481,700	173,600
Corresponding week 1900	111,500	339,800	85,000
Corresponding week 1899	91,700	206.100	81,300
Corresponding week 1898	128,400	365,500	95,600
Corresponding week 1897	143,000	374,100	105,600

18. Compare the total receipts of the week with those of the previous week; of the corresponding week of 1900; 1899; 1898; 1897.

§27. Geography.

1. Lake Titicaca is 12,645 ft. above the level of the sea, and Lake Superior 602 ft. Find the difference in elevation.

2. Mt. Everest is 29,002 ft. above sea level, and Mt. Blanc 15,744 ft. What is the difference in altitude?

3. The Mississippi basin has an area of 1,250,000 sq. mi.; the Amazon, of 2,500,000 sq. mi. How much greater is the basin of the Amazon than that of the Mississippi?

4. The area of the Philippines is 114,410 sq. mi.; that of California, 158,360 sq. mi. The state is how much larger than the islands?

5. Compare the same islands with Texas; with Illinois; with Rhode Island. (See table, p. 30.)

6. Of a population of 6,302,115 in Pennsylvania, 1,151,880 are pupils in elementary and secondary schools. How many persons are not attending those schools?

7. In a certain year the Hawaiian Islands produced 221,694 T. of sugar, while Cuba produced 880,372 T. How much more did Cuba produce than the Hawaiian Islands?

8. In 1890, 25,403 manufacturing establishments in New York City paid for wages \$230,102,167; for materials, \$366,422,722; for miscellaneous expenses, \$59,991,710. The value of the products was \$777,222,721. What was the total profit for all the establishments?

9. The battle of Lexington was fought in 1775, and the battle of Santiago in 1898. How many years elapsed between the two battles?

For the numbers for problems 10-19, see table, p. 30.

10. What is the difference of the areas of the largest and the smallest states?

11. How many more persons are in the state with the largest population than in the one with the smallest?

12. How many more children attend school in one of these two states than in the other?

13. Make and solve other problems, comparing the areas, total populations, and school populations of the different states.

14. Which is the larger, and by how much, the North Atlantic division or the South Atlantic division? the North Central or the South Central? the South Central or the Western?

15. Compare the total populations of these groups.

16. How many persons are not attending elementary or secondary schools in Maine? in Mississippi? How do these two results compare?

17. Find how many persons are not attending these schools in your own state; in bordering states.

18. How many more persons are not attending these schools in New York than in Illinois? How does this result compare with the difference in the total populations of these states?

19. Make and solve other problems of interest to your school.

20. Find, by comparing the populations for 1890 (p. 39) with those for 1900 (p. 30), the increase of population for the ten years in each state of the North Atlantic division; in the whole division.

21. Find the increase of population in your own state from 1890 to 1900. What is the average yearly increase?

22. How many years since Illinois was admitted as a state? since your native state was admitted into the Union?

23. Make similar problems from the table opposite and the table on p. 30, solve your problems, and check your work.

24. Each principal division of states is separated into two or more sections by heavy dotted lines on the map, p. 29. Find how much greater, or less, the growth of population since 1890 has been in your section than in the other section, or sections of the same division.

§28. Commerce.

1. Find the total export trade of the United States with the twelve countries given in the table on p. 40, in 1891 and in 1901.

2. Find the increase, or decrease, for each country, and make a list of these results. Whenever the difference is an increase write + before it. When it is a decrease write – before it. Make the subtractions without rewriting the numbers, recording results.

Dates of Admission and Populations for 1890 of States and Territories Geographically Grouped

	Date of Admission	Population 1890
Me.	Mar. 15, 1820	661,086
N. H.	June 21, 1788	376,530
Vt.	Mar. 4, 1791	332,422
Mass.	Feb. 7, 1788	2,238,043
R. I.	May 20, 1790	345,506
Conn.	Jan. 9, 1788	746,258
N. Y.	July 26, 1788	5,997,853
N. J.	Dec. 18, 1787	1,444,933
Penn.	Dec. 12, 1787	5,258,014
North Atlantic division		
Del.	Dec. 7, 1787	168,493
Md.	April 28, 1788	1,042,390
D. C.	Mar. 30, 1791	230,392
Va.	June 25, 1788	1,655,980
W. Va.	June 20, 1863	762,794
N. C.	Nov. 21, 1789	1,617,947
S. C.	Mar. 23, 1788	1,151,149
Ga.	Jan. 2, 1788	1,837,353
Fla.	Mar. 3, 1845	391,422
South Atlantic division		
Ohio	Feb. 19, 1803	3,672,316
Ind.	Dec. 11, 1816	2,192,404
Ill.	Dec. 3, 1818	3,826,351
Mich.	Jan. 16, 1837	2,093,880
Wis.	May 29, 1848	1,686,880
Minn.	May 11, 1858	1,301,826
Iowa	Dec. 28, 1846	1,911,896
Mo.	Aug. 10, 1821	2,679,184
N. D.	Nov. 2, 1889	182,719
S. D.	Nov. 2, 1889	328,808
Neb.	Mar. 1, 1867	1,058,910
Kan.	Jan. 29, 1861	1,427,096
North Central division		

	Date of Admission	Population 1890
Ky.	June 1, 1792	1,848,635
Tenn.	June 1, 1796	1,767,518
Ala.	Dec. 14, 1819	1,513,017
Miss.	Dec. 10, 1817	1,289,600
La.	April 30, 1812	1,118,587
Tex.	Dec. 29, 1845	2,235,523
Okl.	May 2, 1890	61,834
Ark.	June 15, 1836	1,128,179
Ind. T.	June 30, 1834	180,182
South Central division		
Mont.	Nov. 8, 1889	132,159
Wy.	July 10, 1890	60,705
Col.	Aug. 1, 1876	412,198
N. M.	Sept. 9, 1850	153,593
Ariz.	Feb. 24, 1863	59,620
Utah	Jan. 4, 1896	207,905
Nev.	Oct. 31, 1864	45,761
Idaho	July 3, 1890	84,385
Wash.	Nov. 11, 1889	349,390
Ore.	Feb. 14, 1859	313,767
Cal.	Sept. 9, 1850	1,208,130
Western division		
U. S. without outlying territory		
Alaska	July 27, 1868	32,052
Hawaii	April 30, 1900	89,990
Phil. Is.		
Tutuila		
Guam		
Porto Rico		
Total U. S. with outlying territory		

United States Export Trade

Country	1901	1891	Ten-Year Difference
United Kingdom	$598,766,799	$482,295,796	
Germany	184,678,723	90,326,332	
Canada	107,496,522	41,686,882	
Netherlands	85,643,804	31,261,766	
Mexico	36,771,568	15,371,370	
Italy	34,046,201	14,447,004	
British Australasia	30,569,814	13,564,931	
British Africa	24,994,766	3,511,668	
Japan	21,162,477	3,839,384	
Brazil	11,136,101	15,064,346	
Argentina	11,117,521	1,909,788	
Russia	6,504,867	5,400,357	
Total			

Values of Manufactured and Industrial Products

Manufactured Articles	1902	1897	Difference
Agricultural implements	$2,075,609	$ 243,466	
Books, maps, etc.	988,195	470,358	
Carriages and cars	913,513	80,065	
Copper ingots	198,438	31,583	
Cotton cloths	385,086	1,499,769	
Cotton manufactures, other	1,634,642	983,661	
Cycles, and parts of	98,476	339,563	
Builders' hardware	735,165	377,549	
Sewing machines	182,710	69,756	
Other machinery	894,330	1,222,708	
Total			
Other Articles	**1902**	**1897**	**Difference**
Corn	$1,468,390	$1,770,531	
Wheat	3,769,577	2,548,778	
Wheat flour	638,361	2,415,519	
Coal	5,473,177	6,987,856	
Cotton	4,509,205	2,626,679	
Fruits and nuts	1,345,260	566,584	
Furs and fur skins	667,164	195,534	
Cotton-seed oil	261,688	47,069	
Beef, salted or pickled	240,978	208,195	
Bacon	557,827	365,419	
Hams	218,995	188,116	
Total			

3. The columns on the opposite page show the values of the different kinds of goods exported by the United States to Canada during the nine months ending March, 1902, and March, 1897, respectively. Make a list of differences, marking the difference + whenever it denotes an increase, and – when it denotes a decrease.

§29. Numbers for Individual Work.

SCHOOL STATISTICS FOR THE THIRTY LARGEST CITIES OF UNITED STATES

CITY	POPULATION. CENSUS 1900	POPULATION. CENSUS 1890	SCHOOL ENROLLMENT. 1900	NUMBER TEACHERS. 1900	SCHOOL EXPENDITURES. 1900
New York	3,437,202	2,492,591	559,218	12,212	$21,040,810
Chicago	1,698,575	1,099,850	262,738	5,951	7,929,496
Philadelphia	1,293,697	1,046,964	151,455	3,591	4,677,860
St. Louis	575,236	451,770	82,712	1,751	1,526,140
Boston	560,892	448,477	91,796	2,018	3,664,298
Baltimore	508,957	434,439	65,600	1,600	1,279,936
Cleveland	381,768	261,353	59,635	1,303	1,933,965
Buffalo	352,387	255,664	56,000	1,300	1,408,000
San Francisco	342,782	298,997	48,517	1,017	1,152,631
Cincinnati	325,902	296,908	44,285	993	1,064,047
Pittsburg	321,616	238,617	50,000	1,000	1,757,381
New Orleans	287,104	242,039	31,547	782	455,073
Detroit	285,704	205,876	40,303	966	1,251,825
Milwaukee	285,315	204,468	37,000	900	733,510
Newark	246,070	181,830	41,870	851	1,218,660
Washington	218,196	188,932	40,069	1,043	
Jersey City	206,433	163,003	32,174	586	634,153
Louisville	204,731	161,129	27,626	650	555,811
Minneapolis	202,718	164,738	38,591	892	841,000
Providence	175,597	132,146	23,485	682	682,000
Indianapolis	169,164	105,436	27,334	650	729,106
Kansas City, Mo.	163,752	132,716	28,280	700	524,065
St. Paul	163,065	133,156	26,000	610	672,350
Rochester	162,608	133,896	24,896	692	682,018
Denver	133,859	106,713	27,181	530	750,180
Toledo	131,822	81,434	21,467	455	471,314
Allegheny	129,896	105,287	20,104	377	835,634
Columbus	125,560	88,150	18,855	502	771,132
Worcester	118,421	84,655	19,600	574	529,937
Syracuse	108,374	88,143	21,090	485	409,073
Total					

Problems like the following may be made from the table, and assigned to different pupils.

1. Find the totals of all columns of the table for the cities whose inhabitants numbered over 500,000 at the 1900 census.

2. Find the totals for the cities whose populations in 1900 were between 250,000 and 500,000; for cities with populations between 160,000 and 250,000.

NOTE.—These intervals may be shortened or lengthened at will. Individual pupils may do different parts of the work, thus obtaining a large number of individual problems. In each case the pupil should tell what his total means.

3. Find the increase in population from 1890-1900 of Cleveland, Ohio; of other cities.

4. How many more teachers in 1900 were there in Minneapolis than in Louisville? in New York City than in Chicago? in Boston than in St. Louis? in Chicago than in Boston?

5. How many more pupils in 1900 were enrolled in the schools of Boston than in those of St. Louis? than in those of Baltimore? of Cleveland?

6. How much more money was expended in 1900 on schools in Boston than in St. Louis? than in Cleveland? How much more in Cleveland than in Baltimore? than in Buffalo? How much more in Pittsburg than in Cincinnati?

7. How do the combined school expenditures for New York City, Chicago, Philadelphia, and Boston compare with the combined school expenditures of all the rest of the cities in the table? How do those of New York City and Chicago compare with the total of all the rest?

8. Find the totals for the five columns of the entire table. What is the increase in the total population of these cities from 1890 to 1900?

§30. Subtraction of Literal Numbers.

1. How many five-cent pieces are 7 five-cent pieces – 3 five-cent pieces?

2. How many dimes are 12 dimes – 5 dimes?

3. How many 9's are 16 9's – 9 9's?

4. How many c's are $13c - 7c$?

5. Write the differences:

$18x$	$26y$	$43z$	$68z$	$683a$	$1021z$
$9x$	$8y$	$37z$	$39z$	$97a$	$879z$

6. A lot contains $160x$ sq. ft., and the house covers $40x$ sq. ft.; how many square feet of the lot are not covered by the house?

7. A boy earned $15x$ cts. on Friday and spent $10x$ cts. on Saturday; how many cents did he save?

8. Tell what number x stands for in these problems:

(1) $12 - x = 7$.	(6) $7x - 5x = 10$.	(11) $50x = 100$.
(2) $65 - x = 20$.	(7) $16x - 13x = 15$.	(12) $\frac{1}{2}x = 50$.
(3) $x - 35 = 60$.	(8) $9x - 5x = 20$.	(13) $\frac{3}{4}x = 27$.
(4) $x - 19 = 7$.	(9) $7x = 63$.	(14) $\frac{2}{3}x = 7$.
(5) $3x - 2x = 8$.	(10) $9x = 108$.	(15) $\frac{7}{8}x = 63$.

Literal numbers are numbers denoted by letters.

MULTIPLICATION

§31. Definitions.

ORAL WORK

1. At 75¢ a day, how many dollars does a boy earn in 6 days?

This problem may be solved in two ways. We may say he earned the sum of 75¢ + 75¢ + 75¢ + 75¢ + 75¢ + 75¢, which is $4.50.
Or, we may say he earned 6 times 75¢ (6 × 75¢), or $4.50.
In either case his earnings are $4.50.

2. Which is the shorter way? How do the addends compare in the first solution?

3. Find in two ways how far a vessel will sail in 4 da., making 22 mi. a day.

4. A factory employee, working by the hour, makes the following daily record for a week: 8 hr.; 8 hr.; 8½ hr.; 9 hr.; 9 hr.; 9½ hr. How many hours does he work during the week?

5. How can you solve this problem in more than one way? Give reason for your answer.

6. When the addends are unequal, as in problem 4, what is the only way they can be combined?

7. When the addends are equal, as in the first two problems, in how many ways can they be combined? Which is the shorter

way? By what name do we know this shorter way? What then is multiplication?

Multiplication of whole numbers is a short way of finding the sum of equal addends when the number of addends and one of them are given.

The given addend is the *multiplicand.*

The number of equal addends is the *multiplier.*

The result, or sum, is the *product.*

The multiplicand and multiplier are *factors* of the product.

The sign of multiplication × is read "multiplied by" when it is written after the multiplicand and "times" when it is written before the multiplicand.

Thus 22 mi. × 6 = 132 mi. is read "22 mi. multiplied by 6 equals 132 mi.," and 6 × 22 mi. = 132 mi. is read "6 times 22 mi. equals 132 miles."

An expression like 6 × 22 mi. = 132 mi. is called an *equation.*

8. At $3.50 a pair, how much will 12 pairs of shoes cost?

$3.50 = cost of 1 pair;
$3.50 × 12 = cost of 12 pairs.

$3.50 is the *multiplicand*, 12 is the *multiplier*, and $42.00 is the *product.*

When a problem is expressed in an equation, the equation is called the *statement* of the problem.

§32. WRITTEN WORK

Make statements and solve:

1. There are 128 cu. ft. in 1 cd. of wood. How many cubic feet in 15 cords?

STATEMENT. 15 × 128 cu. ft. = ?
Or, 15 × 128 cu. ft. = x cu. ft.

Find the number which should stand in place of x.

The number which should stand in place of x in an equation is called the *value of x.*

2. There are 5280 ft. in a mile. How many feet in 24 miles?

3. 12 lb. of ham at 17¢ per lb. = how many dollars?

4. A cold wave from the northwest traveling at the rate of 32 mi. an hour moves how many miles in 24 hours?

5. If galvanized telegraph wire weighs 525 lb. per mi., how many pounds of wire will it take to stretch 6 wires from Chicago to Milwaukee, a distance of 82 miles?

6. How much will this wire cost at 8¢ per pound?

7. Find the cost of 275 fence posts at 65¢.

\$.65 = cost of one post;
275 = number of posts;
\$.65 × 275 = \$178.75 (cost of 275 posts) is the equation of the problem.

$$\begin{array}{r} \$.65 \\ \underline{275} \\ 325 \\ 455 \\ \underline{130} \\ \$178.75 \end{array}$$

In this problem, which number is the multiplicand? Which the multiplier?

This problem may be solved also thus: If each post cost 1¢, 275 posts would cost \$2.75; but as each post costs 65¢, the cost of all will be 65 × \$2.75, and the work may be arranged as shown.

Or, we may say 275 times \$.65 is the same as 65 times \$2.75, and multiply as above.

$$\begin{array}{r} \$2.75 \\ \underline{65} \\ 1375 \\ \underline{1650} \\ \$178.75 \end{array}$$

Since the numerical product of factors is the same, whichever is used as the multiplier, the smaller number is generally taken for the multiplier *for convenience.*

8. In 64 pk. there are how many quarts?

9. How many pounds in 494 bu. of corn, if in 1 bu. there are 56 lb.? how many pounds in 25 bu.? how many in x bushels?

10. There are 231 cu. in. in 1 gal. How many cubic inches in 587 gal.? in y gal.? in a gallons?

§33. Tables.

These products must be learned thoroughly:

3 × 3 = 9 = 3 × 3	3 × 4 = 12 = 4 × 3
4 × 3 = 12 = 3 × 4	4 × 4 = 16 = 4 × 4
5 × 3 = 15 = 3 × 5	5 × 4 = 20 = 4 × 5
6 × 3 = 18 = 3 × 6	6 × 4 = 24 = 4 × 6
7 × 3 = 21 = 3 × 7	7 × 4 = 28 = 4 × 7
8 × 3 = 24 = 3 × 8	8 × 4 = 32 = 4 × 8
9 × 3 = 27 = 3 × 9	9 × 4 = 36 = 4 × 9
10 × 3 = 30 = 3 × 10	10 × 4 = 40 = 4 × 10
11 × 3 = 33 = 3 × 11	11 × 4 = 44 = 4 × 11
12 × 3 = 36 = 3 × 12	12 × 4 = 48 = 4 × 12

Learn each of these in the same way:

$3 \times 6 = 18$	$3 \times 7 = 21$	$3 \times 8 = 24$	$3 \times 9 = 27$
$4 \times 6 = 24$	$4 \times 7 = 28$	$4 \times 8 = 32$	$4 \times 9 = 36$
$5 \times 6 = 30$	$5 \times 7 = 35$	$5 \times 8 = 40$	$5 \times 9 = 45$
$6 \times 6 = 36$	$6 \times 7 = 42$	$6 \times 8 = 48$	$6 \times 9 = 54$
$7 \times 6 = 42$	$7 \times 7 = 49$	$7 \times 8 = 56$	$7 \times 9 = 63$
$8 \times 6 = 48$	$8 \times 7 = 56$	$8 \times 8 = 64$	$8 \times 9 = 72$
$9 \times 6 = 54$	$9 \times 7 = 63$	$9 \times 8 = 72$	$9 \times 9 = 81$
$10 \times 6 = 60$	$10 \times 7 = 70$	$10 \times 8 = 80$	$10 \times 9 = 90$
$11 \times 6 = 66$	$11 \times 7 = 77$	$11 \times 8 = 88$	$11 \times 9 = 99$
$12 \times 6 = 72$	$12 \times 7 = 84$	$12 \times 8 = 96$	$12 \times 9 = 108$

§34. ORAL WORK

At current prices, find the cost of the following articles.

1. $\frac{3}{4}$ lb. Oolong tea.
   3 doz. eggs.
   5 lb. ham.
   3 lb. leaf lard.
   2 lb. Java coffee.
   3 lb. best quality of butter.

2. 2 erasers.
   3 tablets linen paper.
   3 packages white envelopes.
   4 doz. pens.
   2 lead pencils.
   2 bottles writing fluid.

3. 2 doz. oranges.
   2 doz. lemons.
   $1\frac{1}{2}$ doz. bananas.
   1 pk. apples.

4. 8 lb. rib roast.
   $2\frac{1}{2}$ lb. porterhouse steak.
   3 lb. lamb chops.
   2 lb. sausage.

5. 3 cd. hard wood.
   2 T. hard coal.
   $1\frac{1}{2}$ cd. pine slabs.
   4 T. soft coal.

6. 2 T. hay.
   3 bu. oats.
   2 bu. corn.
   4 bales straw.

7. 5 lb. tomatoes.
   8 bunches radishes.
   3 pk. turnips.
   1 pk. beets.
   3 qt. Lima beans.

8. 3 pr. shoes.
   $\frac{1}{2}$ doz. tennis balls.
   3 baseball bats.
   2 catcher's gloves.
   1 sweater.

§35. **Farm Account Keeping.**—The forms below show how a certain farmer keeps a systematic account of receipts and expenditures with each of his fields. The fields referred to are those shown in the drawing of Fig. 4, p. 7.

WRITTEN WORK

The items for which money was paid out or received were put down with the dates of payment or of receipt in a day-book thus:

Day-Book for Forty-Acre Cornfield

1. Paid for 4 da. work removing stalks @ $1.15, Mar. 18.
2. Paid for 9 da. plowing @ $1.15, Apr. 30.
3. Paid for 2 da. harrowing @ $1.00, May 1.
4. Paid for 3½ da. planting corn @ $1.20, May 18.
5. Paid for 6 bu. seed corn @ 90¢, May 18.
6. Paid for 2 da. replanting corn @ $1.15, May 30.
7. Paid for 5 da. harrowing corn @ $1.15, May 30.
8. Paid for 7 da. cultivating @ $1.15, June 15.
9. Paid for 6 da. cultivating @ $1.15, June 30.
10. Paid for 6 da. cultivating @ $1.15. July 15.
11. Paid for cutting 240 shocks corn @ 8⅓¢, Sept. 15.
12. Paid for husking 1392 bu. corn @ 3¢, Nov. 20.
13. Paid for shelling 1336 bu. corn @ ¾¢, Jan. 10.
14. Sold 1336 bu. corn @ 38¢, Jan. 10.
15. Received pay for 3 mo. pasture @ $1.50, Feb. 28.

1. These items are here arranged in the form of a receipt and expenditure account. Fill out the vacant columns, and find the totals and the net profit for this forty-acre cornfield:

In Account with Forty-Acre Cornfield

EXPENDITURES				RECEIPTS			
Mar. 18	Removing stalks, 4 da., @ $1.15			Jan. 10	1336 bu. corn @ 38¢		
Apr. 30	Plowing, 9 da., @ $1.15			Feb. 28	3 mo. pasture @ $1.50		
May 1	Harrowing, 2 da., @ $1.00				TOTAL		
" 18	Planting corn, 3½ da., @ $1.20				Expenditures to be deducted		
" 18	Seed corn, 6 bu., @ 90¢				Net profit from 40 acres		
" 30	Replanting, 2 da., @ $1.15				" " per acre		
" 30	Harrowing corn, 5 da., @ $1.15						
June 15	Cultivating, 7 da., @ $1.15				Account closed March 1, 1901		
" 30	" 6 da., @ $1.15						
July 15	" 6 da., @ $1.15						
Sept. 15	Cutting 240 shocks corn @ 8⅓¢						
Nov. 20	Husking 1392 bu. corn @ 3¢						
Jan. 10	Shelling 1336 bu. corn @ ¾¢						
	TOTAL						

2. From the following list of day-book items for the twenty-acre wheatfield the account below is arranged. Fill out the vacant columns and find the totals, the net profit on the whole field, and the net profit per acre.

Day-Book for Twenty-Acre Wheatfield

1. Paid for 4 da. plowing @ $2.50, Oct. 20.
2. Paid for 3 da. harrowing and rolling @ $2.25, Oct. 25.
3. Paid for 25 bu. seed wheat @ $1.25, Oct. 25.
4. Paid for 2½ da. drilling wheat @ $2.50, Oct. 30.
5. Paid for cutting 20 acres wheat @ 75¢, July 15.
6. Paid for 2 da. shocking wheat @ $1.75, July 15.
7. Paid for threshing 480 bu. wheat @ 6¢, Aug. 30.
8. Paid for 3½ da. help in threshing @ $1.50, Aug. 30.
9. Sold 425 bu. wheat @ 68¢, Dec. 22.
10. Received pay for 4 mo. pasture @ $1.50, Mar. 2.

In Account with Twenty-Acre Wheatfield

	Expenditures				Receipts		
Oct. 20	Plowing, 4 da., @ $2.50			Dec. 22	Wheat, 425 bu., @ 68¢		
" 25	Harrowing and rolling, 3 da., @ $2.25			Mar. 2	Pasture, 4 mo., @ $1.50		
" 25	Seed wheat, 25 bu., @ $1.25				Expenditures to be deducted		
" 30	Drilling, 2½ da., @ $2.50				Net profit from the field		
July 15	Cutting 20 acres wheat @ 75¢				" " per acre		
" 15	Shocking wheat, 2 da., @ $1.75						
Aug. 30	Threshing 480 bu. wheat @ 6¢						
" 30	Help threshing, 3½ da., @ $1.50						
	TOTAL						

3. Draw up these facts into an account, and find the totals and the net profit, and treat in the same way as above:

Day-Book for Thirty-Acre Pasture

1. Bought 120 bu. corn @ 37¢, Apr. 23.
2. Sold 5 yearling steers @ $18.00, Apr. 25.
3. Sold 3 yearling heifers @ $15.00, July 6.
4. Sold 2 milk-cows @ $46, July 8.
5. Bought 4 two-yr. old heifers @ $25, July 10.
6. Sold 8 hogs weighing 2064 lb., @ $5 per 100 lb., July 20.
7. 8 head of hogs, worth $12 each, died July 25.
8. Bought 20 pigs @ $2.50, Aug. 10.
9. Sold 3 two-yr. old colts @ $72, Sept. 30.
10. Sold 2 draught horses @ $195, Sept. 30.
11. Sold 9 four-yr. old steers @ $68, Oct. 25.
12. Bought 10 yearling calves @ $12, Nov. 1.
13. Sold 3 Jersey milk-cows @ $45, Nov. 15.
14. Paid 6 mo. wages for one man @ $30, Dec. 28.
15. Sold 8 head of hogs @ $15, Jan. 10.
16. Bought 12 head of hogs @ $9.50, Feb. 14.

4. Fill out this account of the fifty-acre oatfield.

EXPENDITURES				RECEIPTS			
Mar. 20	Removing old stalks 5 da. @ $1.25			Feb. 15	1800 bu. oats @ 22½¢		
" 20	100 bu. seed oats @ 35¢			" 20	85 loads straw @ $2.75		
" 20	6½ da. work sowing oats @ $1.25				Expenditures to be deducted		
Aug. 18	Harvesting 50 acres oats @ 75¢				Net profit from 50 acres		
" 18	4½ da. labor in harvesting @ $1.75				" " per acre		
Sept. 15	Threshing 2148 bu. @ 3¢						
" 15	5 da. help threshing @ $1.50						
	TOTAL						

5. Treat the meadow account similarly.

In Account with Ten-Acre Meadow

EXPENDITURES				RECEIPTS			
Nov. 20	10 bu. timothy seed @ $2.00			Dec. 15	12 tons hay @ $9.50		
Aug. 30	Cutting 10 acres hay @ 35¢				Expenses to be deducted		
" 30	Raking and shocking hay 5 da. @ $1.50				Net profit from 10 acres		
" 30	Stacking 15 tons hay @ 55¢				" " per acre		

6. Draw up the following items into an account like the one above; and find totals and profit or loss to the farmer.

Day-Book for House and Barn Lot (10 acres).

1. Apr. 15. Bought 6 bu. seed potatoes @ $2
2. " 20. Bought 30 pkg. garden seeds @ 10¢
3. " 20. Bought 4 pkg. garden seeds @ 25¢
4. May 15. Paid 5 da. wages @ $1.25
5. " 25. Sold 100 bunches onions @ 2½¢
6. " 30. Sold 148 bunches celery @ 3¢
7. " 30. Sold 200 bunches lettuce @ 2½¢
8. " 30. Sold 240 bunches asparagus @ 3¢
9. June 30. Sold 14 bu. peas @ 4¢ per qt.
10. " 30. Sold 28 bu. green beans @ 4¢ per qt.
11. " 30. Sold 300 bunches asparagus @ 5¢
12. " 30. Paid 18 da. wages @ $1.25
13. July 31. Sold 10 bu. new potatoes @ $1
14. July 31. Paid 15 da. wages @ $1.50
15. " 31. Sold 50 heads cabbage @ 8¢
16. " 31. Sold 20 doz. ears sweet corn @ 8¢ per dozen.
17. Aug. 31. Sold 140 lb. tomatoes @ 8¢
18. " 31. Sold 10 bu. early apples @ 75¢
19. " 31. Sold 25 heads cabbage @ 7¢
20. " 31. Sold 15 bu. peaches @ $1.
21. " 31. 10 doz. ears corn @ 7¢
22. " 31. Paid 20 da. wages @ $1.50
23. Sept. 30. Sold 55 bu. potatoes @ 65
24. " 30. Sold 20 bu. Lima beans @ $1.25
25. " 30. Paid 20 da. wages @ $1.25
26. Dec. 20. Sold 30 bu. apples @ $1
27. " 30. Sold 10 bbl. (40 gal. each) cider vinegar @ 30¢ per gal.

7. Make a statement from these items of general expense, not included in any of above accounts, and find the total profit or loss:

1.	Apr.	20.	Bought 3 sets of harness @ \$28
2.	"	25.	Paid tax on 160 acres @ 25¢
3.	"	30.	Bought 1 wagon @ \$60
4.	"	30.	Bought 2 plows @ \$35
5.	May	10.	Bought 3 cultivators @ \$25
6.	Aug.	20.	Paid 84 rods tile ditching @ 50¢
7.	Aug.	30.	Built corn crib, cost \$112.00
8.	Sept.	10.	Paid for 120 mo. pasturing stock @ \$1.50
9.	Feb.	28.	Paid for 45 mo. pasturing stock @ \$1.25
10.	"	28.	Paid 30 da. fertilizing @ \$1.25
11.	"	28.	Paid 8 da. mending fence @ \$1.25

8. Add all the net receipts and all the net expenditures for the separate fields as given in the records of these fields in problems 1 to 7 above. Find the net earnings for the year of the whole farm.

§36. Problems.

1. A double eagle weighs 516 grains, and a gold dollar 25.8 grains. Find the difference in weight between a double eagle and 2 gold dollars.

2. What will a peck of peanuts cost at 5¢ a pint?

3. At \$50 a front ft., what will 50 ft. of city land cost?

4. If milk costs 6¢ a qt. and I buy 2 qt. a day, what is my milk bill for April?

5. The average milk yield of a cow was 6 qt. a da. for 120 da. If this milk was all sold @ $6\frac{1}{4}$¢ a qt., how much did the owner receive for it?

6. The cow's feed cost the owner \$2.50 per mo. of 30 da. How much profit did the owner receive from the cow's milk during the 120 days?

7. A cow gives 12 lb. of milk a da. and the butter made from this milk equals $\frac{1}{30}$ of the weight of milk. How many pounds of butter does the milk furnish in 30 days?

8. If butter is selling for 35¢ a lb., what is the butter yield of this cow worth in 30 days?

9. 68 qt. of a certain Jersey cow's milk yield 7 lb. of butter. If milk is selling @ 6¢ and butter @ 35¢, is it more profitable to

sell 68 qt. of the milk of this cow, or to make butter of it and sell the butter? How much more profitable is it?

10. The pulse of a healthy man beats 72 times per minute. The heart contracts once for each pulse beat. How many times do the muscles of a healthy man's heart contract in 1 hr.? in 1 da. at the same rate?

11. Count your own pulse beats for 1 min. and find how many times your heart beats in a day at this rate.

12. Tie a heavy ball to a fine string, or split a bullet and fasten it to a thread, and suspend the string to a hook or nail so that $1\frac{1}{2}$ ft. of it may vibrate freely. Start it swinging and count the number of vibrations per minute. At the same rate how many times would the ball vibrate in 24 hours?

13. Solve the same problem with the suspending string 3 ft. long; with the string 3 ft. 3 in. long.

14. My watch ticks 270 times per minute; how many times does it tick in 1 hr.? in 1 day?

15. Light travels 186,600 mi. per second; how far does it travel in 1 day?

16. A man smokes 3 cigars a day, and pays 15¢ apiece for them. How much does this add to his expense account in 365 days?

§37. Multiplying by Factors.

1. $7 \times 8 = 56$ $28 \times 2 = 56$ $14 \times 4 = 56$

7, 8, 28, 2, 14, and 4 are all *factors* of 56.

2. Write the factors of 44, 48, 96, 35, 84, 81, 49, 108.

3. The two equal factors of 144 are 12 and 12. Give the two equal factors of 4, 9, 25, 49, 36, 121.

4. $8 \times 2 = ?$	$8 \times 2 \times 5 = ?$	$8 \times 10 = ?$
5. $12 \times 2 = ?$	$12 \times 2 \times 3 = ?$	$12 \times 6 = ?$
6. $20 \times 4 = ?$	$20 \times 4 \times 3 = ?$	$20 \times 12 = ?$
7. $25 \times 4 = ?$	$25 \times 4 \times 6 = ?$	$20 \times 24 = ?$
8. $64 \times 9 = ?$	$64 \times 9 \times 7 = ?$	$64 \times 63 = ?$

9. Instead of multiplying a number by 15, by what two numbers in succession may I multiply it and get the same product?

10. Multiplying any number by 21 gives the same product as multiplying it by what two numbers in succession?

11. $56 \times 2 \times 4$ gives the same product as 56 multiplied by what single number?

12. Multiplying a number by several factors one after another gives the same product as multiplying it by what single number?

A number which has factors other than itself and 1 is called a *composite* number. A number such as 3, 5, 17, etc., which has no factors other than 1 and itself, is a *prime* number.

13. Instead of multiplying a number by a composite number, in what other way may I multiply it to obtain the same product?

14. Which of these numbers are prime, and which composite: 24, 25, 19, 6, 2, 21, 23, 84, 45, 43, 27, 28?

§38. Multiplying by 10, 100, 1000, 10,000.

1. $84 \times 10 = ?$ $84 \times 100 = ?$ $84 \times 1000 = ?$ $84 \times 10{,}000 = ?$
2. $327 \times 10 = ?$ $327 \times 100 = ?$ $327 \times 1000 = ?$ $327 \times 10{,}000 = ?$
3. How may any whole number be quickly multiplied by 10, 100, 1000, or 10,000?

§39. Multiplying by a Number Near 10, 100, 1000, 10,000.

1. $746 \times 9 = ?$

SOLUTION.—This is once 746 less than 10×746, or $7460 - 746 = 6714$.

2. $965 \times 99 = ?$ *Ans.* $96{,}500 - 965 = 95{,}535$.
3. $965 \times 98 = ?$ *Ans.* $96{,}500 - 1930 = 94{,}570$.
4. $965 \times 101 = ?$ *Ans.* $96{,}500 + 965 = 97{,}465$.
5. $965 \times 102 = ?$
6. $873 \times 11 = ?$
7. $8473 \times 1001 = ?$
8. $8473 \times 999 = ?$

§40. Multiplying by 25, 50, $12\frac{1}{2}$, 75, 500, or 250.

1. $65 \times 25 = ?$ $25 = \frac{1}{4}$ of 100. $\frac{1}{4}$ of $65 \times 100 = 1625$.
2. $83 \times 50 = ?$ $50 = \frac{1}{2}$ of 100. $\frac{1}{2}$ of $83 \times 100 = 4150$.
3. $638 \times 12\frac{1}{2} = ?$ $12\frac{1}{2} = \frac{1}{8}$ of 100. $\frac{1}{8}$ of $638 \times 100 = 7975$.
4. $132 \times 75 = ?$ $75 = \frac{3}{4}$ of 100. $\frac{3}{4}$ of $132 \times 100 = 9900$.
5. $640 \times 500 = ?$ $500 = \frac{1}{2}$ of 1000.
6. $740 \times 250 = ?$ $250 = \frac{1}{4}$ of 1000.

We see from problem 1 that to multiply by 25, we may multiply by 100 and take $\frac{1}{4}$ of the product.

7. Make a rule, different from the ordinary rule, for multiplying quickly by 50; by $12\frac{1}{2}$; by 250; by 500.

8. Make a rule for multiplying quickly by $33\frac{1}{3}$; by $333\frac{1}{3}$. This shows the importance of remembering that:

$\frac{1}{2}$ of $100 = 50$	$\frac{1}{2}$ of $1000 = 500$
$\frac{1}{4}$ of $100 = 25$	$\frac{1}{4}$ of $1000 = 250$
$\frac{3}{4}$ of $100 = 75$	$\frac{3}{4}$ of $1000 = 750$
$\frac{1}{8}$ of $100 = 12\frac{1}{2}$	$\frac{1}{8}$ of $1000 = 125$
$\frac{3}{8}$ of $100 = 37\frac{1}{2}$	$\frac{3}{8}$ of $1000 = 375$
$\frac{5}{8}$ of $100 = 62\frac{1}{2}$	$\frac{5}{8}$ of $1000 = 625$
$\frac{7}{8}$ of $100 = 87\frac{1}{2}$	$\frac{7}{8}$ of $1000 = 875$
$\frac{1}{16}$ of $100 = 6\frac{1}{4}$	$\frac{1}{16}$ of $1000 = 62\frac{1}{2}$
$\frac{1}{3}$ of $100 = 33\frac{1}{3}$	$\frac{1}{3}$ of $1000 = 333\frac{1}{3}$
$\frac{2}{3}$ of $100 = 66\frac{2}{3}$	$\frac{2}{3}$ of $1000 = 666\frac{2}{3}$

9. Make rules for multiplying quickly by the numbers on the right hand side of these equations.

§41. Multiplying When Some Digits of the Multiplier Are Factors of Others.

1. From twice a number, how may you obtain 4 times the same? From 4 times a number, how obtain 8 times the same?

2. Multiply 19,279 by 842.

SOLUTION.—

$$\begin{array}{r l}
19279 & \\
\underline{842} & \\
38558 & = 19279 \times 2, \text{ the first partial product} \\
771160 & = 20 \text{ times the first partial product} \\
\underline{15423200} & = 20 \text{ times the second partial product} \\
16232918 &
\end{array}$$

3. Observe the relations of the digits of each multiplier below, and shorten the work of multiplication, as above. Prove work:

$4,783 \times 363$	$29,847 \times 248$
$12,875 \times 442$	$86,729 \times 393$

§42. Checking Multiplication.

Any of the above ways may be used to test the correctness of products obtained in the usual way.

To check by casting out the nines, cast the nines out of the multiplicand, the multiplier, and the product. Multiply the excesses of the multiplicand and the multiplier. Cast the nines out of this product. If this last excess equals the excess of the original product, the work is probably correct.

ILLUSTRATION.—To check $6848 \times 619 = 4238912$, excess in $6848 = 8$; excess in $619 = 7$; excess in $4238912 = 2$; $7 \times 8 = 56$, excess in $56 = 2$. As excess in 4238912 and in 56 are equal, the work is checked.

A very useful check against blunders is to examine the facts of the problem and to decide about what the answer must be, before beginning to solve it.

§43. Multiplying by Fractional Numbers.

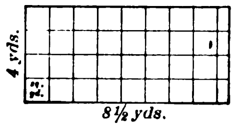


FIGURE 15

1. How many square yards in a room 4 yd. × $8\frac{1}{2}$ yards? (Here × is read "by.")

SOLUTION.—How many sq. yd. fill a strip 1 yd. wide extending along the long side? How many such strips cover the floor? $8\frac{1}{2}$ sq. yd. $\times 4 = x$ sq. yd. What is x?

2. What is the cost of $68\frac{1}{2}$ acres of land @ \$80?

SOLUTION.—$\$80 \times 68 = \$5440$, and $\frac{1}{2}$ of $\$80 = \$40$. Then $80 \times 68\frac{1}{2} = \$5480$. Observe that $\$80 \times 68\frac{1}{2}$ means $80 \times 68 + \frac{1}{2}$ of 80.

3. Tell what these problems mean and then solve them:

$\$64 \times 16\frac{1}{8} = ?$ 81 lb. $\times 25\frac{1}{3} = ?$ 24 ft. $\times 8\frac{3}{8} = ?$

5280 mi. $\times 14\frac{2}{3} = ?$ 897 yd. $\times 19\frac{2}{3} = ?$ $5\frac{1}{2}$ sq. yd. $\times 6 = ?$

160 A. $\times 12\frac{3}{4} = ?$ $5\frac{1}{2}$ sq. yd. $\times 5\frac{1}{2} = ?$

NOTE.—Solve the last problem by drawing a square $5\frac{1}{2}$ units long and $5\frac{1}{2}$ units wide, and dividing it up into square units.

The problems just solved show that $\frac{3}{4} \times 8$ means $\frac{3}{4}$ of 8, or 3 of the 4 equal parts of 8.

4. Give the meaning of these problems and find the value of the letter in each:

$\frac{2}{3} \times 9 = x$. $\frac{6}{7} \times 21 = x$. $\frac{7}{8} \times 56 = y$. $\frac{11}{12} \times 108 = y$. $\frac{13}{15} \times 75 = z$.

§44. Suggestions for Problems.

Make and solve problems based on the following facts:

1. A steel rail weighs 64 lb. per yd. of length. The distance from Washington to Baltimore is 42 mi.; to Philadelphia, 138 mi.; to New York, 227 mi.; to Boston, 459 miles.

NOTE.—There are 1760 yd. in 1 mile.

2. There are double tracks between Washington and each of the cities mentioned.

3. The distance from New York to San Francisco is 3262 miles.

4. The distances in miles from Chicago to 14 railroad centers of the United States are given here:

Indianapolis	184	Rochester	605
St. Louis	283	Baltimore	801
Cincinnati	298	Washington	820
Cairo	364	New Orleans	912
St. Paul	410	New York	913
Omaha	490	Denver	1028
Buffalo	536	San Francisco	2349

5. Galvanized telegraph wire weighing 572 lb. per mi. is used for distances over 400 mi., and for distances under 400 mi., wire weighing 378 lb. per mi. is used.

6. 8 wires run between Chicago and New York.

7. Galvanized iron telegraph wire costs 6¢ per pound.

8. Sound travels in water at the rate of 4708 ft. per second; in air 1130 ft. per second.

9. Silver is worth 56¢ an oz., 12 oz. to the pound.

10. Hay costs $23 per T.; oats 42¢ per bu. A horse is fed 162 lb. of hay and 10 bu. of oats a month. Bedding costs $2 a month.

11. Standard silver is $\frac{9}{10}$ pure silver and $\frac{1}{10}$ copper. A silver dollar weighs 412.5 grains. The total number of silver dollars coined on this basis was 378,166,769.

12. The number of grains in an ounce of silver is 480. The amount of silver bought under the Sherman Law by the United States government to coin into money was 168,674,682 ounces.

13. Light travels 186,600 mi. per second. It requires 448 seconds for light to reach the earth from the sun.

14. It reaches the earth from the moon in $1\frac{7}{25}$ seconds.

15. An oz. of pure gold is worth $20.67. There are 12 oz. in 1 lb. of gold.

16. An Alaskan miner can take away 200 lb. from the mining district.

17. A cu. ft. of granite weighs 170 lb. A granite step measures $\frac{1}{2}' \times 2' \times 8'$.

18. The distance around the driving wheel of a locomotive engine is 22 ft. In going a certain distance the driver turned 5280 times.

19. In the year 1901, 82,305,924 lb. of tea and 511,041,459 lb. of coffee were imported into the United States. Tea was worth 48¢ and coffee 26¢ per pound.

20. A prize-winning steer weighed 15.03 cwt. (1 cwt. = 100 lb.) and sold for $9.00 per hundredweight.

21. Another steer of the same lot weighed 1622 lb. and sold for $8.85 per hundredweight.

22. A boy bicyclist rode *a* miles per da. for *b* days.

23. Pupils should prepare and solve problems based upon price lists obtained from the grocer and the butcher (see p. 4), or from the market reports of the daily papers.

§45. **Rainfall.**—How long is a cubic inch (Fig. 16)? how wide? how high? What is a cubic foot? A cubic yard?

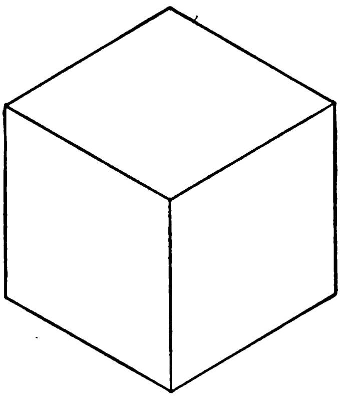


FIGURE 16

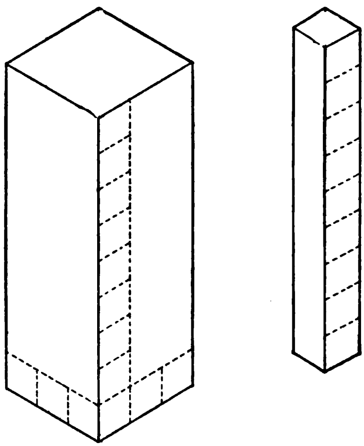


FIGURE 17

A tin box 3 in. square on the bottom and 9 in. high (Fig. 17) was used by a school as a rain gauge. A second tin box 1 in. square on the bottom and 9 in. high was used to measure the depth of water in the large box. After the water was poured

from the large box into the small box, the depth of water in the small box was measured with a thin stick and a foot rule.

1. The rain gauge was placed one evening where the rain could fall freely into its open top. During the night it rained and the next morning the water was poured from the gauge into the small box. It filled the small box to a depth of 9 in. How deep did the water fill the large box? How many cubic inches of water were caught in the large box?

One cu. in. of water for every sq. in. of surface is what is meant by 1 in. of rainfall. What is 6 in. of rainfall?

2. How many cu. in. of water are there in a layer 1 in. deep in the large box (Fig. 17)? 2 in. deep? 5 in. deep? 9 in. deep?

3. How many cu. in. of water fell on 1 sq. ft. of the ground during a rainfall of 1 in. (Fig. 18)? of 2 in.? of 3 in.? of 6 inches?

The number of cubic units (cu. in., cu. ft., cu. yd., etc.), a vessel holds, when full, is called its *capacity*.

4. What is the capacity of a square-cornered box $3'' \times 3'' \times 9''$?

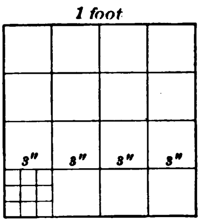


FIGURE 18

5. What was the depth of rainfall during a shower if the large box caught enough water to fill the small box half full? to a depth of 3 in.? of 6 in.? of 1 in.? of 2 in.? of 7 inches?

6. How many cu. in. of water fell on 1 sq. ft. of the ground during a shower giving 1 in. of rainfall? 2 in.? $\frac{1}{2}$ inch?

7. During a rainfall of 1 in. how many sq. ft. of ground received enough water to make 1 cu. ft.? 2 cu. ft.? 12 cubic feet?

8. During June, 1902, the rainfall in the vicinity of Chicago was $6\frac{1}{2}$ in. During this month how many cu. in. of water fell on 1 sq. in. of ground? on 1 sq. ft.? on 12 sq. ft.? on 1 sq. yd.? on 30 sq. yd.? on $\frac{1}{4}$ sq. yd.? on $30\frac{1}{4}$ square yards?

9. Just before a shower set an uncovered bucket where the rain may fall freely into it. After the shower measure the depth of the water in the bucket to find the depth of rainfall.

Find how many cu. in. or cu. ft. of water fell on each sq. ft.? each sq. yd.? each sq. rd. of the ground?

NOTE.—The bucket used must have the same size at the top and bottom, with straight sides. Why?

10. During the first 9 mo. of the year 1902, the region about Chicago received 32 in. of rainfall. How many cu. ft. of water fell during this time on a garden bed $10' \times 14'$? on a garden $48' \times 84'$? over a city block $250' \times 350'$?

11. If the walls of your schoolroom were water-tight, how many cu. ft. of water would it hold if it were filled 1 ft. deep? 2 ft. deep? 5 ft. deep? 8 ft. deep? to the ceiling?

12. Find the areas of these rectangles:

$6'' \times 8''$, $12'' \times 28''$, $40' \times 64'$,
$375' \times 486'$, $9' \times x'$, 9 yd. $\times\, x$ yd.,
a mi. $\times\, b$ mi., x units $\times\, y$ units.

How can you find the number of square units in any rectangle?

NOTE.—$x \times y$ is written xy and read "x, y."

13. How many cubic feet are there in 1 cu. yd.? in 18 cu. yd.?

14. What is the capacity of a square-cornered box $3'' \times 3'' \times 9''$? $3' \times 3' \times 9'$? of a square-cornered room or space, 3 yd. $\times$ 3 yd. $\times$ 9 yd.? 3 rd. $\times$ 3 rd. $\times$ 9 rd.? $3 \times 3 \times 9$? $3 \times 3 \times a$?

15. Find the capacity of a square-cornered box $3'' \times 4'' \times 7''$; $3'' \times 4'' \times 8''$; $3' \times 4' \times 8'$; $3' \times 4' \times 15'$; 3 yd. $\times$ 3 yd. $\times$ 3 yd.; 3 units $\times$ 4 units $\times$ 15 units; $3 \times 4 \times x$; $3 \times x \times y$; a units $\times\, b$ units $\times\, c$ units.

NOTE.—The product $x \times y \times z$ is written xyz and read "x, y, z."

16. How can you find the number of cubic units in any square-cornered vessel?

17. Find the total weight of a snow load of 25 lb. per sq. ft. on a flat rectangular roof $25' \times 48'$. Find the weight of a load of 15 lb. per square foot.

18. Find the total weight of the shingles and sheathings for both sides of the roof shown in Fig. 19, if shingles weigh 3 lb. per sq. ft., and sheathing 5 lb. per square foot.

NOTE.—The eaves project 2 ft. over the plate.

19. Find the weight of the snow load on both sides of the roof, the weight on each sq. ft. of surface being 12 pounds.

20. Find the cost at 3¢ per sq. yd. of lathing and plastering the four walls and the ceiling, no allowances being made.

21. What is the area of the square $ABCD$ of Fig. 19? What is the area of the triangle DEC?

22. A strong gale, giving a pressure of 16 lb. per sq. ft., blows squarely against the end of the building. What is the total wind pressure against the end including the gable?

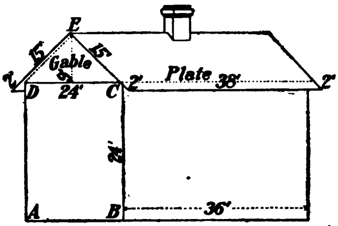


FIGURE 19

23. How many cu. ft. of space are inclosed by the walls to the base, DC, of the gables?

§46. Algebraic Problems.

$4 \times x$ is written $4x$. If $4x = 24$, what is the value of x?

Notice that the equation $4x = 24$ is the statement of this problem. A boy reads 24 pages of a book in 4 days; how many pages does he read a day? What does x stand for in this problem?

1. What number does x stand for in these equations?

(1) $3x = 18$.	(4) $x - 8 = 15$.	(7) $x + 9 = 17$.	(10) $5x + 2x = 70$.
(2) $6x = 48$.	(5) $x - 18 = 4$.	(8) $x + 6 = 22$.	(11) $9x - 3x = 36$.
(3) $8x = 72$.	(6) $x - 9 = 8$.	(9) $x + 16 = 25$.	(12) $8x + 5x = 26$.

2. Write the sum of a and b.

3. Write the difference of a and b.

4. Write the product of a and b; of a and b and c; of x and y and z.

5. Answer the following questions if $a = 9$, $b = 8$ and $c = 6$:

$4a = ?$ $12b = ?$ $9c = ?$ $ab = ?$ $4ab = ?$ $abc = ?$ $3abc = ?$

6. In the shortest way you can, write eight times x; seven times y; twenty-five times a times b; a times x times y.

7. In the shortest way, write and read a times b; c times x; fifteen times a times b times x.

DIVISION

§47. Division and Subtraction Compared.—ORAL WORK

1. A man owes a debt of $12 which he is to pay by work at $2 per day. How much will he owe at the end of the first day? of the second day? of the third day? of the fourth day? the fifth? the sixth?

2. How long will it take to cancel the debt?

3. How many times may 2 be subtracted from 12, leaving no remainder? How many 2's are there in 12?

4. A man buys a horse for $90 and is to pay $15 a month until the horse is paid for. How much does the man owe after the first payment? after the second? the third? fourth? fifth? sixth?

5. How many months will it take to pay for the horse?

6. How many times may 15 be subtracted from 90, leaving no remainder? How many 15's in 90?

7. What is one of the 6 equal parts of 12? of 90?

8. A rectangular plot of ground 8 yd. wide by 18 yd. long is covered with bluegrass sod. How many square yards of sod does it contain? After a strip of sod 1 yd. wide, extending the length of the plot, has been removed, how many square yards of sod remain?

9. How many square yards remain after the removal of 2 such strips? of 3? of 4? of 5? of 6? of 7? of 8?

10. How many times may 18 sq. yd. be subtracted from 144 sq. yd., leaving no remainder? How many 18's are there in 144? What is one of the 8 equal parts of 144?

WRITTEN WORK

1. There are 160 bricks in a pile; find by subtraction how many loads of 20 bricks each there are in the pile. How many 20's are there in 160?

2. Find by successive subtraction how many 88's there are in 616. What is one of the 7 equal parts of 616?

3. Find by subtraction how many months of 30 da. there are in 270 da. What is one of the 9 equal parts of 270?

4. What number may be subtracted 4 times in succession from 120, leaving no remainder?

5. Find by subtraction how many times $1263 is contained in $5052.

6. Tell how to find by subtraction how many times one number is contained in another.

7. Find in a shorter way how many 12's there are in 156.

8. Find in a shorter way than by subtraction one of the 14 equal parts of 224. By what name do you know this short way?

9. In the same way find one of the 15 equal parts of 225; of 210; of 240.

10. A lawn-mower cuts a strip 18 in. wide. How many strips must be cut to mow a lawn 18 ft. wide? 20 ft. wide? 30 ft. wide?

Division is a short way of subtracting one number from another a certain number of times in succession.

§48. Division and Multiplication Compared.—ORAL WORK

1. A horse traveled 72 mi. in 8 hr.; find the number of miles traveled per hour.

2. 4 bu. potatoes cost $2.40. What was the price per bushel?

3. A room 4 yd. wide contains 24 sq. yd.; what is the length of one side?

4. 15 lb. sugar cost 90¢; what is the price per pound?

5. At $6 per ton, how many tons of coal can be bought for $180?

6. A floor containing 132 sq. ft. is 11 ft. wide; what is the length?

7. Find the cost of 10 pk. apples @ 14¢ per peck. If 10 pk. apples cost $1.40, what was the price per peck?

8. How many pecks of apples @ 14¢ can be bought for $1.40?

9. A train runs 420 mi. in 12 hr.; find the average number of miles per hour.

10. I paid 96¢ for veal at 12¢ per pound; how many pounds did I buy?

11. What is the cost of 3 T. hay at $12 per ton?

12. At $12 per ton, how many tons of hay can be bought for $36?

13. A man paid $36 for 3 T. hay; what was the price per ton?

14. Find the cost of 25 bbl. (barrels) flour at $5.

15. Paid $125 for 25 bbl. flour; what was the price per barrel?

16. Paid $125 for flour at $5; how many barrels were bought?

17. The two factors of a number are 13 and 7; what is the number?

18. The product of two numbers is 91 and one of the numbers is 13; what is the other number?

19. The product of two numbers is 240 and one of the numbers is 20; what is the other?

20. $20 \times x = 240$; what number does x stand for?

21. Tell what number the letter stands for in each of these equations:

$12y = 96$; $8x = 56$; $7a = 56$; $9b = 72$; $10a = 150$; $24m = 240$.

Division is a way of finding one of two numbers when their product and the other number are given.

The product is called the *dividend.*

The given number is the *divisor.*

The required number is the *quotient.*

ILLUSTRATION.—The product of two factors is 490, and one of the factors is 7. What is the other factor?

$$7\overline{)490}^{\,70}$$

We may also say that the *dividend* is the number to be divided, the *divisor* is the number by which the dividend is measured or divided, and the *quotient* is the measure.

The sign ÷ of division is read "divided by," as $60 \div 12 = 5$.

$60 \div 12 = 5$ may also be written in the following ways:

$$12\overline{)60}^{\,5} \qquad \begin{array}{l} 12)60(5 \\ \underline{\ \ \ \ 60} \end{array} \qquad \frac{60}{12} = 5 \qquad 60/12 = 5$$

The first and second are in common use in division.

In the third, the line placed between two numbers shows that the number above it is to be divided by the number below it; as

in $\frac{1}{3}$, 1 is the dividend and 3 the divisor. The fraction itself is the quotient. The line between the two numbers is the division sign.

The fourth sign is called the *solidus* (*sol'i-dus*).

§49. Short Division.

1. How many gallons are there in 296 pints?

SOLUTION.—As there are 8 pt. in 1 gal. there are as many gallons in 296 pt. as there are 8's in 296.

$$8\,\overline{)296}\quad 37$$

2. How many days in 120 hours?

3. At $8 per ton, how many tons of coal can be bought for $240?

4. 984 marbles are distributed equally among a certain number of boys. Each boy has 82 marbles. There are how many boys?

5. Selling at 6 for a cent, how much will a dealer receive for 540 marbles?

6. A man paid $2.60 for 4 bbl. of lime. What did each bbl. cost?

7. I bought 6 lb. of butter for $1.62. What was the price per pound?

8. A dressmaker used 84 yd. of cloth for 6 dresses, allowing the same amount for each dress. How many yards in each?

9. A train ran 150 mi. in 6 hr. Not allowing for stops, what was the average number of miles per hour?

Find the value of x in problems 10 and 11:

10. 48 qt. = x gallons.

11. At 6¢ I can buy x lb. of sugar for $1.50.

12. 9 papers of needles cost 72¢. One paper costs how many cents?

13. In 1901, the number of trains entering Chicago every 24 hr. was about 1320; what was the average number per hour?

ORAL WORK

What does x stand for in each of the following equations?

1. $63 \div 7 = x$	4. $45 \div 3 = x$	7. $120 \div 3 = x$
2. $630 \div 9 = x$	5. $96 \div 8 = x$	8. $108 \div 9 = x$
3. $48 \div 4 = x$	6. $960 \div 8 = x$	9. $72 \div 6 = x$

When dividend and divisor are small numbers, the quotient is readily seen. We say it is obtained by *inspection.*

When the divisor is a large number, the quotient is not readily found by inspection.

§50. Applications. WRITTEN WORK

1. In 51,448 qt. how many pecks are there?

SOLUTION.—In 51,448 qt. there are as many pecks as there are 8 qt. in 51,448 quarts.

8 is contained in 51,000, 6000 times, with a remainder of 3000. Write 6 in the thousands place in the quotient. 3 thousands = 30 hundreds; 30 hundreds and 4 hundreds = 3400. 8 is contained in 3400, 400 times, with a remainder of 200. Write 4 in the hundreds place in the quotient. 200 = 20 tens; 20 tens and 4 tens = 24 tens. 8 is contained in 24 tens 3 tens times, without a remainder. Write 3 in the tens place in the quotient. 8 is contained in 8 units 1 unit time. Write 1 in the units place in the quotient.

$$\begin{array}{r} 6431 \\ 8\overline{)51448} \end{array}$$

In 51,448 qt. there are 6431 pecks.

Check: $6431 \times 8 = 51,448$.

2. There are 5280 ft. in a mile; how many yards are there in 1 mi.? in 2 miles?

3. If limestone weighs 160 lb. per cubic foot, how many cubic feet are there in a piece of limestone weighing 6400 pounds?

4. If marble weighs 170 lb. per cubic foot, find the number of cubic feet in a piece of marble weighing 5100 pounds.

5. If sand weighs 120 lb. per cubic foot, find the number of cubic feet in a load of sand weighing 4800 pounds.

6. A train of 12 sleeping cars is 840 ft. long. If the cars are all the same length, how long is each car?

7. A steel rail 30 ft. long weighs 720 lb.; what is its weight per yard of length?

8. An iron beam 24 ft. long weighs 1080 lb.; what is the weight of a piece of the beam 1 ft. long?

9. A steel girder (or beam) weighs 1728 lb. Each foot of length weighs 48 lb. How long is it?

10. There were 487,918 foreign immigrants to the United States in the year 1901. What was the average number per month? per day? (30 da. = 1 month.)

11. During 1900 there were 448,572 immigrants. Find the average number per month.

12. Answer the same question for 1891, 1892, 1893, 1895, and 1897, the numbers for these years being in succession 560,319; 623,084; 502,917; 258,536, and 230,832.

13. In Albany, N. Y., there are 30 mi. of street railway, operated by 600 men. What is the average number of employees per mile?

14. From the data here given answer the same question for these cities:

CITY	MILES	EMPLOYEES
St. Joseph, Mo.	35	175
Memphis, Tenn.	70	490
Oakland, Cal.	80	560
Hartford, Conn.	33	660
Worcester, Mass.	43	473
Peoria, Ill.	50	275

15. Find the value of x in each case:

(1) $3264 \div 6 = x$ (4) $89,765 \div 5 = x$ (7) $24,568 \div 8 = x$
(2) $9432 \div 12 = x$ (5) $78,870 \div 11 = x$ (8) $45,960 \div 12 = x$
(3) $2247 \div 7 = x$ (6) $75,699 \div 9 = x$ (9) $1,241,196 \div 11 = x$

16. How can you prove the correctness of your work in division?

17. Find what x equals in these equations:

(1) $\frac{80}{x} = 10$ (3) $\frac{6300}{x} = 9$ (5) $\frac{490}{x} = 49$ (7) $\frac{640}{x} = 20$

(2) $\frac{63}{x} = 9$ (4) $\frac{5600}{x} = 56$ (6) $\frac{810}{x} = 90$ (8) $\frac{990}{x} = 66$

§51. Long Division.

When dividend and divisor are both large numbers, it becomes necessary to show all the steps of the work.

1. $14,487 \div 33 = ?$

$$\left.\begin{array}{r} 9 \\ 30 \\ 400 \end{array}\right\} = 439, \text{ quotient}$$

$$\begin{array}{rl} 33)\overline{14487} & \\ 13200 & = 400 \times 33 \\ \hline 1287 & \\ 990 & = 30 \times 33 \\ \hline 297 & \\ 297 & = 9 \times 33 \\ \hline \end{array}$$

SOLUTION.—Beginning at the left of the dividend; 33 is not contained in 1, nor in 14, but it is contained in 14,400, 400 times. Write the 400 above the dividend and subtract $400 \times 33 = 13,200$ from the dividend, leaving 1287. This remainder must also be divided by 33.

33 is not contained a whole number of times in 1, nor in 12, but it is contained 30 times in 1280. Write the 30 above the 400 over the dividend and subtract $30 \times 33 = 990$ from 1287, leaving 297.

33 is contained in 297, 9 times, leaving no remainder.

Thus we see 33 is contained in 14,487 $400 + 30 + 9 = 439$ times.

The work may be shortened a little by omitting the zeros and writing numbers in the shorter form below.

SHORTER FORM

$$\begin{array}{rl} & 439, \text{ quotient} \\ \text{divisor, } 33) & \overline{14487}, \text{ dividend} \\ & 132 \\ \hline & 128 \\ & 99 \\ \hline & 297 \\ & 297 \\ \hline \end{array}$$

Check: $439 \times 33 = 14,487$. Since this is the given dividend, the division is probably correct.

When a zero appears in the quotient proceed as follows:

2. $18,722 \div 46 = ?$

$$\begin{array}{r} 407 \\ 46)\overline{18722} \\ 184 \\ \hline 322 \\ 322 \\ \hline \end{array}$$

SOLUTION.—Begin as above. 46 is contained in 187, 4 times, with the remainder 3. Bring down the 2 in the dividend, giving 32. 46 is not contained in 32 a whole number of times. Write 0 in tens place in the quotient and bring down the next 2 of the dividend, giving 322. 46 is contained in 322, 7 times. Write the 7 in units place in the quotient.

Check: $407 \times 46 = 18,722$, which equals the dividend.

Complete these equations and check your work:

3. $3,580 \div 45 =$	7. $33,768 \div 72 =$
4. $15,552 \div 64 =$	8. $35,096 \div 82 =$
5. $18,144 \div 56 =$	9. $62,328 \div 84 =$
6. $20,083 \div 72 =$	10. $44,928 \div 96 =$

§52. **Exercises.**

1. There are 52 wk. in a year. My friend is 1872 wk. old; how many years old is he?

2. There are 160 sq. rd. in 1 A. and a farmer pays 75¢ per acre for cutting and binding wheat. How much will it cost to cut and bind the wheat on a field 68 rd. by 80 rods?

3. A farmer paid a man $21.00 to shock his wheat, wages being $1.50 per day. How many days did the man work?

4. In 1880, 25 farm wagons sold for $2250 and in 1900, 15 such wagons sold for $855. How much less was the average cost of a farm wagon in 1900 than in 1880?

5. In 1880 a Minnesota farmer paid $3900 for 12 twine binders and in 1900, 14 twine binders cost him $1680. How much had the average price of twine binders fallen during these 20 years?

6. A steel rail weighing 72 lb. per yard is 30 ft. long. How many men are needed to carry it, each man carrying 90 pounds?

7. In 25 da. a man earned $56.25 husking corn at 3¢ per bushel. How many bushels per day did he husk?

8. A city lot 175 ft. long, containing 8750 sq. ft., sold for $6000. If the short side fronts the street what was the price per foot of frontage?

9. The force required to draw a street car on a level track is 35 lb. per T. (2000 lb.) of the combined (total) weight of the car and its load. What force is needed to draw a car weighing 5600 lb. when it is loaded with 60 passengers whose average weight is 140 pounds?

10. At the speed of an ordinary horse car the pull of a horse equals about 125 lb. of force in drawing the car. How many horses will be needed to draw the car of problem 9, no horse to draw more than 125 pounds?

11. At a slow walk a horse can exert about 330 lb. of force. The force required to draw a loaded wagon on a level pavement is $\frac{1}{25}$ of the weight of the wagon and load. A coal wagon weighing 4860 lb. is loaded with 4 T. of coal. How many horses will be needed to draw the load over a level pavement, no horse drawing more than 330 pounds?

12. A horse can exert 1540 lb. of force for a few minutes. A box car weighing 30 T. is loaded with 32 T. The force needed to move the loaded car is $\frac{1}{86}$ of the combined weight of the car and load. How many horses will be needed to start the car on a level track?

§53. Larger Numbers.

1. $5{,}128{,}672 \div 9272 = ?$

```
          553
     ————————
9272)5128672
      46360
      —————
       49267
       46360
       —————
        29072
        27816
        —————
         1256
```

SOLUTION.—The divisor, 9272, being too large to use readily, we first use a trial divisor. For the same reason, we select a trial dividend. 92, the trial divisor, is contained in 512, 5 times; but as the whole divisor contains four digits the partial dividend must be enlarged. 9272 is contained in 51,286, 5 times. The quotient figure 5 is of the same order as the last figure of the trial dividend, which is hundreds. We write 5 in hundreds place in the quotient. Multiplying the whole divisor by the quotient figure we have the product 46,360. Subtracting this product from the trial dividend, 4926 remains. 4926 hundreds = 49,260 tens; and 49,260 tens + 7 tens = 49,267 tens. Continue in the same manner with each step that follows.

The last subtraction gives a remainder of 1256. This remainder must also be divided by the divisor 9272.

$$5{,}128{,}672 \div 9272 = 553\tfrac{1256}{9272}$$

```
   9272
    553
  —————
  27816
 46360
46360
———————
5127416
   1256
———————
5128672
```

The 553 is the whole, or integral part of the quotient and the $\frac{1256}{9272}$ is the fractional part.

Check: Multiply the divisor by the quotient, and to the product add the remainder. The result should equal the dividend.

Solve the following problems and check your work:

2. $131{,}320 \div 536 =$	4. $630{,}861 \div 2731 =$
3. $195{,}936 \div 624 =$	5. $1{,}057{,}536 \div 4352 =$

§54. Geography.

1. From the table of §23 the area of Massachusetts is seen to be 8315 sq. mi., and that of Illinois is 56,650 sq. mi. How many states the size of Massachusetts could be made from Illinois?

2. From the same table the area of New England is found to be 66,465 sq. mi. How many states as large as New England could be made from Texas?

3. Make and solve other problems like these, using the table.

4. The same table shows the area of Connecticut to be 4990 sq. mi. and its population for 1900 to be 908,420. How many persons per square mile are there in Connecticut?

NOTE.—In problems such as this, where the fractional part of the quotient has no meaning, drop the remainder if it is less than half the divisor, and add one unit to the whole part of the quotient if the remainder is more than half of the divisor.

5. The table of §28 gives the population of Connecticut for 1890 as 746,258. What was the population of Connecticut per square mile in 1890?

6. Answer questions 4 and 5 for your own state.

7. From the same table answer questions 4 and 5 for Oklahoma territory.

8. Make and solve similar problems for any states you are studying in your geography.

9. The area of Switzerland is 15,781 sq. mi. and its population is 2,933,334. What is the population of Switzerland per square mile?

10. 640 A. = 1 sq. mi. How many square miles in 534,528 acres?

11. The area of the state of Texas is 265,780 sq. mi.; that of New Jersey is 7815 sq. mi. How many states the size of New Jersey could be made from Texas? How many the size of Delaware, which contains 2050 square miles?

12. Porto Rico has an area of 3531 sq. mi., and a population of 953,243. How many inhabitants does it support to the square mile?

13. The greatest ocean depth found is 31,614 ft. near the island of Guam, in the Pacific ocean. The highest mountain in the world is Mt. Everest in Asia, which rises 29,002 ft. above sea level. Find the difference of level in miles between the greatest ocean depth and the greatest land altitude.

14. The state of New York, with an area of 49,170 sq. mi., supports a population of 7,268,894. How many inhabitants does it average to the square mile?

15. Hawaii has an area of 6449 sq. mi., and a population of 154,001. Find the average population per square mile.

16. In 1891 the total wheat area of North and South Dakota was 4,882,157 A.; the yield was 81,819,000 bu. What was the average yield per acre?

17. In the year 1900 Kentucky had 22,488 A. of rye under cultivation. The total yield was 292,344 bushels. What was the average yield per acre?

18. In the year 1890 there were 210,366 persons employed in manufacturing in Chicago. The total wages paid amounted to $123,955,001. What was the average wage paid to each person?

19. In 1880, 3519 factories in Chicago together yielded $249,022,948 worth of products. In 1890, 9977 factories yielded $577,234,446 worth. During which year was the average product per factory greater, and by how much?

20. A comparison of the density of population (population per square mile) may be obtained by finding the density of population of these countries:

COUNTRY	AREA	POPULATION	DENSITY
German Empire........	208,830	56,345,014	
Great Britain..........	120,979	41,454,578	
Austria................	115,903	26,107,304	
France................	204,092	38,641,333	
Italy..................	110,646	32,449,754	
Russia.................	8,660,395	135,000,000	
United States..........	3,688.110	76,212,168	

NOTE.—Only the whole numbers need be found for these quotients.

§55. Division by Multiples of 10.—ORAL WORK

1. Multiply each of these numbers by 10:

568 1268 306 $86.50 $8.65

2. Multiply each of the same numbers by 100; by 1000.

3. Divide each of these numbers mentally by 10:

5680 12680 3060 $865.00 $86.50

4. Make a rule for dividing any number quickly by 10; by 100; by 1000; by 1 with any number of zeros after it.

5. Name these quotients orally:

$60 \div 10 = ?$	$6 \div 2 = ?$	$60 \div 20 = ?$
$860 \div 10 = ?$	$86 \div 2 = ?$	$860 \div 20 = ?$
$\$64.20 \div 10 = ?$	$\$6.42 \div 2 = ?$	$\$64.20 \div 20 = ?$

Examining your answers to the questions just asked, make a rule for dividing a number quickly by 20; by 200; by 2000.

6. $180 \div 10 = ?$	$18 \div 3 = ?$	$180 \div 30 = ?$
$630 \div 10 = ?$	$63 \div 3 = ?$	$630 \div 30 = ?$

From these answers make a rule for dividing any number quickly by 30; by 300; by 3000.

7. Make a rule for dividing a number quickly by 40; by 400; by 4000; by 50; by 800; by 1200; by 1500.

8. Make a rule for dividing any number quickly by any whole number of tens, as 40, 70, 90, 160; by any whole number of hundreds; of thousands.

9. Cutting off zero from the right of a number has what effect on the number? cutting off 2 zeros? 3 zeros?

$$66 \div 10 = 6\tfrac{6}{10}, \text{ or } 6.6.$$
$$165 \div 10 = 16\tfrac{5}{10}, \text{ or } 16.5.$$
$$75 \div 100 = \tfrac{75}{100}, \text{ or } .75.$$
$$478 \div 100 = 4\tfrac{78}{100}, \text{ or } 4.78.$$

10. Using first 10 and then 100 as a divisor, give and show the quotients of the following:

400	500	2200	3300
460	790	4280	4860
287	439	9647	5732

11. There are 10 pk. in a barrel. How many barrels in 1488 pecks?

12. There are 60 lb. in a bushel of potatoes. How many bushels in 486 pounds?

13. 100 lb. = 1 cwt. How many hundredweight in 825 pounds?

14. 200 lb. pork = 1 bbl. How many barrels in 7624 pounds?

15. How many minutes in 4260 seconds?

SOLUTION.—When there are ciphers at the right of both dividend and divisor, cut off an equal number of ciphers from both and divide.

$$6\cancel{0})426\cancel{0} = 71$$

16. How many barrels will be needed for 4800 lb. of beef, allowing 200 lb. to the barrel?

§56. Other Methods of Shortening Division.

1. Solve these problems:

5000)25000 500)2500 50)250 5)25

How do the quotients compare? the dividends? the divisors?

2. Remembering that the dividends stand above the line and the divisors below, solve these problems:

$$\frac{324}{81} = ? \qquad \frac{108}{27} = ? \qquad \frac{36}{9} = ? \qquad \frac{12}{3} = ?$$

How do the quotients compare? the dividends? the divisors?

3. How do the quotients, the dividends, and the divisors compare in these problems:

$$\frac{256}{128} = ? \qquad \frac{64}{32} = ? \qquad \frac{16}{8} = ? \qquad \frac{4}{2} = ?$$

4. To divide 324 by 81 what smaller numbers may I use to get the same quotient? How can I obtain these smaller numbers from 324 and 81?

SOLUTION.—We see that, as $324 = 27 \times 12$ and $81 = 27 \times 3$, we may write

$$\frac{12 \times 27}{3 \times 27} = \frac{12}{3} = 4.$$

This can be indicated thus:

$$\frac{12 \times \cancel{27}}{3 \times \cancel{27}} = 4.$$

Removing these factors is called *cancellation.* It can often be used to simplify the division of products.

5. Answer the same questions for $256 \div 128$.

These problems show that any factor of both dividend and divisor may be dropped from both and the remaining factors divided.

6. Solve these problems by cancellation:

(1) $\frac{34 \times 16}{2 \times 16} = ?$ (2) $\frac{6 \times 8 \times 3}{2 \times 8 \times 3} = ?$ (3) $\frac{15 \times 21 \times 846}{3 \times 7 \times 846} = ?$

(4) $\frac{18 \times 3 \times 4 \times 67}{2 \times 4 \times 3 \times 67} = ?$ (5) $\frac{625}{25} = ?$ (6) $\frac{324}{189} = ?$ (7) $\frac{85}{34} = ?$

To use cancellation effectively, methods of finding factors of numbers are necessary.

§57. Tests of Divisibility.

1. Which of these numbers are exactly divisible (can be exactly divided) by 2:

12 24 36 23 45 18 37 40 59 61

What are the last digits of the numbers which 2 will divide?

Test for the factor 2: If a number ends in 0, 2, 4, 6, or 8, it can be exactly divided by 2.

2. Which of these numbers are exactly divisible by 10:

24 30 45 50 700 640 83 765 6400

What is the last digit of the numbers which 10 exactly divides?

Test for the factor 10: If a number ends in 0, 10 exactly divides it.

3. Make a rule for testing whether 100 divides a number.

4. Which of these numbers does 5 exactly divide:

16 18 35 25 60 20 28 65 460 675 1260

What are the last digits of the numbers which 5 will divide?

Test for the factor 5: If a number ends in 0 or 5, 5 exactly divides it.

5. Which of these numbers does 3 exactly divide:

12 17 24 81 27 93 64 75 126 324 185

Of the numbers which 3 exactly divides, will 3 also exactly divide the sum of the digits? Of the numbers 3 does not exactly divide, is the sum of the digits exactly divisible by 3?

Test for the factor 3: If 3 exactly divides the sum of the digits of a number it divides the number also.

6. Of these numbers what ones does 9 exactly divide:

126 368 453 729 819 639 2358

See whether 9 will exactly divide the sum of the digits of the numbers that 9 exactly divides.

TEST FOR THE FACTOR 9: **If 9 exactly divides the sum of the digits of a number it also exactly divides the number.**

7. Which of these numbers does 4 exactly divide:

113 124 368 560 375 486 1204

See whether of the numbers it exactly divides 4 also exactly divides the number indicated by the last two digits. For example, in the third, 368 can be exactly divided by 4 and so also can 68.

TEST FOR THE FACTOR 4: **If the number denoted by the last two digits of a number can be divided by 4, the entire number can be exactly divided by 4.**

8. Which of these numbers are divisible by 25:

60 175 285 625 1350 1275 8645 8675 8625 8650

25 divides 175 exactly and 25 also exactly divides 75, which is the number denoted by its last two digits. Is this true of all numbers 25 exactly divides? Is it true of any numbers 25 does not exactly divide?

TEST FOR THE FACTOR 25: **If the number denoted by the last 2 digits of any number is divisible by 25 the entire number is divisible by 25.**

TEST FOR THE FACTOR 6: **test for both 2 and 3.**

TEST FOR ANY COMPOSITE FACTOR: **test singly for all the factors of the composite factor.**

9. TEST FOR DIVISIBILITY by such numbers as 36, 216, 27, 49, etc., which contain some factor two or more times.

10. Pick out the numbers of this list which are exactly divisible by 2:

6 81 65 72 86 129 9864 8643 7986 16,835 29,860

11. Pick out those which can be exactly divided by 3; by 9; by 4; by 5; by 10; by 12; by 15.

§58. Checking Division.

Division may be checked by multiplying the divisor by the quotient and adding the remainder to the product. If the sum equals the dividend, the work is checked.

Another check is to divide by the factors of the divisor successively and note whether the final quotient is the same as that given by the complete divisor.

To check by casting out the nines, add the excess in the product of the excesses of divisor and quotient to the excess of the remainder. If the excess of this sum equals the excess of the dividend, the division is probably correct.

ILLUSTRATION.—Check the work of problem 1, §53.
Cast the 9's out of the dividend, 5,128,672. The excess is *4*.
Cast the 9's out of the divisor, 9272. The excess is *2*.
Cast the 9's out of the quotient, 553. The excess is *4*.
Cast the 9's out of the remainder, 1256. The excess is *5*.
The product of the excesses of divisor and quotient is *8*.
The excess of 8 is *8* itself. Add this 8 to the excess of the remainder, giving 13.
The excess of this 13 is *4* and as this equals the excess 4 of the dividend, the division is probably correct.

It is even more important in division than in multiplication to examine a problem carefully before beginning to solve it. Try to foresee about what the answer must be. This often avoids blunders.

ILLUSTRATION.—1. If it requires 480 slates to cover a square (100 sq. ft.) of roof surface, how many squares are there in a roof which requires 13,680 slates to cover it?
Pupil should at once notice that if it required 500 slates to cover a square, there would be a little more than 13,680 ÷ 500, or 136 ÷ 5 = 27 squares and he might guess 28 squares. The actual division of 13,680 by 480 gives 28.5 squares.

First form a rough estimate of the answer and then solve these exercises:

2. $238 was paid for flour @ $3.50; how many bbl. were bought?

SUGGESTION.—How many bbl. would there have been if the price had been $7.00 a barrel?

3. In still air a hawk flew 375 miles in $2\frac{1}{2}$ hr. What was its speed per hour?

4. In 3.9 hr. a crow flew 97.5 mi.; find the rate of flight per hour.

5. From a certain cow $\frac{1}{32}$ of the milk was butter-fat. The cow gave 12 lb. of milk per day. How many pounds of butter-fat will the milk from this cow yield in 70 days?

§59. Town Block and Lots.—Fig. 20 is a drawing, or scale map, of a town block. Scale: 1 in. equals 100 ft. If possible, pupils should measure and draw to scale a block or field in the neighborhood of the schoolhouse, and in all problems use the numbers they obtain from their own measurements in preference to those given in the exercises.

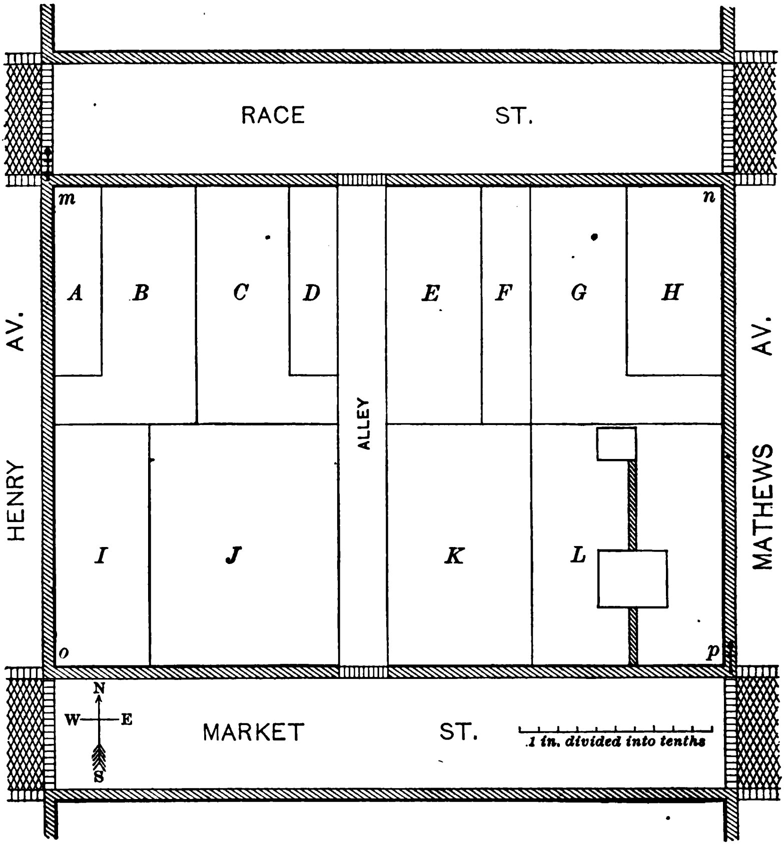


FIGURE 20

Measure the drawing in Fig. 20, and find how long the block is between the sidewalks; how wide.

NOTE.—Use the scale of tenths given in the figure. Lay a strip of paper having a straight edge beside the marked inch and mark short lines on the strip to indicate the tenths.

WRITTEN WORK

1. How many square feet in the area of the block *mnpo*?

2. At $20 per ft. of frontage on (distance along) Market street, what is lot L worth?

3. Make problems like 2 for other lots on Market street, using price per front foot of lots where you live.

4. At $12.50 per ft. of frontage on Mathews avenue, what is lot H worth?

5. Make similar problems for other lots on Mathews avenue.

6. At $18 per ft. of frontage on Race street, what is lot G worth? Similar problems should be made by the pupil.

7. Make problems for any, or all, of the lots, at the price per front foot where you live.

NOTE.—Each property holder is taxed to provide funds for foundation material, brick, and labor to pave in front of his property to the middle line of the street. This is called an assessment.

8. The streets are to be paved with brick. The cost of excavating (digging out) to the proper depth is 30¢ per square yard of surface;* find the cost of excavating a strip 1 yd. wide extending from outside edge of sidewalk to middle of Race street. *Ans.* $3.00.

9. What would be the cost of excavating a similar strip 1 ft. wide? 5 yd. wide? a strip as wide as frontage of lot E? *Ans.* $1.00; $15.00; $50.00.

10. The cost of foundation material is 36¢ per square yard; find the cost of enough such material for the strip 3 ft. × 30 ft. described in problem 8. *Ans.* $3.60.

11. Find the cost of foundation material for each of the three strips (1 ft. × 30 ft., 15 ft. × 30 ft., and 50 ft. × 30 ft.) of problem 9?

12. The cost of the brick to be used is $10 per M. (thousand) and 78 bricks are needed to cover 1 sq. yd. What is the cost of the brick needed for the strip of problem 8? for each of the three strips of problem 9? 1st *Ans.* $7.80.

13. What is the total cost of excavating, of foundation material, and of brick for the strip of problem 8? for each of the three strips of problem 9 (1 ft. × 30 ft., 15 ft. × 30 ft., and 50 ft. × 30 ft.)?

---

* Use prices current in your community whenever they can be obtained, instead of prices given.

14. Find this total for other lots fronting on either Market or Race street.

15. The labor of construction (making) costs 75¢ per sq. yd. What is the cost of labor on a strip of the size mentioned in problem 8? *Ans.* $7.50.

16. What will be the assessment against lot F for paving for each foot of frontage?

17. Make similar problems for other lots, not including the corner lots.

18. Henry and Mathews avenues, which are 30 ft. wide between sidewalks, are to be paved in the same way as Market and Race streets. Rates being the same as above, what will be the total assessment against lot L? (Omit the street crossings.)

19. Make and solve problems like 18 for other lots.

20. The owner builds a house on lot L. The length of the house is 36 ft. and the width is 30 ft. How many square feet of the lot are covered by the house?

21. The southeast corner of the house is located 30 ft. from the south and east lines of the lot. Locate the three other corners.

22. Find the cost, at 45¢ per sq. ft., of the concrete walk 3 ft. wide and 76 ft. long on lot L, as shown in the drawing.

§60. House and Furnishings.—ORAL WORK

1. What is meant by a 1-brick wall?* a 2-brick wall?

2. If it takes 7 bricks to make 1 sq. ft. of wall surface when laid in a 1-brick wall, how many bricks per square foot are needed for a 2-brick wall? a 3-brick wall? a 5-brick wall?

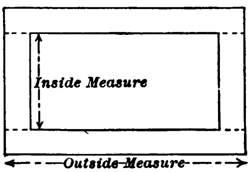


FIGURE 21

3. From the dotted lines in Fig. 21, can you tell how measurements may be taken on a wall to avoid counting corners twice?

4. A brick is 2 in. by 4 in. by 8 in.; how many cubic inches are there in it?

* A 1-brick wall is a wall one brick thick, bricks lying on the largest surfaces.

5. How many square inches in one end? one edge? one of its largest surfaces?

6. In locating the foundation of the house on lot L (Fig. 20), in what direction would you wish the line of the front foundation wall to run with reference to the Market street line?

7. Point out some of the square corners on the plans in Fig. 22.

8. Can you point out any corners that are not square?

WRITTEN WORK

The owner of lot L, which fronts on Market street, builds the house whose foundation plans are given in Fig. 22 and whose first and second floor plans are given in Fig. 23.

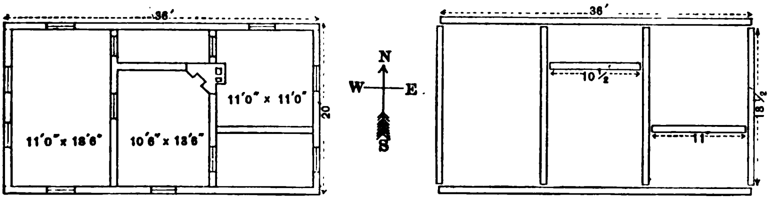


FIGURE 22

1. What will it cost to excavate a rectangle 21′ x 36′ to a depth of 6 ft., for the foundations and cellar, at 20¢ per cubic yard?

Think of the foundation as made up of straight walls such as are shown in the second part of Fig. 22. The mark (′) means *foot* or *feet*, and (″) means *inch* or *inches*.

2. The foundation is inclosed by 2 side walls, each 36′ long, and 2 end walls, each 18½′ long. Point out these walls in both parts of Fig. 22. The foundation walls are all 8′ high. Find the area of the north surface of the north side wall.

Ans. 288 sq. ft.

3. What other wall has an outer surface equal to the surface mentioned in problem 2?

4. Find the area of the outer surface of the east end wall.
Ans. 148 sq. ft.

5. What other outside surface equals this one in area?

6. Masons reckon that it takes 14 bricks for each square foot of outer surface to lay a solid 2-brick wall. If all walls are 2-brick walls, and solid, how many bricks will be needed for the north foundation wall? for the east wall? the south? the west?
1st *Ans.* 4032; 2d *Ans.* 2072.

7. How many bricks will be needed for all four of the outside walls?

8. There are two inside foundation walls each $18\frac{1}{2}$ ft. long, also one $10\frac{1}{2}$ ft. long, and one 11 ft. long. All are 2-brick walls 8 ft. high. If these walls contain no openings, how many bricks will be needed for the inside foundation walls? *Ans.* 6552.

9. The outside walls contain 8 window openings each $1\frac{1}{2}$ ft. by 3 ft. If masons allow for one-half the area of all openings in computing (finding) the number of bricks, how many bricks should be deducted (subtracted) for the outside walls? *Ans.* 252.

10. The inside walls contain 4 door openings each $2\frac{1}{2}$ ft. by 3 ft. How many bricks should be deducted for the inside walls?
Ans. 560.

11. Find the total number of bricks needed for the foundation walls and their cost at $9 per M.*

12. If hauling costs 75¢ per load of $1\frac{1}{2}$ T. and each brick weighs 6 lb., find the cost of hauling the brick for the foundations.†

13. If the mortar costs $1.25 per M. bricks, what will be the cost of the mortar? (Use 18 M.)

14. Four brick piers 2 ft. by 2 ft. and 4 ft. high support the porch columns. How much will the bricks needed for these columns cost at $12 per M., counting $22\frac{1}{2}$ bricks for each cubic foot? (Use here the nearest half thousand, *i.e.* $1\frac{1}{2}$ M.)

15. While the house was building the owner decided to replace weather-boarding by stained shingles on a belt running around the house (which is $30' \times 36'$) and extending to a distance of 9 ft. below the eaves. Each square yard, thus changed, cost $1.75

* In computing the cost of brick use the nearest whole thousand.
† When the last load is fractional, the price for a full load is charged.

extra, no allowance being made for openings. How much does this change add to the cost of the house?

16. Hardwood floors were decided upon later to take the place of pine floors in the dining-room and front hall. This increased the price by 12¢ per sq. ft. How much did this add to the cost of the house, counting the dining-room full 14′ 6″ wide and making no allowance for fireplace in hall. *Ans.* \$51.54.

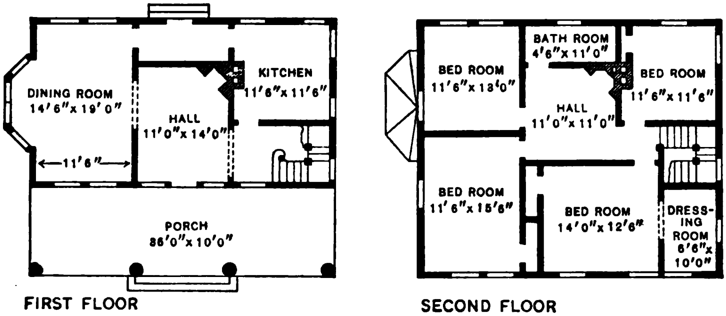


FIGURE 23

17. A room is said to be well lighted when the floor area is not more than 6 times the area of the window surface which admits light. If all windows as shown in the plan are $2\frac{1}{2}$ ft. wide and 6 ft. high, and $\frac{1}{3}$ of the window space is covered by the sash, is the dining-room well lighted? is the kitchen? the hall?

18. Is your schoolroom well lighted? Are the halls of your schoolhouse well lighted?

19. A hall 6 ft. by 8 ft. is lighted by a pane of glass $2\frac{1}{2}$ ft. by 4 ft. in the upper part of a door. Is the hall well lighted?

20. The assembly room of a church is 40 ft. by 60 ft. and is lighted by 18 windows, each containing 24 panes of glass, 1 ft. by $1\frac{1}{2}$ ft., and by a large front window containing 40 such panes. Is the room well lighted?

21. A room 12 ft. by 15 ft. has 3 windows, 2 containing 2 panes of glass each, the panes being $1\frac{2}{3}$ ft. by 3 ft., and 1 containing 2 panes of glass 3 ft. by $4\frac{1}{2}$ ft. The room also has a door $2\frac{1}{2}$ ft. by 7 ft. which admits light when open. Is the room well lighted if the window blinds are up and the door is open? Is it well lighted if the blinds are up and the door closed? If the blinds are half-way up and the door open? If the blinds are $\frac{3}{4}$ of the way up and the door closed?

§61. Applications of Cancellation.

In all the problems of this list indicate your divisions and multiplications, then apply the tests of divisibility, and cancel the factors found in both dividend and divisor. Multiply the uncanceled factors in the dividend together and divide this product by the product of the uncanceled factors of the divisor.

ILLUSTRATION.—A speed of 102 mi. per hour equals how many feet per second? Indicate the work thus:

$$\frac{102 \times 528\cancel{0}}{60 \times 6\cancel{0}} = \frac{\overset{34}{\cancel{102}} \times \overset{22}{\cancel{88}}}{\underset{\underset{5}{\cancel{20}}}{\cancel{60}}} = \frac{34 \times 22}{5} = \frac{748}{5} = 149\tfrac{3}{5}.$$

Ans. $149\frac{3}{5}$ ft. per second.

1. It took 225,280 lb. of steel rails to lay 1 mi. of single-track railroad. What was the average weight per yard of the rails?

	LB. PER CU. FT.	CU. FT. PER T.
Granite	170	
Limestone ..	160	
Marble	170	
Sandstone ..	140	
Slate	170	
Cast Iron ...	450	
Steel	480	

2. Find the number of cubic feet in a ton of 2000 lb. for each of the substances given in the table.

3. The average speed of American express trains is about 35 mi. per hour for long distances. How many feet per second is this?

NOTE.—60 sec. = 1 min.; 60 min. = 1 hour.

SOLUTION.—Put work in this form:

$$\frac{35 \times 528\cancel{0}}{60 \times 6\cancel{0}} = \frac{\overset{7}{\cancel{35}} \times \overset{22}{\cancel{88}}}{\underset{\underset{3}{\cancel{12}}}{\cancel{60}}} = \frac{154}{3} = 51\tfrac{1}{3}.$$ *Ans.* $51\frac{1}{3}$ ft. per second.

4. In both England and America, the average speed of express trains for distances from 100 to 250 mi. is about 40 mi. per hour. How many feet per second is this?

5. For long distances the average speed of English express trains is about 43 mi. per hour. How many feet per second is this?

6. In 1893 an express train in the United States ran 1 mi. at the rate of 98 mi. per hour. How many feet per second is this?

7. A railroad train ran for 1 min. at the speed of 130 mi. per hour. Find the number of feet per second which it traveled.

8. Sound travels in air at the rate of about 750 mi. per hour. How many feet per second is this?

9. Cannon balls have been thrown at a speed of 810 mi. per hour for a few seconds. This is how many feet per second?

10. The moon moves around the earth at a speed of about 1,840,000 mi. in 30 da. How many feet is this per second?

NOTE.—24 hr. = 1 day.

11. In 365 da. the earth moves around the sun through a distance of about 584,000,000 mi. What is the earth's speed in miles per second?

12. The earth, by turning on its axis, carries a place on its equator about 25,000 mi. in 24 hours. How many feet is it carried per second?

13. A farmer bought 18 bu. of wheat at 75¢ and paid the bill with potatoes at 50¢ per bushel. How many bushels of potatoes were required?

14. A farmer bought 80 acres of land at $100 per acre and paid for it with land worth $60 per acre. How many acres were required?

15. A man buys 5 acres of city land at $720 per acre and pays for it with 40 acres of farm land. What price per acre does the farm land bring?

16. How many bricks each 2″ × 4″ × 8″ are there in a straight pile 12″ × 36″ × 20′? In a straight pile 2′ × 6′ × 12′?

17. How many lots each 25′ × 150′ can be made from a town block 250′ × 300′?

§62. Farm Products of the United States in 1900.

In these problems use the tests for divisibility (§57) to discover common factors in divisor and dividend, and then remove these factors by cancellation. The numbers that remain in dividend and divisor are then to be divided in the usual way. Omit fractions in the quotients.

1. There were 512 coffee farms in the United States in 1900,

having a total area of about 70,200 acres. Find the average size of a coffee farm.

SUGGESTION.—Test for the factor 8 in dividend and divisor.

2. The total value of these 512 coffee farms was about $1,933,000. What was the average value of a farm?

3. If the total acreage (number of acres) of the coffee farms was about 70,200, and their total value about $1,933,000, what was the average value of an acre of coffee land in the United States?

SUGGESTION.—Test first for the factor 100 and then for 2.

4. There were 441 farms in the United States devoted to raising a certain food plant, having a total acreage of about 18,900. Find the average size of one of these farms.

SUGGESTION.—Test first for 9, then remove the common factor 7 from both dividend and divisor.

5. The total value of these 441 farms was about $562,500. Find the average value of one of these farms.

6. If the total acreage in these farms was 18,900 and the total value $562,500, find the average value of an acre for raising this plant.

7. There were in our country about 5,720 rice farms, having a total acreage of about 108,800 and a total value of about $17,834,000. Find the average acreage and the average value of a rice farm, also the average value of an acre for raising rice.

8. About 2,025 nursery farms had a total acreage of about 165,800 and a total value of $19,146,000. What were the average acreage, the average value of a nursery farm and the average value of land per acre for nursery purposes?

9. For sugar farms these numbers were nearly correct: 7,340 farms, of 2,669,000 acres, worth $150,420,000. Solve problems like 7 and 8 for sugar farms.

10. The numbers for farms devoted to raising flowers and plants were about 6,160 farms, of 42,660 acres, worth $52,460,000. Solve problems like 7 and 8 for flower- and plant-raising farms.

11. From these numbers, solve like problems for fruit farms: number of farms 82,200; total acreage, 6,150,000; total value, $440,000,000.

12. Further problems may be made on this table, which is a fairly complete list of the farm products of the country.

PRODUCT	NUMBER OF FARMS	ACRES IN FARMS		VALUE OF FARMS	
		Total	Average	Total	Average
Tobacco	106,000	9,570,000		$ 215,500,000	
Vegetables	186,000	10,160,000		547,000,000	
Dairy produce.	358,000	43,300,000		1,693,500,000	
Cotton	1,072,000	89,600,000		1,108,000,000	
Hay and grain.	1,320,000	210,260,000		6,380,000,000	
Live stock	1,565,000	355,000,000		7,505,300,000	
Miscellaneous..	1,060,000	113,200,000		2,383,700,000	

§63. **The Thermometer.**—The official record of hourly temperatures for Chicago from 6 p.m. February 27 to 6 p.m. March 1, 1902, as given in a daily paper, was as follows:

FEB. 27; NIGHT	FEB. 28; DAY	FEB. 28; NIGHT	MARCH 1; DAY
6 p. m. ......38°	6 a. m. ......40°	6 p. m. ......39°	6 a. m. ......34°
7 p. m. 38	7 a. m. 41	7 p. m. 38	7 a. m. 34
8 p. m. 37	8 a. m. 41	8 p. m. 37	8 a. m. 33
9 p. m. 37	9 a. m. 41	9 p. m. 37	9 a. m. 33
10 p. m. 39	10 a. m. 43	10 p. m. 37	10 a. m. 32
11 p. m. 39	11 a. m. 44	11 p. m. 37	11 a. m. 32
12 midnight ..38	12 m. 38	12 midnight ..36	12 m. 32
Average......	Average......	1 a. m. 36	1 p. m. 31
12 midnight ..38	12 m. 38	2 a. m. 35	2 p. m. 31
1 a. m. 38	1 p. m. 35	3 a. m. 34	3 p. m. 30
2 a. m. 38	2 p. m. 35	4 a. m. 34	4 p. m. 29
3 a. m. 38	3 p. m. 37	5 a. m. 34	5 p. m. 28
4 a. m. 37	4 p. m. 39	6 a. m. 34	6 p. m. 28
5 a. m. 39	5 p. m. 40	Sum	Sum ....
6 a. m. 40	6 p. m. 39	Average	Average......
Average......	Average		

NOTE.—The *average* of temperatures, or numbers, is found by adding them and dividing the sum by the number of temperatures, or numbers, which have been added. *To average* means to find the average.

1. Average the temperatures on Feb. 27 from 6 p.m. to midnight; from midnight to 6 a.m.

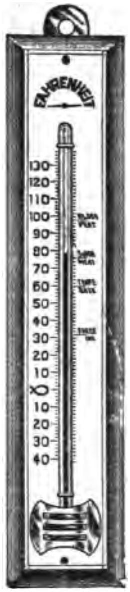


FIGURE 24

2. What was the average temperature on Feb. 28 from 6 a.m. to 12 m.? from 12 m. to 6 p.m.?

3. What is the difference between the two averages of problem 1? of problem 2? What do these differences mean?

4. What is the difference between the lowest and highest temperatures from 6 p.m. Feb. 27 to 6 p.m. Feb. 28? This is called the *range* of temperature for the day.

5. The average temperature for Feb. for 30 yr. is 26°. How much does the average temperature for the 24 hr. following 6 p.m. Feb. 27 exceed the thirty-year average?

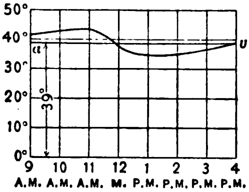
6. Find the average temperature for the night of Feb. 28; for the day of March 1.

7. Find the difference between these averages. What does this difference show?

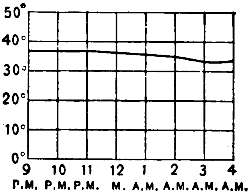
8. Make and solve such problems from outdoor thermometer readings at your schoolhouse.

9. What is the reading of the thermometer of Fig. 24? What would the thermometer read if the top of the mercury column stood at "freezing"? at "summer heat"? at "blood heat"?

10. Average the thermometer readings taken at your own school for each hour from 9 a.m. to 4 p.m. for a month, and keep in a notebook a record of your readings and averages.



February 28
FIGURE 25



February 28—March 1
FIGURE 26

11. Lay off on cross-lined paper the readings of problem 10 as the readings of the table from 9 a.m. to 4 p.m. of Feb. 28 are laid off in Fig. 25; draw a smooth curve, free-hand, through the points. Such a curve gives a picture of temperature changes.

12. From a daily paper obtain the hourly readings from 9 p.m. to 4 a.m. and lay them off to scale as the readings of the table from 9 p.m. Feb. 28 to 4 a.m. March 1 are laid off in Fig. 26. Draw the free-hand curve as before and find whether the day or night temperatures are the *steadier* (more regular).

13. Average your readings from 9 a.m. to 4 p.m. and draw a straight, horizontal line on your diagram, at a distance above the horizontal zero line equal to the average, as the line *av* of Fig. 25 is drawn. This is the *average line* for these 8 hours.

EXERCISES FOR PRACTICE

Check results in division by multiplying divisor by quotient and adding the remainder if there is one.

SHORT AND LONG DIVISION

1. Divide 1260 by 2; by 3; by 6; by 5; by 7; by 8; by 10.
2. Divide 2880 by 3; by 4; by 5; by 6; by 9; by 10; by 12.
3. Divide 3402 by 3; by 7; by 21; by 4; by 12; by 28.
4. Divide 2520 by 4; by 5; by 20; by 6; by 7; by 30; by 8.
5. Divide 33264 by 6; by 7; by 8; by 42; by 56; by 48; by 9.
6. Divide 50400 by 10; by 30; by 40; by 50; by 60; by 90.
7. Divide 201600 by 200; by 300; by 400; by 500; by 600; by 700; by 900.
8. Divide 3360 by 7; by 8; by 56.
9. Divide 2880 by 8; by 9; by 72.
10. Divide 57672 by 6; by 2; by 12; by 72.
11. How may you divide a number by 35 by short division? by 56? by 63? by 80? by 72? by 54? by 48? by 84? by 96?

LONG DIVISION

12. Divide 360360 by 120; by 210; by 336; by 416; by 1248.
13. Divide 5045040 by 42; by 105; by 315; by 540; by 702; by 862; by 1404.
14. Divide 6387624 by 81; by 320; by 576; by 736; by 826.
15. Divide 8736824 by 632; by 8261; by 42125; by 143216.
16. Divide 30200610 by 700; by 1764; by 1908; by 36821.
17. Divide 23642901 by 21; by 86; by 364; by 4081; by 19881; by 98631.
18. Divide 38083584 by 672; by 816; by 2688; by 8064.

BILLS AND ACCOUNTS

§64. **Exercises.**

	Mon.	Tu.	Wed.	Th.	Fri.	Sat.	Wages	
	hours	hours	hours	hours	hours	hours	$	cts.
Adams............	7⅓	8	8	8⅓	9	8¾		
Benson............	8	8½	8¼	8¾	8½	9		
Boyd..............	0	4	6	6	6½	7		
Claussen	8	8⅓	8½	8⅓	9	8⅓		
Denning	8⅓	9	8½	8¼	9	9		
Doan..............	8	8	8	8	8	8		

FIGURE 27

1. Find the number of hours each man worked.
2. Find the number of hours all the men worked.
3. At 30¢ an hour, find the daily wages of each man.
4. Find the amount due each man at the close of the week.
5. Find the amount due to the six men at the close of the week.
6. Which man worked the greatest number of hours? the smallest number?
7. Counting 9 hr. to the day, what is the greatest amount any one man could earn in a week?
8. Counting 8 hr. to the day, how much "overtime" did each man work?
9. For how many extra hours did five of the men work, counting 8 hours to the day?
10. If double wages were paid for "overtime," how much did each man earn during the week? How much did all earn?
11. How many hours did the third man lose?

§65. ORIGINAL PROBLEMS

1. Adams paid $4 on a doctor's bill, and put aside $2 for rent.
2. Benson has a wife and two children. Average amount for each one in the family.

3. How much does Boyd's loss of time reduce his wages for the week?

4. Claussen paid $10 for clothing, and $3.50 for the week's board.

5. Denning saves $3 a week to pay the premium (yearly charge) on a life insurance policy, pays $3.50 a week for board, and 35¢ for laundry.

§66. Accounts.

A man has on hand at the beginning of the month of February, 1902, $250. He receives during the month $200 for salary and $60 for the rent of two houses. During the month he pays $72.50 on an insurance policy, $110 for household expenses, $25.50 for incidentals, and $5 for books and stationery. His account book shows the following entries:

Dr. (*Receipts*) CASH (*Expenditures*) *Cr.*

1902				1902			
Feb. 1	On Hand	$250	00	Feb. 1	By Insurance	$ 72	50
" 15	To Salary..........	200	00	" 28	" Household Exp.	110	00
" 27	" Rent	60	00		" Incidental "	25	50
					" Books and Stat'y	5	00
					" Balance	297	00
		510	00			510	00
" 28	On Hand	297	00				

1. With what items is cash the man's debtor? With what items is cash his creditor?

2. During the month, how much money did the man receive? How much money did he actually possess between February 1 and 28?

3. How much was spent? What is the difference between receipts and expenditures? How much more was spent than was received as salary?

Draw forms and prepare cash accounts for the following:

4. March, 1902. On hand first of the month $1000. March 5, sold land for $1500; received on the 15th, $150 for rent; and sold a team for $225 on the 25th. Bought real estate on the 8th for $1000; paid $65.25 for repairs on the 15th; taxes on the 16th

amounted to $17.50; household expenses for the month amounted to $100, and personal expenses to $50.

5. June, 1902. A boy's receipts and expenditures were as follows: Received 25¢ for mowing the lawn on the 1st; 50¢ a day for running errands on the 4th and 5th; 25¢ for mowing the lawn again on the 15th. On the 17th he paid $1.25 for a hat. The same day he earned 50¢ for delivering packages for the grocer.

6. July, 1902. A farmer sold on the 13th two loads of hay at $20 a load; 4 hogs weighing in all 1000 lb. at 6¢; 15 lb. butter at 20¢. On the 13th he bought 26 lb. granulated sugar at 6¢; 3 lb. coffee at 35¢; and dry goods to the amount of $15. On the 26th he sold vegetables to the amount of $1.25, and 9 doz. eggs at 10¢. The same day he bought clothing to the amount of $10.

7. Find cost of each purchase, also the amount due, on the bill below:

E. S. WICKWIRE,

In account with MANNHEIMER BROS., *Dr.*

1902					
Jan. 4	To 1 Rug .... at $75.50				
" 4	" 1 Overcoat........... " 35.00				
" 12	" 7 yd. Dress Goods .... " 2.00				
" 14	" $3\frac{1}{8}$ " Silk............. " 1.50				
	Amount due	......	......		

Received payment,

Jan. 31, 1902. MANNHEIMER BROS.

Draw forms and prepare similar bills:

8. Edward Morris in account with J. W. Bowlby, Dr., St. Paul, Minn., May 30, 1902. 1 doz. collars at 25¢ a piece; $\frac{1}{2}$ doz. pr. cuffs at 25¢ a pair; 2 neckties at 50¢; 3 shirts at $1.25; 1 hat at $3.50; 1 pr. shoes at $3.50; 1 pr. gloves at $1.25. Paid, May 30.

9. A. H. Simons bought of Yerxa Bros., Chicago, June 6, 1902, 1 sack appleblossom flour at $1.30; 8 lb. ham at 17¢; 15 lb. granulated sugar at 6¢; $\frac{1}{2}$ lb. oolong tea at $1.60; 3 lb. Mocha and Java coffee at 40¢; 1 bunch of celery at 30¢. Paid in full the same day.

10. Edward Ryan bought of M. J. Doran, February 7, 1902, 2 cords of hard wood at $8.75; 3 T. of hard coal at $8.25; 2 cd. of pine slabs at $3.25.

11. May 4, 1902, Alvin Johnson bought of the John Martin Lumber Co., cash payment, 1000 cedar shingles at $3 per M; 1 bbl. of lime at 68¢; 1000 ft. of pine flooring at 55¢ per M; 3000 laths at $1.50; 15 lb. shingle nails at 7¢.

Accounts are not always settled in full at the close of the month. When a portion of the amount due is carried over to the next month, this item appears as the first item on the bill, under the title "accounts rendered."

12. Find amount of this bill:

J. FIRESTONE,

To ALLEN, MOON & CO., *Dr.*

1902					
Feb. 1	To Acc't rendered	......	......	24	85
" 10	" 2 Sacks of Flour....... at $2.60				
" 10	" 25 lb. Ham................ .18				
" 10	" 4 doz. Eggs............... .22				
" 10	" 6 lb. Butter............... .25				
" 15	" 25 lb. Granulated Sugar... .06				
" 15	" Vegetables.................5.00				
	By Cash........................				

13. A. L. Parker, in account with Marshall Field, lacks $12.75 of paying the entire bill of February, 1902. His March purchases were as follows: March 5, $\frac{1}{2}$ doz. towels, @ 50¢ a piece; 2 table cloths, @ $6.50; 1 doz. napkins, @ $5.50 a doz.; 2 bed spreads @ $2.50; $\frac{1}{2}$ doz. sheets @ 75¢ a piece; $\frac{1}{2}$ doz. pillow cases @ 20¢ a piece; 7 yd. dress goods @ $1.50; findings, $5.25. Paid in full March 28.

14. Edward D. Young in account with F. W. Salisbury, wood and coal dealer, has left over from a previous account $8.25. On the 3d of January, 1902, he bought 2 T. of hard coal @ $8.25; 1 cd. of hard maple wood, $8.50; 1 cd. of pine slabs, $3.50. Paid the whole amount January 29.

15. On Nov. 15, I purchased 1 can lima beans @ 12¢; 1 can peas @ 12¢; 1 qt. cranberries @ 12¢; 4 lb. pork chops @ 12¢; $4\frac{1}{2}$ lb. chicken @ 12¢; 2 lb. butter @ 32¢, and paid the account with a $5.00 bill. What change was due me?

§67. Problems of the Grocery Clerk.—A grocery boy made the following transactions (sales and purchases), including the filling of the orders, and returned the correct change without an error all in one hour. What change did he return to each customer?

1. The first customer bought

Celery	$0.05	Potatoes	$0.15
Eggs	.22	Sugar..10 lb. @ $4\frac{1}{2}$¢.	

and paid the clerk with a $1.00 bill.

2. The second customer bought

Salt	$0.10	Apples	$0.25
Flour	.55	Breakfast food	.13
Yeast	.02	Rice..3 lb. @ $8\frac{1}{3}$¢.	
Milk	.07		

and he paid with $1.50.

3. The third customer bought

Tomatoes	$0.20	Beans......4 qt. @ 7¢.	
Soap	.25	Tea........2 lb. @ 60¢.	
Butter..3 lb. @ 27¢.		Cauliflower..3 heads @ 18¢.	
Coffee..3 lb. @ $33\frac{1}{3}$¢.			

and paid with a $5.00 bill.

4. The fourth customer bought

Eggs	$0.22	Lampwicks	$0.10
Bacon	.18	Cheese	.16
Peas	.13		

and paid with a $5.00 bill.

5. The fifth customer bought

Cornmeal	$0.25	Melon	$0.25
Sweet potatoes	.27	Pineapple	.20
Candy	.10	Lard..3 lb. @ $13\frac{1}{3}$¢.	
Grapes	.18		

and paid with a $2 bill.

6. The sixth customer was a farmer, who sold to the grocer

Eggs......6 doz. @ 18¢.	Tomatoes...2 bu. @ 75¢.
Butter.....15 lb. @ 22¢.	Sugar corn..6 doz. @ 10¢.
Potatoes....5 bu. @ 35¢.	

and bought of the grocer

Sugar......20 lb. @ 4½¢.	Tea.........2 lb. @ 55¢.
Coffee......3 lb. @ 25¢.	Flour......1 sack, $1.15
Cheese......2 lb. @ 18¢.	Rice.........6 lb. @ 8⅓¢.

How much money should the grocer pay the farmer to balance the account?

7. A grocer's daily sales for 4 weeks are given below. Without rewriting the numbers find the total sales (*a*) for each week; (*b*) for the 4 weeks; (*c*) the average daily sales for the month.

MON.	TUES.	WED.	THUR.	FRI.	SAT.	WEEKLY TOTALS
$28.75	$37.25	$35.18	$68.12	$20.13	$86.58	....
18.62	9.68	21.83	40.28	37.60	75.76	....
30.18	29.95	23.61	9.61	10.84	68.94	....
19.27	39.13	28.16	38.19	5.63	98.56	....

§68. Family Expense Account.

Foot the following problems as rapidly as you can work accurately.

1. The household expenditures (money spent) of a family are here given for each of the first 7 da. of Oct., 1900. Find the daily expenditures and the total expenditure for the week.

Oct. 1.		Oct. 2.		Oct. 3.	
Oatmeal......	$0.11	Laundry......	$0.48	Bread	$0.15
Meat	.18	Sweet potatoes	.18	Baking powder	.25
Peaches	.25	Apples	.25	Cheese.......	.10
Grapes........	.13	Bread.	.10	Macaroni	.15
Dried beef....	.10	Meat.........	.20	Bananas	.15
Tea..........	.30	Sundries	.63	Honey.	.20
Bread........	.20	Bread	.20	Peanuts	.05
Celery	.05	Crackers	.15	Tomatoes	.15
Soap.	.25		——	Shoe polish ..	.10
	——	Total....		Car fare......	.10
Total....				Onions	.10
				Total....	

DATE	APR.	MAY	JUNE	JULY	AUG.	SEP.
1	$ 52.86	$ 53.48	$ 56.75	$ 56.73	$ 55.10	$ .65
2	3.87	2.50	.67	1.28	2.50	54.10
3	3.68	2.18	1.20	2.10	6.82	4.06
4	6.36	1.28	.85	3.50	.04	.60
5	1.86	.25	.75	4.05	.00	2.75
6	2.15	8.12	3.80	1.10	1.28	.25
7	.75	3.62	2.37	.05	.75	1.52
8	1.95	.67	2.12	.12	4.25	.68
9	3.28	4.10	.55	.15	1.18	2.12
10	2.18	.10	.96	.35	2.60	.39
11	4.10	1.01	2.84	3.60	.00	1.86
12	2.15	.86	3.69	.10	3.10	3.24
13	1.00	1.52	2.50	2.18	.69	3.27
14	.10	.78	.55	.00	1.58	1.68
15	1.61	2.58	1.86	1.57	3.82	.55
16	3.21	3.65	1.68	.76	.55	1.68
17	.57	.25	2.75	.56	6.53	2.17
18	3.76	2.51	.99	3.65	.28	.69
19	.15	.86	6.24	.05	.12	3.00
20	4.25	1.76	3.26	8.29	.17	.12
21	.86	2.08	.25	.00	2.16	6.81
22	.95	1.15	4.86	.00	2.60	.05
23	.87	3.12	.28	1.68	.05	.10
24	1.16	1.28	.83	1.00	8.16	7.06
25	3.15	3.75	1.96	2.28	.15	.19
26	1.82	.15	1.76	.05	.08	2.60
27	2.78	1.67	3.75	6.28	1.20	2.97
28	.60	2.18	1.28	.00	2.00	3.60
29	3.25	1.01	8.14	3.16	3.60	.10
30	.96	3.87	1.00	2.15	.25	2.65
31	.00	.75	.00	1.58	1.68	.00
Monthly Totals						
					TOTAL	

6. Find the daily average per month for each of the 6 months.

7. Find the monthly average for 6 months from April to September.

8. Find the monthly average for the whole year.

9. Find by adding horizontally the total expenditure for the first day of these 6 months; for the second day.

§69. The Equation.

1. y lb. of sugar are placed on the left scale pan, and a weight of 10 lb. on the right pan balances it (see Fig. 28). How heavy is y?

The balance of these two weights is expressed thus: $y = 10$. (I)

This expression is called an *equation*, and is read "y equals 10."

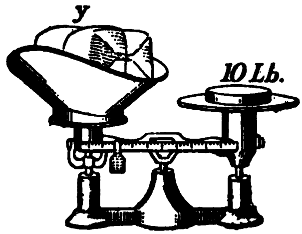


FIGURE 28

2. If a 4 lb. weight is now added to the right pan, how many additional pounds of sugar must be placed upon the left pan to balance the scales? Write an equation to express the relation between the weights now in the pans.

3. If x lb. on the right balance y lb. on the left, how would you state the fact in an equation?

4. If 20 lb. are added to the x lb. already on the right, how many pounds must be added on the left to balance the equation?

5. If 35 is added on the right side of equation (I), what change must be made on the left side to balance the equation? Write the equation thus changed.

Just as the horizontal position of the scale beam shows that there is a balance of the weights on the pans, so the equality sign, $=$, shows that there is a *balance of value* of the numbers between which it stands. It must never be used between two numbers, or combinations of numbers, that are not equal in value.

The number on the left of the sign of equality is called the *first member*, or the *left side* of the equation. The number on the right is called the *second member*, or the *right side* of the equation.

6. Four equal weights and a 5 lb. weight just balance 21 lb. (Fig. 28.). How heavy is one of the equal weights?

SOLUTION.— $4x + 5 = 21$. Subtract 5 from both sides

$$\begin{array}{r} 4x + 5 = 21 \\ 5 \quad\; 5 \\ \hline 4x \quad\; = 16 \end{array}$$

$4x = 16$. If $4x = 16$ what is x? What is the answer to the problem?

7. $3y + 2 = 14$; find y. 8. $5x + 12 = 32$; find x. 9. $7x + 8 = 29$; find x. 10. $9z + 3 = 75$; find z. 11. $15a + 3 = 48$; find a.

CONSTRUCTIVE GEOMETRY

§70. Problems with Ruler and Compass.

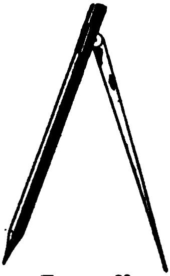


FIGURE 29

Problems I. to XI. are to be solved with ruler and compass. Keep all the pencil points sharp while drawing and work carefully.

The simple instrument shown in Fig. 29 is a form of the compass which will do for these problems.

The pencil point of the compass will be called the pencil foot, or pen foot. The other point will be called the pin foot.

PROBLEM I.—Draw a circle with $\frac{1}{2}$ inch radius.

EXPLANATION.—First Step: Place the pin foot on an inch mark of your foot rule and spread the compass feet apart until the pencil foot just reaches to the next half inch mark.

Second Step: Without changing the distance between the compass feet, put the pin foot down at some point, as *A*, Fig. 30, of your paper and, with the pencil foot, draw a curve entirely round the point *A*.

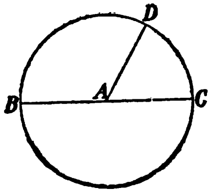


FIGURE 30

A curve drawn in this manner is called a *circle*. How far is it from *A* to any point of the curve?

Any part of the whole circle, as the part from *C* to *D*, or from *D* to *B* is an *arc* of the circle.

The point *A*, where the pin foot stood, is the *center*.

The distance straight across from *B*, through *A* to *C*, is a *diameter*. *BC* is a diameter.

The distance from *A* straight out to the circle is a *radius*. The plural is radii (rā′-dĭ-ī). *AC*, *AB* and *AD* are all radii.

What part of the diameter equals the radius?

EXERCISES

1. Draw circles with these radii:

$\frac{3}{4}''$; $1''$; $1\frac{1}{4}''$; $1\frac{1}{2}''$; $1\frac{7}{8}''$; $2''$.

2. Draw circles with the same center *A*, and with these radii:

$\frac{1}{4}''$; $\frac{1}{2}''$; $\frac{3}{4}''$; 1; $1\frac{1}{4}''$.

Circles whose centers are all at the same point are called *concentric* circles.

PROBLEM II.—Draw (or lay off) a line equal to a given line.

EXPLANATION.—Let the given line AB (Fig. 31) have the length a units.

With ruler and pencil draw any straight line, as CX, longer than a.

Place the pin foot on A and spread the compass feet until the pencil foot just reaches to B. Without changing the distance between the feet, put the pin foot on C and with the pencil foot draw a short arc across the line CX as at D.

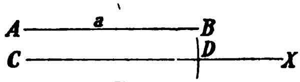


FIGURE 31

Then CD is the desired line; for if we call its length x, we have made $x = a$.

EXERCISES

1. Draw lines equal in length to these given lines:

2. Draw lines having these lengths:

1″; $1\frac{1}{2}$″; $2\frac{1}{2}$″; $2\frac{3}{4}$″; $3\frac{7}{8}$″.

PROBLEM III.—Draw a line equal to the sum of two or more given lines.

EXPLANATION.—Let the two given lines be a and b, Fig. 32. First step: Draw the indefinite line CX longer than the combined length of a and b, and make CD equal to a as in Problem II.

Second Step: Spread the compass feet apart as far as the length of b. Then put the pin foot on the crossing point (intersection) of the arc and line at D and draw another short arc at E. How long is DE? How long is CE?

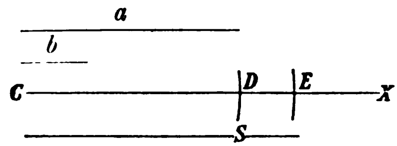


FIGURE 32

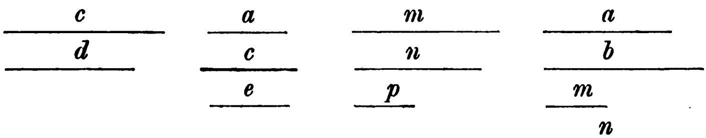
CE is the desired sum. If we call its length s, we may write this equation:

$$\text{(I.)} \quad s = a + b.$$

With ruler and compass how can you find a line equal to the sum of 3 given lines? of 4? of any number of given lines? This is called constructing the sum of lines.

EXERCISES

1. Construct the sum of the lines in each column, denote the constructed line by s, and write an equation like (I.) for each case:



2. Construct these sums:

$$1'' + \tfrac{3}{4}'' + \tfrac{1}{2}'' + \tfrac{3}{8}''; \quad 2'' + \tfrac{1}{8}'' + \tfrac{3}{4}'' + \tfrac{7}{8}''; \quad \tfrac{1}{4}'' + 3\tfrac{1}{2}'' + \tfrac{1}{2}'' + \tfrac{1}{4}''.$$

Problem IV.—Draw a line equal to the difference of two given lines.

Explanation.—Let the two given lines be *a* and *b*. First step: Draw the indefinite line *CX* longer than the minuend line *a*. As above, make the minuend *CD*, equal to *a*.

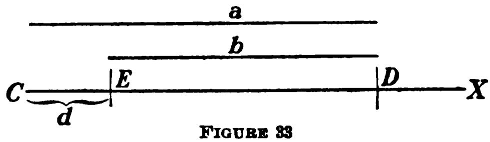


Figure 33

Second Step: Spread the compass points apart, as before, as far as the length of the subtrahend line *b*. This is called "taking *b* as a radius." Place the pin foot on *D*, swing the pencil foot back toward *C*, and draw the arc at *E* across the line *CX*. This makes *DE* how long? What line now equals the difference *d* between *a* and *b*? The equation for this case is:

$$\text{(II.)} \quad d = a - b.$$

EXERCISE

Construct the differences of these pairs of lines, call each difference *d*, point it out in the construction and write the equation for each case:

m	*c*	*f*	*a*
s	*n*	*g*	*a*

Problem V.—Draw a line equal to 2, 3, or 4 times a given line.

Explanation.—Let *a* be the given line. We are simply to draw a line equal to the sum of *a* and *a*. (See Problem III.)

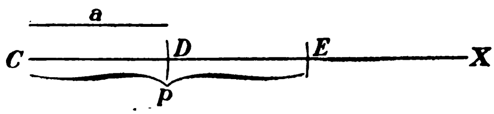


Figure 34

We may write

$$\text{(III.)} \quad p = 2a.$$

How would you construct $3a$? $4a$? $5a$? $7a$? $8x$? $8y$? $6z$?

Write and read the equation for each construction.

EXERCISE

Construct three times these lines and give their equations (like III., above):

a	*c*	*d*	$2a$	$3m$

Problem VI.—Divide a given line into two equal parts.

DEFINITION.—Dividing a line into two equal parts is called *bisecting* the line.

EXPLANATION.—Let the given line be AB, Fig. 35, call its length a. First step: Spread the compass feet apart a little farther than $\frac{1}{2}$ the length of AB. Place the pin foot first on A and bring the pencil foot, by estimate, somewhere above the middle of the line a, and draw the arc 1. Then carry the pencil foot down, by estimate, below the middle of a and draw the arc 2. These arcs should both be drawn long enough to make sure that arc 1 passes over arc 2, under the middle point.

Second step: Now change the pin foot to B, and without changing the distance between the feet, draw arc 3 cutting arc 1 and 4 cutting arc 2. Call the crossing points C and D.

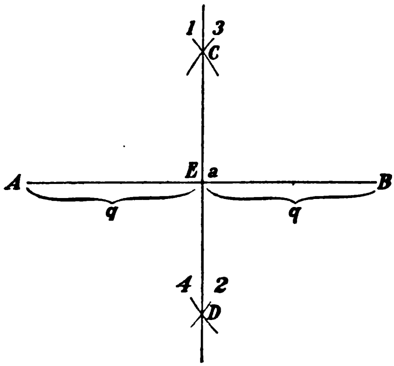


FIGURE 35

Third step: Place the edge of the ruler on C and D and draw the straight line CD, crossing a at E. Either AE or BE is equal to $\frac{1}{2}$ of a. Test by measuring with the compass.

The equation for this case is:

$$\text{(IV.)} \quad q = \tfrac{1}{2}a, \text{ or } q = \frac{a}{2}.$$

The first is read "q equals one-half a," meaning q equals $\frac{1}{2}$ of a, and the second is read "q equals a divided by 2." Do they, therefore, mean the same thing?

EXERCISES

1. Draw a line $\frac{3}{4}''$ long and bisect it.
2. Bisect lines of these lengths:

$\frac{7}{8}''$; $1\frac{1}{2}''$; $3''$; $5''$; $5\frac{1}{2}''$; $3\frac{1}{4}''$.

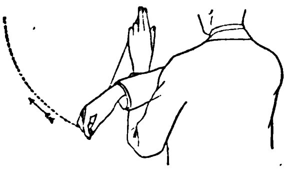


FIGURE 36

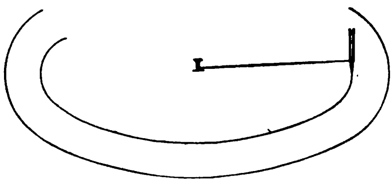


FIGURE 37

3. Longer lines may be bisected at the blackboard with crayon and string (Fig. 36), or on the ground with a cord and a sharp stake (Fig. 37).

PROBLEM VII.—Construct an equilateral (equal sided) triangle with each side equal to a given line.

EXPLANATION.—Let a Fig. 38 be the given line.

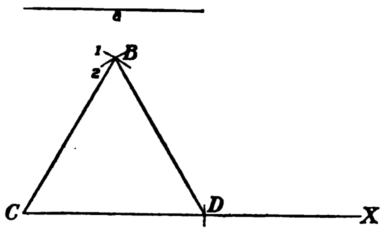


FIGURE 38

Draw CX longer than a and make $CD = a$, as in Problem II.

With length a between the compass feet, place the pin foot on C and draw arc 1, by estimate, above the middle of CD.

Without changing the distance between the compass feet, place the pin foot on D and with the pencil foot cut arc 1 with arc 2. Call B the intersection (crossing point) of arc 1 and arc 2. With the ruler draw a line from C to B and another from D to B. Then CDB is the desired equilateral triangle.

EXERCISES

1. Construct equilateral triangles having sides of these lengths:

b c d e

2. Construct equilateral triangles having these sides:

1″; 2″; 3″; 2½″; 4″.

3. With crayon and string construct these equilateral triangles on the blackboard:

6″; 1′; 14″; 18″; 24″.

4. With cord and nail or stake construct these equilateral triangles on the floor or ground:

6′; 10′; 18′; 1 rod; 30′.

PROBLEM VIII.—Construct an isosceles (ī-sŏs′-sē-lēs) triangle with the base and the two equal sides equal to given lines.

DEFINITION. — An isosceles triangle has at least two sides equal.

EXPLANATION.—Let b, Fig. 39, equal the base, and e, one of the equal sides.

Make $CD = b$ (Problem II). With C as center and radius equal to e draw arc 1. With the same radius and with D as center draw arc 2. Connect their intersection E with C and with D.

Then CDE is the desired isosceles triangle; for we made $CD = b$, $CE = e$ and $DE = b$.

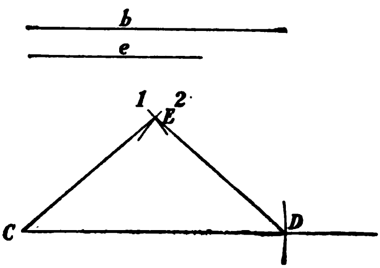


FIGURE 39

DEFINITION.—The side, CD, which is not equal to either of the other two sides is the *base*.

EXERCISES

1. The figures in Fig. 40 represent all forms of the triangle. What is a triangle?

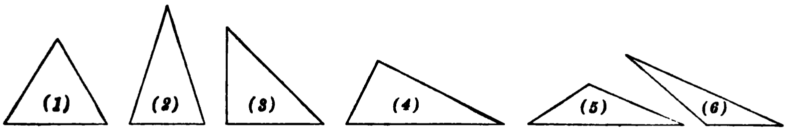


FIGURE 40

2. What triangle has all its sides equal? What is an *equilateral* triangle?

3. What triangles have at least 2 sides equal? What is an *isosceles* triangle?

4. What triangles have no two sides equal? What is a *scalene* (skā-leen′) triangle?

PROBLEM IX.—Draw a scalene triangle with sides equal to given lines (no two being equal).

EXPLANATION.—On the line CX make $CD = a$ as in Problem I. With C as center and b as radius, draw arc 1. Then with D as center and with c as radius, draw arc 2 across arc 1. Call their intersection E.

With the ruler draw line EC and ED.

Then CDE is a scalene triangle with the sides equal in length to a, b and c.

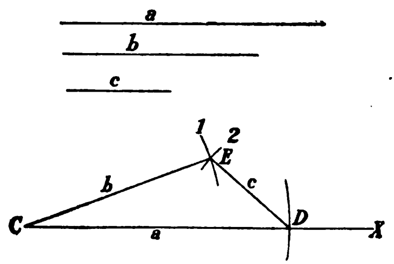


FIGURE 41

EXERCISES

1. Draw a scalene triangle having sides of these lengths:

a ______ *b* ________ *c* ______

2. Draw a scalene triangle having sides of 3″, 4″, and 5″; of 1″, $1\frac{1}{2}$″, and 2″.

PROBLEM X.—With the compass draw a three-lobed figure inside of a circle of $\frac{1}{2}$″ radius.

EXPLANATION.—Draw a circle with $\frac{1}{2}''$ radius around some point, as *O*, as a center. Set the pin foot at any point on the circle, as at *1*, Fig. 42, and without changing the distance between the compass feet draw the arc from *A* on the circle through *O* to *B* on the circle.

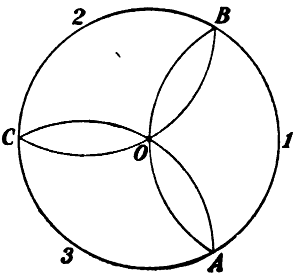


FIGURE 42

With same radius and with pin foot on *B*, draw a short arc at *2*.

Put the pin foot on *2* and with the same radius as before draw the arc *BOC*, *C* being on the circle first drawn.

Put the pin foot on *C* and draw a short arc at *3*. Then place the pin foot on *3* and draw an arc from *C* through *O* to *A*.

EXERCISES

1. Draw a three-lobed figure using a radius of $1''$.

2. Draw a three-lobed figure using as radius this line ________.

3. Making the distance between the compass points $\frac{1}{2}''$, and using the points 1, 2, 3, 4, 5, and 6 in turn, draw a six-lobed figure like Fig. 43.

4. Color the lobes of your figure with a red lead pencil or with water colors and the spaces *a*, *b*, *c*, *d*, *e*, and *f* with a green or yellow pencil or water colors.

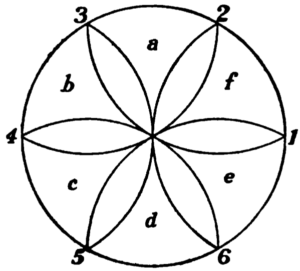


FIGURE 43

PROBLEM XI.—Draw a regular 6-sided figure (regular hexagon) within a circle of $\frac{1}{2}''$ radius.

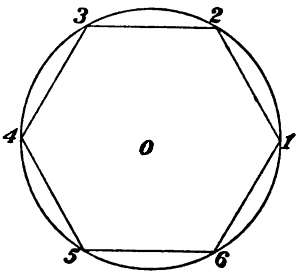


FIGURE 44

EXPLANATION. — Draw a circle with center at *O* and with $\frac{1}{2}''$ radius.

Starting at any point on the circle as at *1*, put the pin foot on *1* and draw the short arc *2* across the circle, keeping the radius $\frac{1}{2}''$. Then put the pin foot on *2* and draw the arc *3*. Then with pin foot on *3* draw arc *4*; and so on around.

How many steps do you find reach once round?

Now with the ruler and pencil connect *1* with *2*, then *2* with *3*, *3* with *4*, and finally *6* with *1*.

The figure made by the 6 straight lines is the regular hexagon desired.

EXERCISES

1. Draw regular hexagons within circles of these radii: 1″; 2″; $\frac{3}{4}$″; $2\frac{1}{2}$″.

2. How long is one side of each of the hexagons drawn in exercise 1? How long is the sum of all the bounding lines of each hexagon?

DEFINITION.—The sum of all the bounding lines of any figure is called the *perimeter* of the figure.

MEASUREMENT

§71. Measuring Value.

Money is the common measure of the value of all articles that are bought and sold.

The unit on which United States money is based is the *dollar*. The dollar sign is $; thus 5 dollars is written $5 or $5.00. This unit is called the U. S. *Standard of value.*

1. A dollar is worth as much as how many dimes? nickels? quarters? cents?

To measure the value of one amount of money by another is to find how many times one of the amounts is as large as the other.

2. Measure $1 by 50¢; by 25¢; by 20¢; by 40¢; by 75¢.

3. Measure $10.50 by 50¢; by 75¢; by $1.50; by $5.25.

4. A farm is worth $1000 and a city lot $2000. Measure the value of the farm by the value of the lot.

5. Measure the value of a $150 horse by the value of a $15 pig.

6. Measure $75 by $5; by $25; by $15; by $150; by $225.

7. Measure the value of 160 A. of land worth $75 per acre by $100; by $500; by $1000; by $8000; by $24,000.

8. Which is worth the more, 80 A. of land @ $75, or 50 A. @ $112? how much more?

9. Measure $400 by the value of an $8 calf; of a $25 colt.

§72. Measuring Length and Distance.

Answer these problems, first, by estimate, writing your estimates in a notebook; then answer by actual measurement. Finally, compare your estimates with the results of your measurements and tell how much your estimates differ from the measurements.

1. How wide is the page of this book? how long? How wide are the margins?

2. How wide is a pane of glass in your schoolroom window? how long is it?

3. How wide is the top of your desk? how long? How high is the top of your desk from the floor? How high is your seat?

4. How high is the bottom of the blackboard from the floor? How wide is your blackboard? how long is it?

5. How tall are you? How far can you reach by stretching both arms as far apart as possible?

6. How far is it around your waist? How far is it around your chest, just below your arms, when as much of the air as possible is exhaled from your lungs? What is your chest measurement when as much air as possible is drawn into your lungs?

7. The difference between these two chest measures is called your chest expansion. What is your chest expansion? How does your chest expansion compare with the average for the pupils of your room?

NOTE.—You may easily increase your chest expansion by a little practice in deep breathing.

8. How many steps wide is your schoolroom? how many steps long?

9. How many feet long is your room? how many feet wide?

10. How many yards long is it? how many yards wide?

11. How many inches long is your step? how many feet long? how many yards long?

12. How many inches long and wide is your schoolroom?

13. How many feet long is your schoolhouse? how many yards long?

14. How many steps wide and how many steps long is your school yard? how many feet wide and how many feet long? how many yards?

15. How many steps is it from your home to the school-house? how many feet? how many yards? how many rods (1 rd. = $5\frac{1}{2}$ yd.)?

16. How far can you walk in 1 min.? How many miles could you walk in 1 hr. at the same rate?

17. How far is it from your home to a neighboring large city? Answer this by using a map and the scale given with it. How long would it take you to walk to that city at the rate of walking in problem 16?

18. How long will it take a train to run from your nearest station to that city at 24 mi. per hour?

19. Using the scale map of your state (see Geography) find the length and the width of your state; of your county; of the U. S.

20. How long and how wide is a two-cent postage stamp?

21. How long is the *diameter* (distance across) of a copper cent (see Fig. 30)? of a dime? of a nickel? of a quarter dollar? of a half dollar? of a silver dollar?

22. How long is the *radius* of each of these coins?

23. Wrap a strip of paper, or a string, around each of these coins and find how far it is around each.

24. What kind of unit do you need to measure and express long distances? medium distances? very short distances?

NOTE.—Such a number as three-eighths, or $\frac{3}{8}$, of an inch means three units each one-eighth of an inch long. Such a unit is called a *fractional unit*, and such numbers as $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{11}{16}$, $\frac{17}{16}$, $\frac{25}{27}$, are all said to be numbers expressed in fractional units. Name the unit of each of the six fractions just given.

25. How long is the distance around (*circumference*) a bicycle wheel (wrap a string around the wheel)? How long is its diameter?

26. Answer the same questions for a carriage wheel; for the bottom of a bottle; of a can; for the head of a barrel; for any other circles you can find.

27. Arrange these measures thus:

OBJECT	DIAMETER	CIRCUMFERENCE
Silver dollar........	....	....
Half dollar..........	....	....
Quarter dollar.......	....	....
Nickel	....	....
Dime.................	....	....
Cent	....	....
Bicycle wheel	....	....
Carriage wheel	....	....
Bottle...............	....	....
Can..................	....	....
Barrel head..........	....	....

Fill out columns 2 and 3 of a table like this with your measurements and keep them for later use.

§73. Measuring Surfaces.

A *square unit* is a square 1 unit long and 1 unit wide.

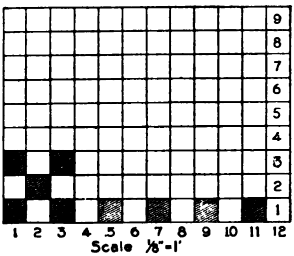


FIGURE 45

1. The side wall of a room is 9 ft. by 12 ft. What will it cost to lath and plaster the wall at 3¢ per square yard?

2. Using your own measures of the side and the end walls of your schoolroom, answer the same question for its walls and ceiling.

3. How many square feet of blackboard surface are there in your room?

4. How many square inches are there in a pane of glass in one of your windows? How many square feet of window surface admit light into your room?

5. This should be at least as great as $\frac{1}{8}$ of the floor surface of your room. Is it?

6. $\frac{1}{16}$ of an inch in the drawing, Fig. 46, stands for one foot in the room. Measure and find the number of square feet of plastered surface in the whole interior of the room, deducting the part covered by the baseboard and the floor.

Such a representation of the room as that of Fig. 46, showing the walls and the ceiling spread out on a flat surface, is called a *development* of the room.

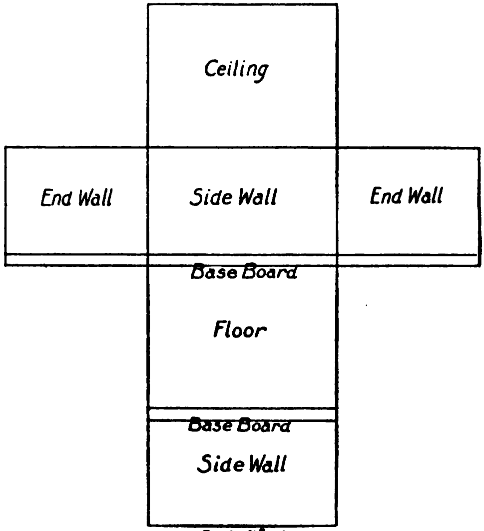


FIGURE 46

7. Draw to scale, from your own measures, the development of your own schoolroom.

8. A schoolroom is 24′ square and 15′ high. How many sq. yd. of flooring are needed for the room? How many sq. yd. of plastering are needed for the ceiling?

9. One side wall of the room (prob. 8) contains two windows, each 6′ × 8′; an end wall contains a window 6½′ × 8′; and the other side wall has a window and a door, each 4′ × 8′. How many sq. yd. of window and door surface in the entire room?

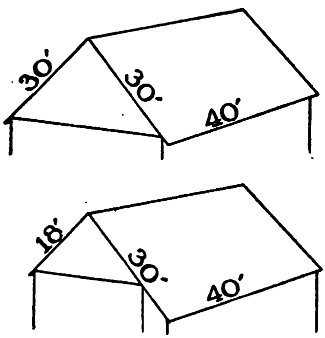


FIGURE 47

10. The rest of the wall surface is plastered. How many square feet of plastering are there in the room?

11. A barn roof is made up of two rectangles, each 30′ × 40′. How many square feet are there in the roof? How many squares of roofing? (A square of roofing is a 10-ft. square = 100 sq. ft.).

12. A house roof is made up of two rectangles, one 18′ × 40′ and the other 30′ × 40′. How many squares of roofing in the whole roof? How many sq. yd. of roof?

13. How many acres (1 acre = 160 sq. rd.) are there in a field 40 rd. × 80 rd.? 60 rd. × 80 rd.? 80 rd. × 80 rd.? 80 rd. × 160 rd.?

14. What is the cost at $2.20 a sq. yd. of paving a street 60′ wide and $\frac{5}{8}$ of a mile long?

15. What is the cost at 90¢ per sq. ft. of making a cement sidewalk $1\frac{2}{3}$ yd. wide and $\frac{3}{8}$ of a mile long?

16. How many square feet of glass are there in a window containing 4 panes of glass, each 18″ × 1 yard?

17. A rug 9′ × 12′ is placed in a room 14′ × 18′, with its long edge parallel to the long side of the room and with its center at the middle of the floor. How many square feet of the floor remain uncovered?

18. The streets of a city cut out a row of blocks like that of Fig. 48. The numbers along the sides are the lengths in feet. How many square feet are there in block *A*? in *B*? in *C*?

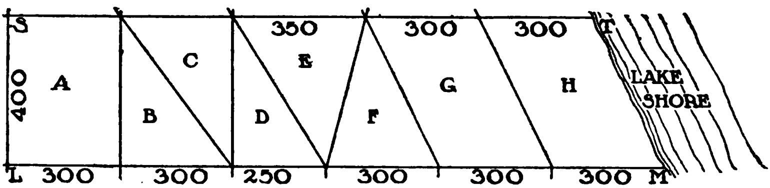


FIGURE 48

DEFINITIONS.—A four-sided figure, like *A*, having *square corners*, is called a *rectangle*. A four-sided figure, like *G* or *H*, having *two pairs of parallel sides*, whether the corners are square or not, is a *parallelogram*.

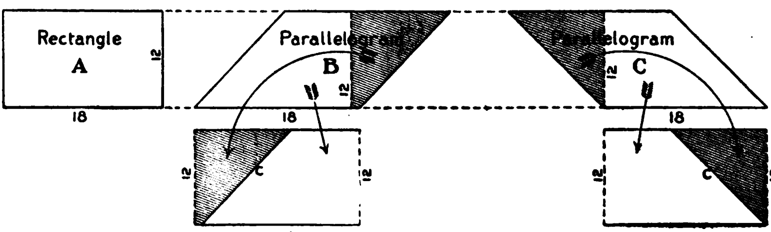


FIGURE 49

19. What is the area of the rectangle of Fig. 49?

20. Examine the parallelograms and the rectangles beneath them and find the areas of the parallelograms.

21. If the length of any parallelogram and its distance square across (*altitude*) are given, how can we find a rectangle with the same area as the parallelogram?

22. Point out in Fig. 48 a rectangle that has the same area as the parallelogram G.

23. The length of a parallelogram is b ft. and its altitude is a ft.; what is its area?

24. Call P the area, b the length, and a the altitude of a parallelogram; write an equation to show how you would use b and a to find P.

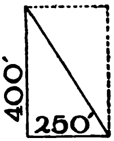


FIGURE 50

25. The streets, ST and LM, are parallel, Fig. 48. How many square feet are there in block G? in H? in E? in F?

26. Study Fig. 50 and find how many square feet there are in D of Fig. 48.

27. How can you find the area of a triangle when you know the length of its base and its altitude (shortest distance to the base from the opposite corner)?

28. In the triangle, ABC (Fig. 51), call the area T, the altitude (CE) a, the base (AB), b. Write an equation to find T from a and b.

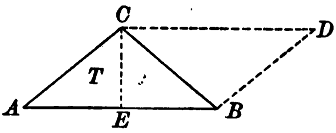


FIGURE 51

29. Find the area in square feet of the following triangles of Fig. 48: B; C; D; E; F.

30. The part of a roof in the shape of a parallelogram has a length (*base*) of 18 ft. and a breadth (*altitude*) of 12 ft. What is the area of the parallelogram?

31. A city lot has the shape of a parallelogram of 400 ft. base and 300 ft. altitude. What is its area?

32. A field has the form of a triangle of 68 rd. base and 46 rd. altitude. What is its area?

33. The triangular gable (see Fig. 19, p. 59) of a house is 24 ft. at the base and is 14 ft. high. What is the area of the gable?

34. Find the missing part (base, or altitude, or area) of the following parallelograms:

	Base	Altitude	Area
(1)	28 ft.	18 ft.	______
(2)	40 rd.	______	960 sq. rd.
(3)	______	40 rd.	1600 sq. rd.
(4)	160 rd.	$60\frac{3}{4}$ rd.	______
(5)	______	88 rd.	28160 sq. rd.
(6)	m rd.	n rd.	______
(7)	p mi.	q mi.	______
(8)	x in.	______	$x \times y$ sq. in.

§74. Measuring Volume (Bulk) and Capacity.

ORAL WORK

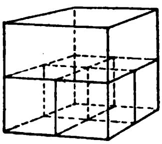


FIGURE 52

1. How many cubic inches are there in a 2-inch cube? a 4-in. cube? an 8-in. cube? a 16-in. cube?

2. How many cubic feet are there in a cubic yard? in a 6-ft. cube? in a room 3 yd. × 3 yd. × 3 yd.?

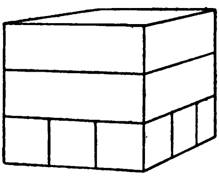


FIGURE 53

3. How many cubic inches are there in a 5-in. cube? in a 10-in. cube?

4. How many cubic inches are there in a 9-in. cube? in an 18-in. cube?

5. How many cubic inches are there in a cubic foot?

WRITTEN WORK

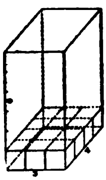


FIGURE 54

1. What is the capacity of a square cornered box 3″ × 4″ × 6″ (see Fig. 54)? 3′ × 4′ × 6′? of a room 3 yd. × 4 yd. × 5 yd.? a yd. × b yd. × c yards?

2. A box-car 8′ × 34′ can be filled with wheat to a height of 5 ft. When full how many cubic feet of grain does it hold?

3. If a bushel of wheat = $\frac{5}{4}$ cu. ft., how many bushels does the car hold?

4. Noticing that a cubic foot of grain (not ear corn) is $\frac{4}{5}$ bu., make a rule for finding the number of bushels a wagon-box or a granary will hold when full.

5. A wagon-box is $2' \times 4' \times 10'$. How many bushels of ear-corn will it hold if $\frac{9}{4}$ cu. ft. = 1 bu. of ear-corn?

6. How many rectangular solids $4'' \times 3'' \times 7''$, will fill a box $16'' \times 12'' \times 28$ inches? (See Fig. 55.)

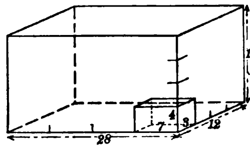


FIGURE 55

7. A box $30'' \times 24'' \times 12''$ will contain how many blocks $6'' \times 4'' \times 3''$ inches?

8. 128 cu. ft. = 1 cd. How many cords in a straight pile of wood 80 ft. $\times$ 4 ft. $\times$ 4 feet?

9. The volume of a rectangular bin is 1500 cu. ft. If it is 25 ft. long and 6 ft. deep, what is its width?

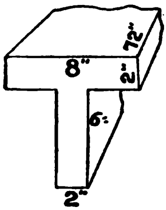


FIGURE 56

10. How many bricks $8'' \times 4'' \times 2''$ are there in a regular pile $218'' \times 24'' \times 48''$ inches?

11. Measure the length, the width, and the height of your schoolroom, and find how many cubic feet of air it contains.

12. How many pounds does the steel T-beam of Fig. 56 weigh if 1 cu. in. of steel weighs $4\frac{1}{2}$ ounces?

13. A steel I-beam 24′ long has a cross section of the form and size shown in Fig. 57. How much does it weigh if steel weighs 486 lb. per cubic foot (metal 1″ thick)?

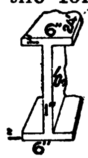


FIGURE 57

14. A water tank is $3' \times 6' \times 10'$. How many cubic feet of water does it hold when full?

15. There are 231 cu. in. in a gallon; how many gallons does the tank of problem 14 hold?

16. How many cubic inches of grain does a box hold if it is 8 in. square and $4\frac{1}{8}$ in. deep? how many gallons?

17. How many cubic inches does a box hold if it is 16 in. square and $8\frac{1}{2}$ in. deep? how many bushels (2150.2 cu. in. = 1 bushel)?

18. How many cubic inches does a box hold if it is $9'' \times 10'' \times 12''$? about what part of a bushel?

19. Answer similar questions for a box 10 in. square and $10\frac{3}{4}$ in. deep; for a box $10'' \times 12'' \times 18''$.

§75. Measuring Weight.

Review the oral work of §74.

1. A 3-in. cube of soil weighs 20 ounces; how many pounds does 1 cu. ft. of the same soil weigh? How many pounds does a cubic yard weigh?

2. A 4-in. cube of sand weighs 48 ounces; how many pounds does 1 cu. ft. of the same sand weigh? How many pounds does a cubic yard weigh?

3. A cubic foot of water weighs 1000 ounces; how much does a 6-in. cube of water weigh? a 3-in. cube?

4. A cubic foot of iron weighs 480 pounds; how many ounces does 1 cu. in. weigh?

SUGGESTION.—Notice that 1 cu. ft. $= 12 \times 12 \times 12$ cu. in., then use cancellation, thus:

$$\frac{\overset{\overset{10}{\cancel{40}}}{\cancel{480}} \times \overset{4}{\cancel{16}}}{\cancel{12} \times \underset{3}{\cancel{12}} \times \underset{3}{\cancel{12}}} = \frac{10 \times 4}{3 \times 3} = \frac{40}{9} \times 4\tfrac{4}{9}.$$ Ans. 1 cu. in. weighs $4\frac{4}{9}$ oz.

5. A cubic foot of oak wood weighs 64 lb.; how many ounces does 1 cu. in. of oak wood weigh? How many cubic inches of oak weigh just 1 pound?

6. A cubic foot of copper weighs 552 lb.; how many ounces does 1 cu. in. of copper weigh? How many cubic inches of copper weigh a pound?

7. A cubic foot of gold weighs 1200 lb.; how many ounces does 1 cu. in. of gold weigh? How many cubic inches of gold weigh a pound?

8. A cubic foot of zinc weighs 436 lb.; how many ounces does 1 cu. in. of zinc weigh? How many cubic inches of zinc weigh a pound?

9. A bushel of coal weighs 80 lb., a short ton weighs 2000 lb., and a long ton weighs 2240 lb. How many bushels of coal are there in a car load of 36 short tons? of 36 long tons?

10. Find the cost of $3\frac{3}{4}$ lb. of wire nails @ 4 cents.

11. What is the cost of 1 lb. of sugar selling 22 lb. for a dollar?

12. Postage on first-class mail matter is 2¢ an ounce. What would the postage be on a $2\frac{3}{4}$ lb. package of first-class matter (16 oz. = 1 pound)?

13. Find the cost of $2\frac{3}{4}$ lb. butter @ 32 cents.

14. New York merchants bought in 1 da. the following:

6324 lb. butter at an average price of $22\frac{1}{2}$ cents;
1988 lb. cheese at an average price of $14\frac{3}{4}$ cents;
6840 lb. sugar at an average price of $4\frac{3}{4}$ cents;
2780 lb. sugar at an average price of 4.8 cents.

Find the total weight and the total cost.

15. Find the total number of pounds and the total value of these purchases:

650 lb. cut loaf sugar @ $5.74 per hundredweight (cwt.);
825 lb. granulated sugar @ $4.80 per hundredweight;
400 lb. powdered sugar @ $5.24 per hundredweight;
700 lb. confectioner's sugar @ $4.89 per hundredweight;
350 lb. extra white sugar @ $4.78 per hundredweight.

16. Find the weight and total value of this shipment:

5400 lb. plain beeves @ $5.70 per hundredweight;
6375 lb. choice beeves @ $5.80 per hundredweight;
8450 lb. fair beeves @ $4.90 per hundredweight;
8625 lb. medium beeves @ $4.60 per hundredweight;
40,700 lb. veal calves @ $6.50 per hundredweight;
43,000 lb. western steers @ $8.50 per hundredweight;
8450 lb. Texas steers @ $3.70 per hundredweight;
25,200 lb. beef cows @ $2.85 per hundredweight.

17. How many tons did the purchases of problem 14 weigh (2000 lb. = 1 T.)? How many tons in the shipment of problem 16?

18. A troy ounce of pure gold is worth $20.67. How much is a troy pound of pure gold worth (12 troy oz. = 1 troy pound)?

19. An avoirdupois pound equals $1\frac{31}{144}$ of a troy pound. About what is the value of an avoirdupois pound of gold?

NOTE.—Add to the value of a troy pound of gold 31 times $\frac{1}{144}$ of its value.

20. An avoirdupois pound = 16 avoirdupois oz.; what is the value of an avoirdupois ounce of gold?

21. What is the value of your weight in gold? (Your weight is given in avoirdupois pounds.)

22. A grain dealer received during January 130 carloads of grain, averaging $25\frac{1}{2}$ T. each. Counting 60 lb. to the bushel, how many bushels did he receive during the month?

§76. Measuring Temperature.

On the Fahrenheit thermometer the point where water begins to freeze is marked 32°, and the point where water begins to boil is marked 212°. (See Fig. 24, p. 86.)

1. How many degrees are there between the boiling point and the freezing point?

2. At 11 p.m. on a certain date a thermometer read 32°. The mercury then fell $\frac{1}{2}$° an hour for 4 hr. What was the reading at 3 a.m. the next day?

3. The mercury then fell 1° an hour for 5 hr. What was the reading at 8 a.m.?

4. At 3 a.m. on a certain day the reading was 26°. The mercury fell 2° an hour for 5 hr. What was the reading at 8 a.m.?

5. On a certain date the reading was 10°, and the mercury fell on the average $1\frac{3}{7}$° an hour for 7 hr. What was the reading at the end of the 7 hours?

6. It then fell $1\frac{1}{4}$° an hour for 8 hr. What was the reading then?

7. The mercury fell from the reading 12° above zero to 3° below zero. How many degrees did it fall?

8 We might write readings *above zero* thus: A. 12°; A. 6°; A 32°; and readings *below zero* thus: B. 3°; B. 6°; B. 30°.

How many degrees does the mercury fall from the first of these readings to the second?

(1) A. 8° to A. 3°; (4) A. 5° to B. 7°; (7) B. 2° to B. 11°;
(2) A. 8° to B. 3°; (5) A. 2° to B. 12°; (8) B. $7\frac{1}{2}$° to B. 13°;
(3) A. 30° to B. 2°; (6) A. $4\frac{1}{2}$° to B. $4\frac{1}{2}$°; (9) B. $9\frac{1}{4}$° to B. $12\frac{1}{2}$°.

9. How many degrees of rise or fall are there from the first of these readings to the second? If the change is a *rise*, mark it R; if a *fall*, mark it F.:

(1) B. 7° to B. 2°; (5) A. 9° to B. 9°; (9) A. $6\frac{1}{2}$° to A. 12°;
(2) B. $2\frac{1}{2}$° to B. 1°; (6) A. $2\frac{1}{2}$° to B. $2\frac{1}{2}$°; (10) A. 3° to B. $30\frac{1}{2}$°;
(3) B. $2\frac{1}{2}$° to A. 1°; (7) B. 15° to B. $6\frac{1}{4}$°; (11) A. 18° to A. $67\frac{1}{2}$°;
(4) A. $3\frac{1}{2}$° to B. 1°; (8) B. 15° to A. $6\frac{1}{4}$°; (12) B. $22\frac{1}{2}$° to A. $67\frac{1}{2}$°.

10. Instead of writing an A. for "above zero" readings, it is customary to use the sign (+). What sign would you then suggest for "below zero" readings? Tell whether a change from the first to the second of these readings denotes a *rise* or a *fall* in each case and by how much?

(1) + 16° to + 100°; (4) + 32° to − 3°; (7) − 18° to − 34°;
(2) + 32° to + 212°; (5) + 16° to − 17°; (8) − 18° to − $6\frac{1}{2}$°;
(3) + 2° to + 161°; (6) + 8° to − 30°; (9) − 6° to − $27\frac{1}{2}$°.

11. The 12 o'clock (noon) readings for 4 successive days were as follows: + $82\frac{1}{2}$°; + $78\frac{3}{4}$°; + $61\frac{1}{2}$°; + $53\frac{1}{4}$°. What is the average of these 12 o'clock readings.

12. Find the average of these 9 a.m. readings for 6 da.: + 9°; + 4°; + 5°; + $12\frac{1}{2}$°; + $14\frac{1}{4}$°; + $9\frac{1}{4}$°.

13. Find the average of these 6 readings: − 4°; − 6°; − 2°; − 5°; − 13°; − 12°.

14. Find the average of these 2 readings: + 8° and − 2°. Show on Fig. 24, p. 86, what point on the thermometer is midway between the readings + 8° and − 2°.

15. What point is midway between the readings + 13° and − 5°? between the readings + 4° and − 6°? + 12° and − 12°? − 4° and − 12°?

§77. **Measuring Time.**

The primary unit used in measuring time is the *mean solar day*. This day is the average time interval during which the rotation of the earth carries the meridian of a place eastward from the sun back around to the sun again. It is the average length of the interval from noon to the next noon.

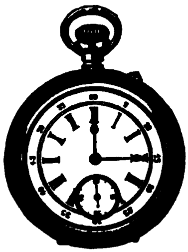


FIGURE 58

1. If the time piece is running correctly, how many times does the short hand (the hour hand) of a watch or clock turn completely around from noon to the next noon?

2. How much time is measured by one complete turn of the hour hand?

3. What part of a day of 24 hr. is measured by the rotation (turning) of the hour hand from XII to VI? from XII to III? from XII to I? to II? to IX?

4. What name is given to the time interval in which the hour hand moves from XII to I?

5. For measuring shorter periods the motion of the long hand (the minute hand) is used. What part of a day is measured by the movement of the minute hand from XII around to XII again? from XII to VI? from XII to III? from XII to IX? from XII to I?

6. What name is given to the interval of time required for the long hand to make one complete turn?

7. How many times does the minute hand turn around while the hour hand turns around once?

8. The small and rapidly moving hand covering the VI of the watch face is the second hand. How much time is measured by one whole turn of the second hand?

9. Over how much space does the tip of the minute hand move while the second hand moves around once?

10. The second hand circle is divided into how many equal parts? What name is given to the time interval in which the second hand moves over one of these equal spaces?

11. How many times does the minute hand turn around in one

day? in a week? in a month of 30 da.? in 365 da. (1 common year)?

12. Answer the same questions for the second hand.

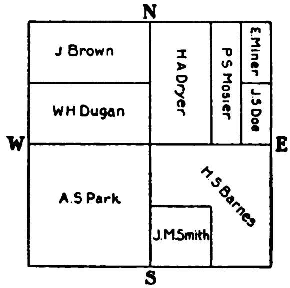
13. There are about 365¼ da. in 1 yr. If the hour and the minute hands of a watch start at XII and the second hand at 60 at the beginning of a year, how will the hands point at the instant the year ends, if the watch runs correctly and without stopping?

14. Answer question 13 supposing the length of the year to be 365 da. 5 hr. 48 min. 46 seconds.

§78. Measuring Land.

1. A *section* of land is a tract 320 rd., or 1 mi., square. It is usually divided into halves, quarters, eighths and sixteenths, as shown. How many square rods make a section of land?

2. An acre = 160 sq. rd.; how many acres in a section of land? in a quarter section?



SECTION OF LAND—FIGURE 59

3. A section of land is divided up into farms, as shown in the cut; how many acres are there in the farm belonging to A. S. Park? to H. S. Barnes? to J. Brown? H. A. Dryer? J. M. Smith? P. S. Mosier? J. S. Doe?

4. Point out the south half of the section; the east half; the N ½; the W ½.

5. Point out the SE ¼; the NW ¼; the SW ¼; the NE ¼.

6. Point out the N ½ of the SE ¼; the E ½ of the SE ¼; the W ½ of the NE ¼; the S ½ NW ¼; the W ½ E½.

7. Point out the SW ¼ of the SE ¼; the NE ¼ SW ¼; the NW ¼ NW ¼; the NE ¼ SW ¼.

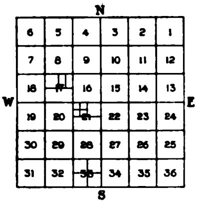
8. The farm of E. Miner would be described as the N ½ E ½ E ½ NE ¼; describe the farms of the 8 other owners of this section.

9. How many rods of fence would be required to enclose the section and separate it into farms as shown?

10. How many rods long is a ditch starting from the NE corner of H. A. Dryer's farm, thence running due south to H. S. Barnes's

N line, thence due W to Barnes's W line, thence S to J. M. Smith's N line, thence due W across Park's farm to his W line?

11. Each small square in Fig. 60 represents a section. The large square represents a *township;* how long is a township? how wide?



TOWNSHIP—FIGURE 60

12. How many square miles in 1 Tp.? how many acres?

13. Point out in Fig. 60 the following:

(1) NW $\frac{1}{4}$ Sec. 33.

(2) S $\frac{1}{2}$ Sec. 17; NE $\frac{1}{4}$ Sec. 17; E $\frac{1}{2}$ NE $\frac{1}{4}$ Sec. 17.

(3) NW $\frac{1}{4}$ Sec. 21; NE $\frac{1}{4}$ NW $\frac{1}{4}$ Sec. 21; SE $\frac{1}{4}$ NW $\frac{1}{4}$ Sec. 21.

14. How many farms of 160 A. each could be made of a township?

15. How long and how wide is a square 160-acre farm?

§79. Plotting Observations and Measurements.

1. The hourly temperatures from 6 a.m. to 6 p.m. December 26, 1902, were:

6 a.m.	7 a.m.	8 a.m.	9 a.m.	10 a.m.	11 a.m.	
7°	7°	10°	10°	12°	13°	
12 m.	1 p.m.	2 p.m.	3 p.m.	4 p.m.	5 p.m.	6 p.m.
14°	14°	15°	16°	15°	14°	14°

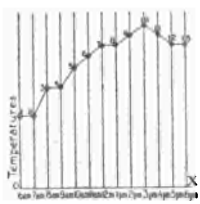


FIGURE 61

Draw 13 equally spaced vertical parallels one for each hour. Draw a horizontal line as OX across the parallels. Using $\frac{1}{8}''$ to represent 1°; measure off on the first vertical the distance O1 to represent 7°; on the second vertical the distance 7-2 to represent the second 7°; on the third parallel measure off the distance 10°, and so on. Mark the top of each measured vertical distance with a dot. Draw lines connecting these dots as in Fig. 61. *This is called plotting the readings.*

This line shows the temperature change during the 12 hrs.

This plotting is very much aided by the use of cross-lined paper, ruled into small squares. A horizontal side of one of the small squares might be used to represent 1 hr. and a vertical side to represent 1° of temperature.

NOTE.—Pupils in arithmetic should have notebooks containing several pages of squared paper for such work as this.

2. Read the out-door temperatures from hour to hour at your schoolhouse and plot your readings as above.

3. The average hourly temperatures from 6 a.m. to 6 p.m., December 26 to January 2, were:

6 a.m.	7 a.m.	8 a.m.	9 a.m.	10 a.m.	11 a.m.
21.5°	22.1°	22.2°	22.2°	22.5°	23.2°

12 m.	1 p.m.	2 p.m.	3 p.m.	4 p.m.	5 p m.	6 p.m.
23.9°	24°	24.5°	25.2°	25°	24.7°	24.3°

Plot these readings on squared paper, or by measurement, as above in Figure 61. Plot the tenths by estimate.

4. Does the line for the averages for a week agree in a general way with the line for December 26? What does a comparison of the two lines show?

This table gives the heights in feet and weights in pounds of boys and girls of ages given in the first column:

	HEIGHT IN FEET		WEIGHT IN POUNDS	
AGE	BOYS	GIRLS	BOYS	GIRLS
0	1.6	1.6	7.1	6.4
2	2.6	2.6	25.0	23.5
4	3.0	3.0	31.4	28.7
6	3.4	3.4	38.8	35.3
9	4.0	3.9	50.0	47.1
11	4.4	4.3	59.8	56.6
13	4.7	4.6	75.8	72.7
15	5.1	4.9	96.4	89.0
17	5.4	5.1	116.6	104.4
18	5.4	5.1	127.6	112.6
20	5.5	5.2	132.5	115.3

5. Using squared paper, or making a drawing such as is indicated in Fig. 62, plot the numbers in the first column horizontally and those in the second column vertically. Draw free-hand a

smooth line through all the plotted points and obtain the curve for growth in stature of boys. At what age do boys cease growing rapidly in height?

6. Using the numbers of columns 1 and 3 obtain a similar curve for girls. At what age do girls cease growing rapidly in stature?

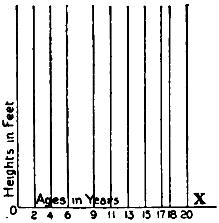


FIGURE 62

7. Compare the two curves and note when boys and when girls grow fastest?

8. Using the numbers of columns 1 and 4 draw the curve for growth of boys in weight.

9. Use numbers of columns 1 and 5 similarly for girls.

10. Compare the curves of problems 8 and 9 and note any likenesses or differences in the two curves. What do the curves show about the growth of boys and girls?

11. Twelve different rectangles each 12 in. long have the widths given in the first line and the areas in the second line.

Widths in inches..	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″	12″
Areas in sq. in....	12	24	36	48	60	72	84	96	108	120	132	144

Two lines, OX and OY, are drawn at right angles. 12 equally spaced lines are drawn parallel to OY. The horizontal distances O1, O2, O3 and so on denote the heights, 1″, 2″, 3′ and so on. On a scale of $\frac{1}{8}$″ (or the vertical side of a small square) to 12 sq. in. of area, mark off the lengths 1a, 2b, 3c, and so on, to denote the numbers in the second line above. This gives the points a, b, c, and so on.

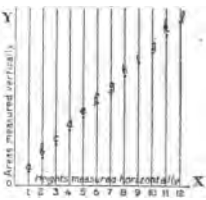


FIGURE 63

Make the construction to the scale indicated or to some other convenient scale, and place the straight edge of a ruler along the points, such as a, b, c, and so on. On what kind of line do the points seem to lie?

12. The bases of 12 triangles are each 16 in. long and the altitudes (heights) and the areas in square inches are:

1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″	12″
8	16	24	32	40	48	56	64	72	80	88	96

Using any convenient scale, plot these numbers. On what kind of line do all the points lie?

13. The lengths of the sides and the areas in square inches of 12 squares are:

Sides..	1″	2″	3″	4″	5″	6″	7″	8″	9″	10″	11″	12″
Areas..	1	4	9	16	25	36	49	64	81	100	121	144

Using any convenient horizontal and vertical scales plot these observations. Do the plotted points lie on a straight line?

14. Plot these data and draw a smooth free-hand curve through the plotted points:

Date of Census	Population of U. S. in Millions	Date of Census	Population of U. S. in Millions
1790	3.9	1850	23.2
1800	5.3	1860	31.4
1810	7.2	1870	38.6
1820	9.6	1880	50.2
1830	12.9	1890	62.6
1840	17.1	1900	76.3

Plot dates on horizontal, and populations on vertical, lines.

Draw a smooth free-hand curve through the plotted points. By continuing the curve can you predict (tell) about what the population will be in 1910?

Is the line through the plotted points a straight line?

§80. Measuring By Hundredths. PERCENTAGE

1. What is $\frac{1}{100}$ of 500 mi.? $\frac{3}{100}$ of 500 mi.? $\frac{8}{100}$ of 500 miles?
2. What is $\frac{3}{100}$ of 200 A.? $\frac{5}{100}$ of 200 A.? $\frac{12}{100}$ of 200 acres?

NOTE.—First find $\frac{1}{100}$.

3. What is $\frac{50}{100}$ of 100 ft.? $\frac{37}{100}$ of 100 ft.? $\frac{33\frac{1}{3}}{100}$ of 100 feet?
4. $\frac{1}{2}$ of anything equals how many hundredths of it?
5. How many hundredths of anything are the following fractional parts of it?

$\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$; $\frac{5}{5}$; $\frac{1}{10}$; $\frac{3}{10}$; $\frac{7}{10}$; $\frac{9}{10}$; $\frac{1}{20}$; $\frac{1}{25}$; $\frac{3}{20}$; $\frac{8}{25}$.

DEFINITION.—One hundredth is often written 1% and read 1 *per cent*. The sign (%) is a short way of writing "one one-hundredth." *Per cent* is a short way of speaking "one one-hundredth."

6. Read and give the meaning of the following:

2%; 6%; 8%; 12%; $12\frac{1}{2}$%; 25%; $33\frac{1}{3}$%; $87\frac{1}{2}$%; 100%.

7. Give the simplest fractional equivalents of (fractions that are equal to) these per cents:

8. How many lb. are 50% of 8 lb.? of 18 lb.? of 24 pounds?

9. How many square feet are 25% of 16 sq. ft.? of 48 sq. ft.? of 88 sq. ft.? of 400 square feet?

10. Heating an iron rod 100 in. long increased its length 2%. How many inches was the length increased?

11. One boy threw a stone 100 ft., and another threw it 12% farther. How many feet farther did the second boy throw the stone?

12. Referring to Fig. 12 (page 31) point out the following per cents of it: 20%; 40%; 60%; 80%; 2%; 4%; 10%; 100%.

13. Draw a square and divide it by a line so as to show 50% of it; 25% of it; 75% of it; $12\frac{1}{2}$% of it.

14. Similarly, show the same per cents using a circle; a rectangle.

15. Draw a line of any length and show $33\frac{1}{3}$% of it; $66\frac{2}{3}$% of it; 100% of it; $12\frac{1}{2}$% of it; 10% of it; 20% of it.

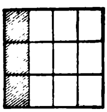


FIGURE 64

16. Draw a square and divide it into small rectangles as shown in the figure. Point out the following per cents of it:

25%; $33\frac{1}{3}$%; 50%; $66\frac{2}{3}$%; 75%; $12\frac{1}{2}$%; 100%.

17. A pair of shoes costing $2 was sold at a gain of 50%. What was the amount gained?

18. A boy was 50 in. in height 2 years ago. Since then he has grown 4% higher. How many inches taller is he now? How tall is he now?

19. The height of a 14 year old boy is 60 in., and a girl of the same age is 2% taller. How tall is the girl?

20. An umbrella was marked \$1.50, and was sold at a reduction of 20%. At what price did it sell?

21. Eighty per cent of the cost of a girl's suit is \$4.00. What is 1% of the cost? What is the whole cost, or 100% of the cost?

22. An article sold for \$4.00 after having been marked down 20%. What was the price before it had been marked down?

23. A lot was sold for \$800, which was 60% more than it cost the man who sold it. How much did it cost him?

NOTE.—The 60% means 60% of what it cost the seller. This cost is how many per cent of itself? \$800 is, then, how many per cent of the original cost. 1% of the original cost equals what? 100% of this cost equals what?

24. A thermometer reading was 120°, which was 20% higher than the reading taken 10 min. before. What was the previous reading?

NOTE.—In such questions as this answer first the question, "20% of what?" In this case 20% of the previous reading.

25. A man sold his farm for \$12,000, which was 25% more than he paid for it. What did he pay for it?

26. This year a flat rented for \$55 a month, which is 10% more than it rented for last year. For what did it rent last year?

27. Out of 80 games played by a championship team, 20 were lost. What per cent of the games were lost? What per cent were won?

28. The chest measure of a boy at the close of the school year was 27½ in., which was 10% greater than at the beginning of the year. What was the boy's chest measure at the beginning of the year?

29. A man paid \$18 for the use of \$300 for a year. What per cent was the sum he paid of the sum he borrowed?

§81. Simple Interest.

An extensive use of the system of measurement by hundredths is made with problems in Interest.

DEFINITION.—*Interest* is money to be paid for the use of money. Two per cent interest means that 2¢ is to be paid every year for the use of 100¢ or \$1; \$2 for the use of \$100, and so on at this rate. What then would 6% interest mean? 4% interest? 10% interest?

The amount of money to be paid each year for the whole sum borrowed is called the *Interest* for 1 year.

1. What is the interest on $200 at 4% for 1 year?

2. What is the interest at 6% on $1 for 1 year?

NOTE.—In computing interest a year means 12 months of 30 days each.

3. What is the interest at 6% on $1 for ½ yr., or 6 mo.? for 2 mo.? for 1 month?

4. What is the interest at 6% on $1 for 3 mo.? for 5 mo.? for 7 mo.? for 14 mo.? for 16 mo.? for 34 mo.? for any number of months?

5. From the answer to problem 2, find the interest at 6% for 1 yr. on $8; on $25; on $48; on $85; on $124; on $450.

6. If you know the interest at 6% for 1 yr. on $1, how can you find the interest at 6% for 1 yr. on any number of dollars?

7. From the last answer to problem 5, find the interest at 6% on $450 for 2 yr.; for 3 yr.; for 4½ yr.; for 12¼ yr.; for any number of years.

8. Using the third answer to problem 4, find the interest at 6% for 7 mo. on $48; on $450; on $75.

9. Tell how to find the interest at 6% on any number of dollars for any number of months.

10. The interest on a certain sum of money for 1 yr. at 6% is $24. What is the interest on the same sum for the same time at 12%? at 18%? at 30%? at 60%? at 1%? at 3%? at 2%?

11. ⅚ of the interest on a sum of money for a given time at 6% equals the interest on the same sum for the same time at what rate per cent?

NOTE.—⅚ of a number may be easily found by subtracting from it ⅙ of itself. How may ⅞ of a number be found similarly?

12. Knowing the interest on any sum of money at 6%, how can you quickly find the interest on the same sum for the same time at 7%? at 8%? at 11%? at 15%? at any rate per cent?

13. Find the interest on $1200 at 6% for 2½ yr.; for 2¾ years.

14. A man has $350 in a bank, which pays 3% interest. To how much interest is the man entitled if his money has been in the bank 2 yr. and 4 mo.? 6 yr. and 9 mo.? 8 yr. and 2 months?

§82. **Pressure of Air.** ORAL WORK

1. When a glass is filled level with water, covered with a piece of writing paper, and carefully inverted, why does not the water fall out when the glass is held mouth downward?

2. When a soft leather sucker is moistened and spread out on a smooth, flat surface, why does it cling to the surface even when the sucker is raised by being lifted at its middle?

3. When one end of a glass tube is placed in water and you draw with the mouth at the other, why does the water rise in the tube?

4. Fill a bottle with water, leave the cork out, and invert the bottle in a vessel of water. Why does the water not run out?

5. When a glass tube with one end closed is partly filled with mercury and the open end dipped under the surface of mercury in a cup, why does not all the mercury run out of the tube?

NOTE.—In connection with 5, examine a mercury barometer.

6. When a sheet of thin rubber, or paper, is held over the mouth of a funnel, and the air is sucked out of the funnel through its neck, why does the rubber curve inward? Will it do this in all positions of the funnel? Why?

These experiments show that the air presses downward, upward, and in all directions upon surfaces. Careful measurements have shown that the pressure of the air on every square inch of surface is about 15 pounds.

NOTE.—This pressure is equal on all sides of surfaces, upper sides and lower sides, outside and inside, toward the right and toward the left.

WRITTEN WORK

1. Measure the length and the width of the cover of your book and find the downward pressure in pounds on the upper surface of your book when it lies flat upon the desk. Why is the book so easily moved about under this pressure?

2. Measure and compute the downward pressure of the air on the top of your desk. Can you lift this number of pounds? Why can you lift the desk?

3. A room is 10′ × 20′ × 25′. Find the pressure of the air, in pounds, on the floor, on the ceiling, and on each of the four walls of the room. Why does not this pressure tear the walls apart?

4. The average surface of the human body equals 20.6 sq. ft. What is the total pressure of the air on the outside surface? Why does not this pressure crush the body?

5. Measure the length and the width of the door of your room, and find the air pressure on one side of the door. Why can you open and close the door so easily?

6. Measure and find the air pressure in pounds on the outside surface of a pane of window glass. Why does not this pressure break the glass?

§83. **Passenger and Freight Trains.**

1. A fast train runs from Chicago to a station 356.4 mi. distant in exactly 9 hr. What is the average rate (miles per hour) of the train?

2. The train left Chicago at 6:10 a.m. At what time did it arrive at the station?

3. Another train runs 10½ hr. at an average rate of 36 mi. per hour, including stops; how far does it run?

4. If the train (problem 3) started at 2:30 a.m., at what time did it reach the end of the run?

5. A traveler went by rail from Chicago to Los Angeles, California, in 4 da. 19 hr., at the average rate of 37.4 mi. an hour, including stops. How far is it from Chicago to Los Angeles by this route?

6. A train ran 361 mi. in 9½ hr. What was its average rate?

7. A freight train is running 21 mi. an hour while a brakeman on top of the cars walks toward the engine at the rate of 2½ mi. an hour. How fast does the brakeman actually move forward? how fast, if he walks from front to rear at the rate of 2½ mi. an hour?

8. A conductor walks from the front to the rear of a train at the rate of 3½ mi. an hour while the train is running 38 mi. an hour.

At what rate per hour does the conductor actually move forward? At what rate does he move in the direction of the running train if he walks from the rear to the front at the rate of $3\frac{1}{2}$ mi. an hour?

9. How may the conductor (problem 8) suddenly change his rate of motion from $34\frac{1}{2}$ mi. to $41\frac{1}{2}$ mi. an hour? How many miles an hour does this change of rate amount to?

10. A passenger train running 32 mi. an hour meets a freight train running in the opposite direction on a parallel track $18\frac{1}{2}$ mi. an hour. At what rate do the trains approach each other?

11. At what rate does the passenger train (problem 10) pass the freight if both are running in the same direction on parallel tracks?

12. A boy tries to overtake a street car by running up behind it. If the boy runs 10 ft. a second and the car runs 6 ft. a second, how soon will the boy overtake the car if he is now 20 ft. behind it?

13. Two friends, coming from opposite directions, have arranged to meet at a certain railway station. Their trains are running at 35 mi. and 25 mi. an hour. Not allowing for time lost in stops, how soon will their trains be together at the station, if they are now 30 mi. apart and both trains reach the station at the same time?

14. A passenger train is made up of a postal and baggage car. an express car, 3 common coaches, 2 chair cars, a dining car, and 2 sleepers. The average length of a car is $61\frac{2}{3}'$, and the length of the engine and tender together is 65′. How long is the train?

15. The weight of the engine (problem 14) is 142,780 lb.; of the tender, 43,200 lb.; and the average weight of a car is 83,480 lb. What is the total weight of the train, in tons?

16. To draw a train on straight, level track at a speed of 40 mi. an hour, requires a horizontal pull of $\frac{1}{210}$ of the weight of the train. What force in tons would be needed to draw the train of problem 14 at a speed of 40 mi. an hour?

17. An engine, 62′ long and weighing 240,000 lb., draws a tender, 22′ long, weighing 64,500 lb., and a train of 72 empty freight cars, averaging $36\frac{1}{2}'$ in length and 32,700 lb. in weight. How long is the train? how heavy?

18. If it requires $\frac{3}{1250}$ of the weight of a train to draw it on straight, level track at a speed of 20 mi. an hour, how many pounds of force must the engine of problem 17 exert to draw the train, on such track, 20 mi. an hour? How many tons of force?

NOTE.—First find $\frac{1}{1250}$ of the weight of the train including engine and tender.

19. How much force would be needed to draw 38 cars, each weighing $16\frac{1}{2}$ T., and each loaded with $18\frac{1}{2}$ T. of coal, on straight, level track at a speed of 20 mi. an hour? (See problem 18.)

20. When a train is moving 5 mi. an hour it takes a horizontal force of about $\frac{1}{700}$ of the weight of the train to draw it along straight, level track. On such track what force must an engine exert to draw a train of 68 empty freight cars, each weighing 34,300 lb., at a speed of 5 mi. an hour?

21. Under the same conditions as in problem 20, what force would be needed to draw a train of 38 cars, each weighing 34,000 lb. and carrying a load of 58,600 lb. of coal in addition to its own weight?

22. A railroad company purchased 6 locomotive engines, weighing, in pounds, 142,780, 142,630, 158,670, 139,790, 146,890, and 138,960. The tenders weighed, in pounds, 43,750, 44,200, 45,280, 42,920, 43,650, and 44,280. The cost per pound of the combined weight was $13\frac{4}{5}$¢. What was the total cost?

23. The greatest horizontal pulling force that an engine can exert on dry, unsanded steel track in starting a train is about $\frac{2}{9}$ of the part of the engine that is carried by the driving wheels. What is the greatest horizontal pull the engine of problem 17 can exert in starting a train on such track, supposing the entire weight of the engine is carried by the driving wheels?

24. Answer a similar question for the engine of problem 15, supposing 105,750 lb. of the weight of the engine is carried by the driving wheels.

§84. Train Despatcher's Report.

A copy of a train despatcher's sheet, for a test run, showing the distance between stations, the schedule time and the exact running time of the train, the number of the cars in the train, the number of passengers carried each trip for 7 week days.

PERFORMANCE OF TRAIN NO. 25, FROM JULY 1ST TO 8TH, 1896.

STATION	DISTANCE MI.	SCHEDULE TIME	1ST	2D	4TH	5TH	6TH	7TH	8TH
1......	0	3:50	3:50	3:49¾	3:50¾	3:50	3:50½	3:50½	3:51¼
2......	3.1	3:55	3:55	3:55½	3:56	3:55	3:55	3:55¼	3:56
3......	5.5	3:57	3:57	3:58	3:58½	3:57¼	3:58¾	3:57¾	3:58¼
4......	7.9	3:59	3:59	4:00	4:00½	3:59¼	4:00¼	4:00	4:00
5......	12.0	4:02	4:02	4:03¾	4:03¾	4:03¼	4:04	4:03	4:03
6......	17.0	4:06	4:06	4:07¾	4:07½	4:06¼	4:08½	4:07½	4:07½
7......	19.9	4:09	4:08	4:10	4:09½	4:08½	4:11	4:10	4:10
8......	24.5	4:12	4:11½	4:14	4:13	4:12	4:14½	4:14	4:13¾
9......	27.6	4:15	4:13½	4:16½	4:15	4:14	4:17¾	4:16	4:15
10......	33.8	4:20	4:18	4:21¼	4:20	4:18½	4:22	4:20¾	4:20¾
11......	38.7	4:24	4:22	4:25	4:24	4:22	4:26½	4:24¼	4:23
12......	43.5	4:28	4:25½	4:28½	4:27½	4:26	4:30	4:28	4:27
13......	50.5	4:33	4:30½	4:34½	4:32½	4:31	4:35½	4:33	4:32
14......	53.8	4:38	4:32½	4:37½	4:35	4:34	4:38	4:36	4:35½
15......	55.5	4:40	4:35¼	4:40	4:37¾	4:36	4:40¼	4:37¾	4:37
Number of cars........			5	7	6	5	5	5	5
Passengers carried......			201	441	11	79	107	113	118
Running time..........									
Mi. per hour (average)..									

PROBLEMS

1. By "running time" is meant the difference between the time of leaving* station 1 and the time of arriving at station 15. On a separate sheet write the "running time" for each day.

2. Find the difference between "schedule time" and the time the train actually reached station 6 on the 2d, 4th, 6th, 7th, 8th.

3. The train is "on time" if it reaches a station just at "schedule time." Find how much the train was ahead of or behind time in reaching station 12 on each of the 7 da. Mark "ahead of time" results with an "A," thus: A. 2½ min., and "behind time" results with a "B," thus: B. 1¼ minutes.

4. Make and answer similar questions for other stations.

5. How far is it from station 1 to station 15? from 2 to 9? 4 to 11? 3 to 15? Make and answer similar questions for other stations.

6. What is the difference in schedule times between stations 1 and 15? 2 and 9? 4 and 11? 3 and 15? any other two stations?

* Stops are so short that leaving and arriving times are regarded as the same.

7. From your answers to problems 5 and 6 find the average* rate of running (in miles per minute) between stations 1 and 15, by a train running exactly on schedule time; between stations 2 and 9; 4 and 11; 3 and 15.

8. Find the average rates of running between the same stations (as in problem 7) on the 6th.

9. Find the average rates of running between stations 1 and 15 on each day, that is, the "mi. per hour."

10. What is the total number of cars drawn during the whole period?

11. Find the total number of passengers carried.

12. Make and solve other problems on the table.

§85. **Areas of Common Forms.** ORAL WORK

1. If one side of an inch square represents 80 rd., how many square rods will the inch square represent? how many acres?

2. In a drawing of a rectangular farm to a scale 1 in. = 80 rd., how could you find the number of acres in the farm?

3. If a square field containing 40 A. is cut across diagonally by a railroad, how many acres does each of the triangular parts contain, supposing the railroad itself covers 3 acres?

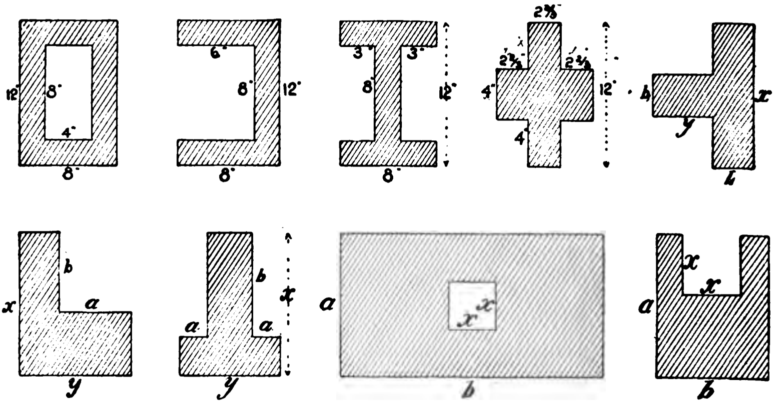


FIGURE 65

4. Give the areas of the cross-lined portions of the 9 forms of Fig. 65.

* "Average rate" here means *distance run divided by time of running.*

PROBLEMS

Each inch in Fig. 66 represents 80 rods.

1. How many acres are in the tract *ABCDE*? What is the farm worth at \$95 per acre?

2. The field is entirely enclosed, except along the river, by a barbwire fence having 4 lines of wire. Barbwire weighs about 1 lb. per rod. Find the cost of the wire at \$3.50 per bale of 100 pounds.

3. If posts cost 35¢ apiece and are set 2 rd. apart, find the cost of the posts.

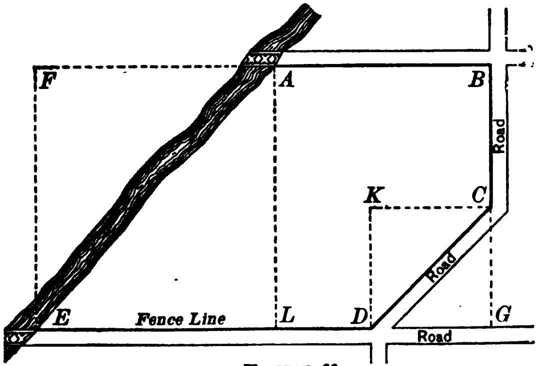


FIGURE 66

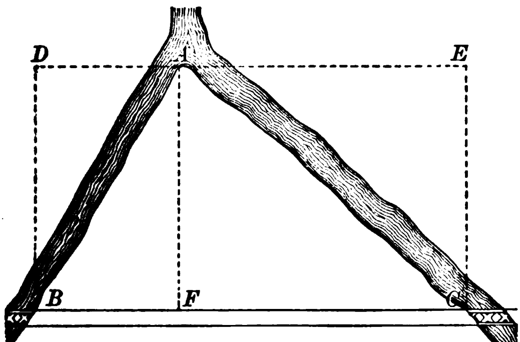


FIGURE 67

4. In Fig. 67 1 in. represents 80 rd., or *the scale is 1″ to 80 rd.* How many acres are in the field ABF? What is the field worth at \$75 per acre?

5. Find how many acres there are in the field AFC and what is its value at \$87½ per acre.

6. What part of the rectangle ADBF is the triangle ABF? What part of the rectangle EAFC is the triangle AFC?

7. What is the value of the field ABF at \$80 per acre?

8. What part of the rectangular tract DBCE is the triangular field ABC?

9. How many rods of fence would be needed to enclose the field ABF? AFC? the field DBCE?

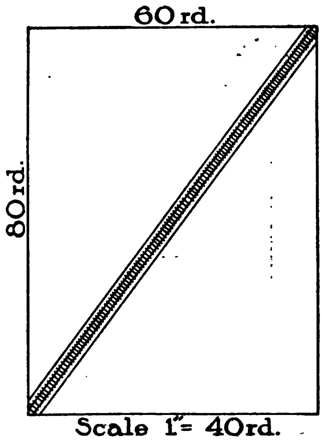


FIGURE 68

10. A railroad runs straight across a rectangular field, dividing it into two triangles as shown in Fig. 68. How much is each triangular part worth at $120 per acre, supposing the road takes off equally from each part a strip containing $2\frac{1}{2}$ acres?

11. The railroad company fences both sides of its strip with 4-wire fences, each 96 rd. long. Posts are set a rod apart, and cost 25¢ each. The wire used costs $3.25 per bale of 100 rd., and 2 men at $1.50 per day work 3 days to build the fence. What did it cost the company to fence its road?

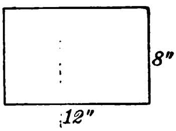


FIGURE 69

12. If 1 sq. in. of the rectangle of Fig. 69 represents $\frac{5}{8}$ of an acre, how many acres are represented by the whole rectangle?

13. If one sq. ft. of a sheet of zinc weighs $2\frac{1}{2}$ oz., how much will pieces of the same sheet of the sizes shown in Fig. 70 weigh?

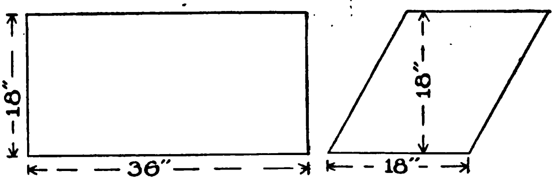


FIGURE 70

14. If 15 sq. ft. of sheet copper weigh 2.5 lb., how much will 25 sq. ft. of the same sheet weigh? How many sq. ft. in 2.5 lb.?

15. If 5 lb. of rope cost 75¢, how much would $37\frac{1}{2}$ lb. of rope cost at the same rate? 47 lb?

16. If the pendulum of a clock swings 40 times in 15 seconds, how many times will it swing in 1 minute? in 17 minutes? in 1 hour?

17. Two square feet of a 1-in. board weigh 9 lb.; what would 9 sq. ft. of the same board weigh? 72 square inches? 16 square inches?

INTRODUCTION TO RATIO AND PROPORTION

Ratio

Definition.—The *ratio* of one number to another is the quotient of the first number divided by the second.

1. What is the ratio of A to O? of O to A? of B to A? of A to B? of B to O? of O to B? of C to B? of C to A? of C to O? of O to C? of D to C? of D to B? of D to O? of O to D?

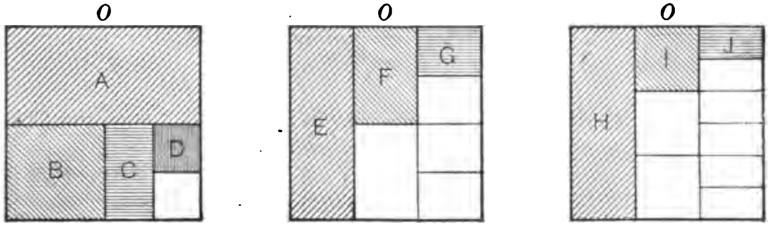


Figure 71

2. In the second square, compare each of the parts E, F, and G with O; compare O with each of these parts.
3. Compare each division with each of the others, separately.
4. In the third square, $\frac{1}{3}$ of H is how many times J? $\frac{1}{3}$ of I is what part of H? $\frac{1}{3}$ of O is how many times I?
5. What is the ratio of 3″ to 1″? of 1″ to 3″? of 1″ to $\frac{1}{3}$″? of $\frac{1}{3}$″ to 1″?
6. What is the ratio of 6 to 6? of 50 to 50? of $\frac{3}{8}$ to $\frac{3}{8}$? of a to a? of x to x?
7. What is the ratio of any two equal numbers?

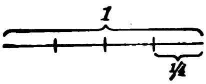


Figure 72

$\frac{1}{4}$ is a short way of writing the following expressions:

The ratio of 1 to 4,

$1 \div 4$,

$1 : 4$,

$\frac{1}{4} : 1$.

$1:4$ is read "1 to 4," and $\frac{1}{4}:1$ is read "$\frac{1}{4}$ to 1."

8. What is the ratio of 1 ft. to 1 yd.? of 1 hr. to 1 da.? of 1 nickel to 1 dime? of 1 in. to 12 in.? of 1 ft. to 3 ft.? of 12 in. to 1 in.? of 1 mi. to 1 mile?

The ratio of a 12-in. line to a 1-in. line is called, also, the *measure* of the 12-in. line by the 1-in. line.

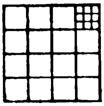


FIGURE 73

9. What is the ratio of 1 sq. ft. to a 3-in. square (see Fig. 73)? of 1 sq. ft. to 1 square inch?

The ratio of a square foot to a 3-in. square is the *measure of a sq. ft. in terms of the 3-in. square*, that is, how many 3-in. squares in a square foot.

Finding the ratio of the foot to the inch is the same as measuring the foot by the inch. Furthermore, to find the ratio of any number, or quantity, to any other number, or quantity, is to find the quotient of the first number, or quantity, divided by the second.

The result of measuring one number by another is called the *numerical measure* of the first by the second.

The ratio of one number to another, or the measure of one number by another, can be found only when both numbers are expressed *in the same unit.*

10. Measure the avoirdupois pound by the ounce; the foot by the inch; the inch by the foot; the ounce by the pound; the gallon by the quart; the bushel by the quart.

11. Measure the rod by the yard; the mile by the rod; the square yard by the square foot; the quart by the gallon.

12. Measure 8 ft. by 4 ft.; 16 ft. by 64 ft.; a square mile by a square rod; an acre by a square rod; a square mile by an acre.

13. Measure 80 by 8; 80 by 4; 3 by 4; 4 by 3; 9 by 18; 18 by 9; 125 by 25.

14. Measure a by b; a by $2a$; b by a; $2a$ by a; $6x$ by $2x$; $2x$ by $6x$.

15. If 5 apples cost 4¢, what will 35 apples cost?

SOLUTION.—How many fives of apples in 35 apples?
How much does one five of apples cost?
How much then do 35 apples, or 7 fives of apples, cost?

This analysis may be thought of in the form of ratios.

$$\frac{\text{Cost of 35 apples}}{\text{Cost of 5 apples}} = \frac{x}{4¢}, \text{ or more briefly } \frac{35}{5} = \frac{x}{4¢} \qquad (1)$$

and we have to find a number which has such a ratio to 4¢ as 35 apples has to 5 apples.

This leads us to an equation of ratios.

DEFINITION.—An equation of ratios is called a *proportion.*

§86. Proportion.

In all these problems use the equation form of statement like (1) in the solution of problem 15 of the preceding section.

1. If 1 doz. oranges cost $0.30, what will 4 oranges cost at the same rate?

2. If a yard of cloth costs $1.50, at the same rate what will $\frac{1}{3}$ yd. cost?

3. If a 3-in. square of tin costs $\frac{1}{2}$¢, what will a square foot cost at the same rate?

4. If 6 qt. of oil cost 15¢, what will 3 gal. cost at the same rate? (4 qt. = 1 gal.)

5. Two rectangular flower beds have the same shape. One is 3 ft. wide and 4 ft. long; the other is 6 ft. wide. How long is it?

6. Two books are of the same shape but of different sizes. One is 5″ wide × $7\frac{1}{2}$″ long. The other is 15″ long. How wide is it?

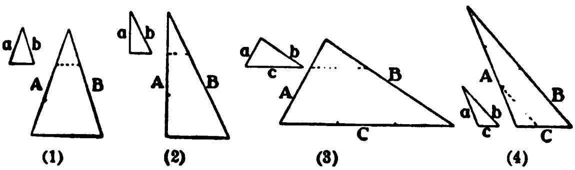


FIGURE 74

7. In two triangles of the same shape, like those of Fig. 74, (1), if $a = 4″$, $A = 12″$, and $b = 4″$, how long is B?

8. In triangles of the same shape, like those of Fig. 74, (2), if $a = 4″$, $A = 12″$, and $b = 5″$, how long is B? If $a = 6″$, $b = 8″$, and $B = 32″$, how long is A? If $b = 7″$, $B = 35″$, and $A = 30″$, how long is a?

9. In triangles of the same shape, as those of Fig. 74, (3), if $A = 21″$, $b = 9″$, and $B = 27″$, how long is a? If $a = 5\frac{1}{2}″$, $A = 22″$, and $B = 30″$, how long is b?

10. In triangles of the same shape, like those in Fig. 74, (4), if $a = 3″$, $A = 9″$, and $C = 6″$, how long is c? If $A = 24″$, $c = 4″$, and $C = 12″$, how long is a?

11. Denoting the sides of two triangles having the same shape by letters as in problem 10; if $a = 8″$, $b = 12″$, $c = 17″$ and $B = 30″$, how long are A and C?

INTRODUCTION TO COMMON FRACTIONS

§87. Fractions as Ratios and as Equal Parts.

1. Into how many equal parts is *A* (Fig. 75) divided? What is one of the parts called?

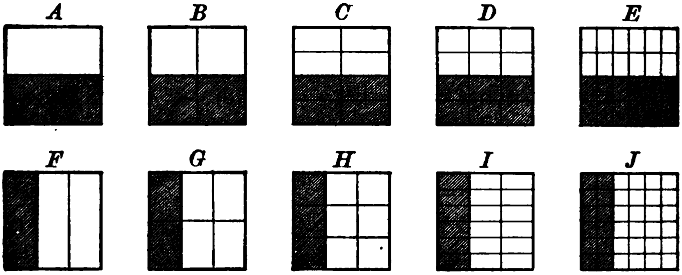


FIGURE 75

3. One part of *A* equals how many of the smallest parts of *B*? of *C*? of D? of *E*? of *G*? of *I*? of *J*?

4. One part of *F* equals how many of the smallest parts of *G*? of *H*? of *I*? of *J*? of *D*? of *E*?

5. Write the fraction that names one of the equal parts of *A*, as divided in the illustration.

6. Measure *A* by one of its equal parts. *Ans.* 2 halves or $\frac{2}{2}$.

7. Measure the shaded part of *B* by one of its 4 equal parts.
Ans. $\frac{2}{4}$, read "2 fourths."

8. Measure the unshaded part of *B* by the same unit.

9. Using one of the 8 equal parts of *C* as a unit, express the shaded part of *C* in this unit; the right half of the shaded part.

10. Express *E* in terms of its smallest unit.

11. Express each figure in terms of its smallest unit, as follows:

(1) The whole square, *F*, *G*, *H*, *I*, *J*.
(2) The shaded part of " " " " "
(3) The unshaded part of " " " " "
(3) " " *H*; " " ; " "
(4) The shaded part of *G* in the smallest part of *I*;
(5) The unshaded part of *I* and of *G* in the smallest part of *J*.

DEFINITION.—$\frac{3}{4}$ means 3 of the $\frac{1}{4}$'s (one fourths) of some number, or asured quantity. The $\frac{1}{4}$ is called the *fractional unit.* The fractional of any fraction is one of the equal parts expressed by the fraction.

12. What is the fractional unit of each of the following fractions: $\frac{2}{3}$, $\frac{2}{5}$, $\frac{3}{6}$, $\frac{1}{2}$, $\frac{4}{7}$, $\frac{5}{6}$, $\frac{8}{9}$, $\frac{7}{9}$, $\frac{10}{11}$, $\frac{3}{5}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{5}{7}$, $\frac{7}{8}$, $\frac{8}{13}$, $\frac{6}{35}$, $\frac{8}{35}$, $\frac{7}{13}$?

13. What is the fractional unit of a fraction having the number 7 below the line? the number 6? 9? 12? 15? 25? 18? 11? 64?

14. How many fractional units are expressed by the 1st fraction of question 12? by the 2d fraction? the 3d? 4th? 5th? 6th? 7th? Draw a square and divide it free-hand by lines showing the meaning of each of the first 8 fractions of problem 12.

15. What does the number written below the line of a fraction express?

16. What does the number written above the line of a fraction express?

DEFINITIONS.—The number above the line is called the *numerator* (meaning *numberer*).

The number below the line is called the *denominator* (meaning *namer*).

17. What are the fractional units of fractions having the following denominators:

3? 7? 12? 18? 24? 125? 75? 19? a? x?

18. Point out the fractions of question 12 that have the same fractional units.

DEFINITION.—The numerator and the denominator are together called the *terms* of a fraction.

19. If each half of a square is divided into 2 equal parts, into how many equal parts is the whole square divided?

20. Into how many equal parts is a whole divided, if each half of it is divided into 3 equal parts? into 4 equal parts? into 5? 6? 7? 8? 9? 10? 11? 25?

21. Into how many equal parts is a whole divided, if each third of it is divided into 2 equal parts? into 3 equal parts? into 4? 5? 6? 7? 8? 9? 10? 12? 15? 20? 30?

22. Into how many equal parts is a whole divided by dividing each of its fifths into 2 equal parts? 3 equal parts? 4? 5? 6? 7? 8? 9? 10? 20? 30?

Give solutions of such as the following and explain:

23. Express as sixths, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, $\frac{1}{2}$, $\frac{3}{2}$.

24. Express as twelfths, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{6}$, $\frac{5}{6}$.

25. In question 12 point out pairs of fractions that are not expressed in the same fractional unit, but may easily be so expressed.

26. Express the following pairs of fractions as equivalent fractions having the same fractional unit, or, what is the same thing, having a *common denominator:*

$\frac{1}{2}$ and $\frac{3}{5}$; $\frac{1}{3}$ and $\frac{1}{5}$; $\frac{3}{4}$ and $\frac{5}{6}$; $\frac{2}{3}$ and $\frac{6}{7}$; $\frac{2}{5}$ and $\frac{5}{9}$; $\frac{7}{8}$ and $\frac{5}{11}$.

COMMON FRACTIONS

REDUCTION OF COMMON FRACTIONS

§88. To Reduce Fractions to Higher, Lower, and Lowest Terms.

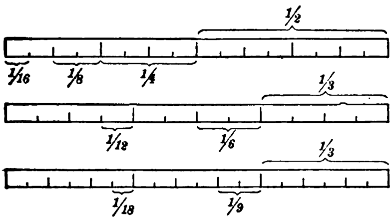


FIGURE 76

1. Express $\frac{1}{2}$ as 4ths; 8ths; 16ths.

2. Express $\frac{1}{3}$ as 6ths; 9ths; 12ths; 18ths.

3. Express as 3ds $\frac{4}{6}$; $\frac{6}{9}$; $\frac{14}{21}$; $\frac{20}{30}$; $\frac{140}{210}$; $\frac{26}{39}$.

4. Express as 7ths $\frac{4}{28}$; $\frac{35}{49}$; $\frac{24}{56}$; $\frac{54}{63}$.

5. Compare the values of these fractions after reducing each to its lowest terms (that is, to the smallest possible whole numbers for numerators and denominators).

$$\frac{3}{4};\ \frac{6}{8};\ \frac{9}{12};\ \frac{21}{28};\ \frac{60}{80};\ \frac{75}{100};\ \frac{300}{400}.$$

6. Express $\frac{2}{5}$ as 10ths; 15ths; 20ths; 35ths; 75ths; 100ths.

7. By what must you multiply the numerator of $\frac{3}{4}$ to make the numerator the same as that of $\frac{6}{8}$? By what must you multiply the denominator of $\frac{3}{4}$ to make the denominator the same as that of $\frac{6}{8}$? Draw a square and divide it free-hand to show the comparative sizes of $\frac{3}{4}$ and $\frac{6}{8}$ (see square *C*, Fig. 75, §87).

8. By what must you multiply each term of $\frac{3}{4}$ to obtain $\frac{9}{12}$? $\frac{21}{28}$? $\frac{60}{80}$? $\frac{75}{100}$? $\frac{300}{400}$? What, then, is the value of each fraction of problem 5?

9. How then may the value of a fraction be expressed in higher terms?

A fraction may be expressed in higher terms, without changing its value, by multiplying both terms by the same number.

10. Express the following fractions in higher terms:

$$\tfrac{3}{8};\ \tfrac{6}{7};\ \tfrac{5}{9};\ \tfrac{13}{15};\ \tfrac{12}{24};\ \tfrac{6}{12};\ \tfrac{20}{50}.$$

11. How can you obtain the terms of $\frac{3}{4}$ from the terms of $\frac{6}{8}$? of $\frac{9}{12}$? of $\frac{21}{28}$? of $\frac{75}{100}$?

12. What fraction do you obtain by dividing each term of $\frac{4}{28}$ by 4. What, then, is the value of $\frac{4}{28}$ as compared with the fraction $\frac{1}{7}$?

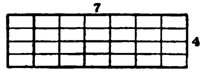


FIGURE 77

13. By using a rectangle, divided as in Fig. 77, show that $\frac{4}{28}$ of the rectangle equals $\frac{1}{7}$ of it.

14. In what lower terms can $\frac{20}{30}$ be expressed? $\frac{40}{100}$? $\frac{30}{42}$? Show that $\frac{20}{30} = \frac{2}{3}$ by means of a properly divided rectangle.

15. What effect on the value of a fraction is made by dividing each of its terms by the same number?

16. How may the value of a fraction be expressed in lower terms?

PRINCIPLE I.—*Multiplying or dividing both terms of a fraction by the same number changes the form of the fraction without altering (or changing) its value.*

Fractions are most easily used when their numerators and their denominators are the smallest possible whole numbers.

DEFINITION.—The fractions are then said to be *in their lowest terms.*

Among the many ways in which $\frac{12}{24}$ can be written are these:

$$\tfrac{12}{24};\ \tfrac{6}{12};\ \tfrac{3}{6};\ \tfrac{2}{4};\ \tfrac{1}{2}.$$

17. Show how $\frac{1}{2}$ is obtained from $\frac{12}{24}$ by the use of Principle I.

18. Give the values of these fractions in their lowest terms:

$$\tfrac{5}{10};\ \tfrac{6}{9};\ \tfrac{21}{28};\ \tfrac{8}{16};\ \tfrac{35}{49};\ \tfrac{32}{56}.$$

To obtain the value of a fraction in the smallest possible terms, it is necessary to divide both terms by the largest exact common divisor of both terms.

DEFINITION.—This divisor is called the *greatest common divisor* of the terms. It is indicated by the initial letters G. C. D.

Dividing both terms of a fraction by their G. C. D. reduces the fraction to its lowest terms.

19. Why are these fractions in their lowest terms?

$$\tfrac{2}{3};\ \tfrac{3}{4};\ \tfrac{4}{5};\ \tfrac{1}{2};\ \tfrac{5}{6};\ \tfrac{3}{7};\ \tfrac{13}{15};\ \tfrac{15}{23};\ \tfrac{10}{27}.$$

Among these fractions is there a factor common to any numerator and its denominator?

DEFINITION.—Two numbers that have no common factor, except 1, are said to be *prime to each other*.

20. Reduce these fractions to their lowest terms:

$$\tfrac{16}{24};\ \tfrac{27}{81};\ \tfrac{60}{80};\ \tfrac{25}{75};\ \tfrac{39}{52};\ \tfrac{74}{111};\ \tfrac{169}{351}.$$

§89. Factors, Prime and Composite.

NOTE.—Review tests of divisibility §57, pp. 73, 74, and use them through this section and the next, when searching for factors.

1. Write down all the factors, or exact divisors, of 36 and of 48.

DEFINITION.—By factor is here meant *exact* divisor, or a divisor that is contained without a remainder.

CONVENIENT FORM

$$\begin{array}{r|l} 2 & 36,\ 48 \\ \hline 2 & 18,\ 24 \\ \hline 3 & 9,\ 12 \\ \hline 2 & 3,\ \ 4 \\ \hline & 3,\ \ 2 \end{array}$$

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, and 36.

The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.

2. What factors are common to both 36 and 48?

3. What is the *greatest* factor that is common to 36 and 48?

4. Arrange the factors of 42 and 105 as the factors of 36 and 48 are arranged in problem 1, and answer questions like 2 and 3 for the factors of 42 and 105.

5. Give the factors that are common to the 3 numbers, 18, 24, and 36. Give the *greatest* factor common to all three numbers.

6. Answer questions like those of problem 5 for these numbers: (1) 12, 72, and 84; (2) 42, 98, and 168; (3) 36, 84, 96, and 108.

7. Are there factors of 5 other than itself and 1? of 7? of 13?

DEFINITIONS.—A number that has no factors except itself and 1 is a *prime* number. A number that has factors beside itself and 1 is a *composite* number.

8. Name the prime numbers from 1 to 100; the composite numbers from 1 to 100.

QUERY.—Is 2 a prime or a composite number?

DEFINITIONS.—A number that can be exactly divided by 2 is an *even number*. All numbers that can not be exactly divided by 2 are *odd numbers*.

9. How can an even number be quickly recognized? (See test of divisibility by 2, p. 73.)

10. Name the even numbers from 0 to 50; the odd numbers.

11. Tell what numbers of these are (1) even, (2) odd, (3) prime, (4) composite: 5, 8, 9, 2, 21, 15, 19, 26, 27, 38, 41, 42.

12. Mention some numbers that are both odd and composite; even and composite; odd and prime.

13. Factors of 28 are 7 and 4. As 7 is a prime number it is called a *prime factor*. What is a prime factor?

14. Write the prime factors of 21; of 24; of 25; of 27; of 30.

NOTE.—Write out the prime factors of these numbers as they are here written out for 96:

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3.$$

15. How many times does 2 occur as a prime factor in 96? This may be indicated by writing 96 thus: $2^5 \times 3$. The small 5 written to the right and above the 2 is to show how many times 2 is to be used *as a factor*.

§90. Greatest Common Divisor by Prime Factors.

1. What are the prime factors of 36 and 48? (See problem 1, §89.) Write 36 in the form given for 96 in problem 15 of the last section. Write 48 also in this form.

2. Will each of the *common* prime factors of 36 and 48 divide the G. C. D. of 36 and 48? Will the product of all the common prime factors divide the G. C. D. exactly?

3. Answer questions like 1 and 2 for the numbers 42 and 105; for 96 and 216; for 75 and 250.

4. When any common prime factor occurs oftener than once in one or both of the numbers, how often does it occur in the G. C. D. of those numbers? Answer by examining these pairs of numbers:

(1) 12 and 48; (3) 75 and 250;
(2) 54 and 405; (4) 98 and 343.

5. Make a rule for finding the G. C. D. of two or more numbers from their common prime factors when no common prime factors are repeated in any of the numbers. Test your rule by finding the G. C. D. of 30 and 42; of 105 and 231; of 30 and 70.

6. Make a rule that will give the G. C. D. of two numbers when one or more of the common prime factors is repeated in one or both of the numbers, and test your rule by finding the G. C. D. of 72 and 108; of 288 and 648; of 675 and 1125.

7. Find the G. C. D. of 792 and 1080.

CONVENIENT FORM

$$\begin{array}{r|rr}
2) & 792 & 1080 \\ \hline
2) & 396 & 540 \\ \hline
2) & 198 & 270 \\ \hline
3) & 99 & 135 \\ \hline
3) & 33 & 45 \\ \hline
 & 11 & 3)\,15 \\ \hline
 & & 5
\end{array}$$

$$792 = 2^3 \times 3^2 \times 11$$
$$1080 = 2^3 \times 3^3 \times \ 5$$
$$\text{G. C. D.} = 2^3 \times 3^2 = 8 \times 9 = 72$$

PRINCIPLE II.—*The G. C. D. of two or more numbers equals the product of all the prime factors common to all the numbers, each common prime factor being used the smallest number of times it occurs in any one of the given numbers.*

8. Find the G. C. D. of the sets of numbers in problem 6, §**89**.

9. Find the G. C. D. of the following, applying tests for divisibility:

(1) 100 and 120; (5) 108, 162, and 216;
(2) 54 and 144; (6) 66, 176, and 286;
(3) 225 and 315; (7) 315, 630, and 756;
(4) 215 and 1935; (8) 162, 1134, and 1458.

10. Reduce these fractions to their lowest terms by dividing both terms by their G. C. D.:

(1) $\frac{16}{18}$; (4) $\frac{180}{210}$; (7) $\frac{360}{416}$; (10) $\frac{90}{315}$;
(2) $\frac{36}{49}$; (5) $\frac{108}{168}$; (8) $\frac{63}{147}$; (11) $\frac{68}{748}$;
(3) $\frac{40}{75}$; (6) $\frac{240}{264}$; (9) $\frac{102}{136}$; (12) $\frac{96}{672}$.

§91. Problems.

In each case when the answer to the problem is a fraction, or ratio, it must be expressed in its *lowest* terms.

1. A man works 48 da. out of 64 da. What part of 64 da. does he work?

2. A grocer bought 24 boxes of oranges and sold 16. What part of his purchase was sold?

3. Out of 56 bu. of potatoes a huckster sold 49 bu. What part of his potatoes was sold?

4. A street car on one line makes a weekly average of 72 trips. A car on another line makes an average of 144 trips. What is the ratio of the former to the latter?

5. A man pays \$50 for wood and \$125 for coal in one season. Find the ratio of the cost of the wood to the cost of the coal.

6. Out of 1000 ft. of lumber purchased, 500 ft. were used for the flooring of two rooms. Let x equal the ratio of the number of feet of lumber used to the total number of feet purchased. Find the value of x.

7. Out of 126 bu. of oats, a livery man fed 63 bu. in 1 wk. Let y equal the ratio of the quantity of oats bought to the quantity of oats fed. Find y.

8. In 667 lb. of sandy loam there were 377 lb. of sand and gravel. What part of the soil by weight was sand and gravel?

9. The human body needs about 94.5 oz. of water and solid food each day, of which 64.8 oz. should be water. The water is what part, by weight, of the total quantity of solid food and water?

SUGGESTION.—Both terms of the fraction may be multiplied by 10 without changing the value of the fraction. Then find the G. C. D. of the new terms and divide both new terms by it.

10. The total population of a certain city is 36,729, of which 12,243 are colored and 10,017 are foreign. What part of the entire population is colored? What part is foreign?

11. What part of the total population is made up of colored and of foreign persons?

The solution of problem 10 will show that $\frac{1}{3}$ of the population is colored, and that $\frac{3}{11}$ of it is foreign. We can solve problem 11 if we can find the value of $\frac{1}{3}+\frac{3}{11}$.

12. A boy spent $\frac{2}{5}$ of his money and gave away $\frac{1}{3}$ of it; what part of it did he have left?

To solve this problem we must know how to add and subtract fractions. This is what we shall study next.

§92. Fractions Having a Common Fractional Unit (a Common Denominator).

1. Complete these equations:

$\frac{4}{9}$ yd. + $\frac{3}{9}$ yd. =	$\frac{3}{5}$ gal. + $\frac{2}{5}$ gal. =	$\frac{6}{b}+\frac{9}{b}=$
\$ $\frac{5}{10}$ + \$ $\frac{8}{10}$ =	$\frac{5}{17}+\frac{6}{17}=$	$\frac{7}{x}+\frac{1}{x}=$
$\frac{5}{8}$ hr. + $\frac{1}{8}$ hr. =	$\frac{6}{19}+\frac{9}{19}=$	$\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}$
$\frac{3}{7}$ wk. + $\frac{2}{7}$ wk. =	$\frac{7}{12}+\frac{1}{12}=$	$\frac{x}{z}+\frac{y}{z}=$
$\frac{3}{5}$ bu. + $\frac{1}{5}$ bu. =	$\frac{5}{a}+\frac{6}{a}=$	$\frac{m}{n}+\frac{p}{n}=$

NOTE.—$\frac{5}{a}$ is read, "5 divided by a," and $\frac{x}{z}$ is read, "x divided by z." $\frac{a+b}{c}$ is read, "a plus b divided by c."

2. Make a rule for adding fractions having a common denominator.

3. Complete these equations:

$\frac{4}{9}-\frac{3}{9}=$	$\frac{8}{9}-\frac{5}{9}=$	$\frac{7}{15}-\frac{3}{15}=$	\$ $\frac{9}{10}$ − \$ $\frac{5}{10}$ =
$\frac{5}{6}-\frac{3}{6}=$	$\frac{7}{12}-\frac{3}{12}=$	$\frac{10}{12}-\frac{3}{12}=$	$\frac{11}{24}$ hr. − $\frac{5}{24}$ hr. =
$\frac{7}{10}-\frac{4}{10}=$	$\frac{5}{7}-\frac{2}{7}=$	$\frac{7}{8}-\frac{5}{8}=$	$\frac{13}{14}$ da. − $\frac{5}{14}$ da. =
$\frac{a}{c}-\frac{b}{c}=$	$\frac{m}{x}-\frac{n}{x}=$	$\frac{c}{r}-\frac{x}{r}=$	$\frac{a}{n}-\frac{x}{n}=$

NOTE.—$\frac{a-b}{c}$ is read, "a minus b divided by c."

4. Make a rule for finding the difference between two fractions having a common denominator.

5. Denoting any two fractions having common denominators by $\frac{x}{y}$ and $\frac{z}{y}$, state in symbols:

PRINCIPLE III.—*The sum, or the difference, of any two fractions having common denominators equals the sum, or the difference, of their numerators, divided by the common denominator.*

§93. Fractions Easily Reduced to Common Fractional Unit.

1. $\frac{1}{4}$ equals how many 12ths? 8ths? 16ths? 20ths? 64ths? 100ths?

2. $\frac{7}{12}$ hr. + $\frac{1}{4}$ hr. = x hr. What is the value of x?

SOLUTION.— $\frac{7}{12}$ hr. + $\frac{3}{12}$ hr. = $\frac{10}{12}$ hr. = $\frac{5}{6}$ hr. $x = \frac{5}{6}$.

3. $\frac{7}{12}$ hr. − $\frac{1}{4}$ hr. = y hr. What is y?

4. Find these sums and differences:

$\frac{2}{5} + \frac{3}{10} =$	$\frac{6}{7} - \frac{3}{14} =$	$\frac{9}{11} + \frac{3}{22} + \frac{2}{33} =$
$\frac{1}{2} + \frac{3}{8} =$	$\frac{3}{8} - \frac{3}{16} =$	$\frac{13}{32} + \frac{5}{16} + \frac{3}{8} =$
$\frac{7}{9} - \frac{1}{3} =$	$\frac{7}{15} - \frac{1}{6} =$	$\frac{7}{14} + \frac{3}{28} - \frac{5}{42} =$

Fractions, whose fractional units are not the same, must be expressed in a common unit before they can be added or subtracted. To do this we must find a fractional unit, whose denominator can be exactly divided by each of the given denominators. It will be the simplest always to select the fractional unit whose denominator is the *least* number that each of the given numbers will divide.

DEFINITION.—A denominator which is common to two or more fractions is called a *common denominator*. When this common denominator is the least number that can be found which may be used as a common denominator of the fractions, it is called the *least common denominator*, and is written L. C. D.

Before studying more difficult fractions we must learn how to find least common denominators. We begin by a study of multiples, common multiples, and least common multiples of numbers.

5. Five measures of last year's growth of twigs from the lower branches on the north side of an oak tree are: $5\frac{1}{2}''$, $6\frac{1}{4}''$, $4\frac{3}{4}''$, $5\frac{1}{2}''$ and $6\frac{1}{4}''$. What is the average growth?

6. From the lower branches on the east and west sides of the same tree the measures are: $6\frac{3}{4}''$, $4\frac{3}{4}''$, $6\frac{1}{4}''$, $6\frac{1}{4}''$ and $6\frac{1}{2}''$. What is the average growth of twigs on the east and west sides of the tree?

7. For twigs on the south side of the same tree the measures are: $6\frac{1}{4}''$, $6\frac{3}{4}''$, $6\frac{1}{4}''$, $8\frac{3}{4}''$ and $4\frac{1}{2}''$. What is the average growth of twigs on the south side of the tree?

8. From the middle branches of the oak tree 5 measures of branches from the north side are: $5\frac{3}{4}''$, $4\frac{3}{4}''$, $7\frac{1}{2}''$, $3\frac{3}{4}''$ and $5\frac{1}{8}''$; of branches from the east and west sides are: $4\frac{3}{4}''$, $7\frac{1}{2}''$, $5\frac{1}{2}''$, $7\frac{1}{4}''$ and $6\frac{3}{4}''$; of branches from the north side: $5\frac{1}{2}''$, $6\frac{1}{2}''$, $7\frac{3}{4}''$, $6\frac{5}{8}''$ and $7\frac{1}{2}''$. How much greater is the average growth of twigs on the south side than the average growth of those on the north side of the tree? than twigs from the east and west sides?

§94. Multiples.

ORAL WORK

42 yd. is exactly measured by 7 yd., 3 yd., and 2 yards.

$21 is exactly measured by $7 and $3.

56 pk. is exactly measured or divisible by 7 pk. and 8 pecks.

42 is a multiple of 7, 3, and 2. 21 is a multiple of 7 and 3. 56 is a multiple of 7 and 8. What are some of the multiples of 3 and 5; of 5 and 7; of 2 and 11?

1. What then is a multiple of a number?

DEFINITION.—A number that can be exactly divided by another number is called a *multiple* of the latter number, or a *multiple* of a number is found by *multiplying* it by some whole number.

Following is a list of multiples of 2, from 2 to 36, inclusive:

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36.

Following is a list of all the multiples of 3, to 36:

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36.

2. Why is each number in the first row a multiple of 2?

3. Passing along the rows underscore the numbers that are the same in both rows.

4. In what way are these multiples of 2 and 3 different from the rest? *Ans.* They occur in both rows.

5. What is a common multiple of two numbers?

DEFINITION.—A number that can be exactly divided by two or more numbers, is called a *common multiple* of those numbers.

6. Rewriting the common multiples of 2 and 3 we have:

6, 12, 18, 24, 30, 36.

Can you supply a few more common multiples here without extending the rows above problem 2?

7. What number, besides 2 and 3, will exactly divide all the common multiples of 2 and 3?

8. On the blackboard write out rows of multiples of 3 and of 5, like those above, underscoring, or writing in colored chalk, the multiples that are common to both rows.

9. Will the least of the common multiples of 3 and 5 divide all the other common multiples? What is the least common multiple of two numbers?

DEFINITION.—The *least common multiple* of two or more numbers is the least number that is exactly divisible by each of the numbers. It is usually written L. C. M. for brevity.

§95. Finding the L. C. M.

WRITTEN WORK

1. Make lists of the multiples of 2, 5, and 7, and find their L. C. M.
2. In the same manner, find the L. C. M. of 3, 4, and 5.
3. Note that all the given numbers in problems 1 or 2 are prime to each other. In such a case how can the L. C. M. be found from the numbers?

The L. C. M. of two or more numbers, all prime to each other, is their product.

4. Illustrate the truth of this statement by two sets of numbers of your own selection, one to contain two numbers, and the other three, the numbers of each set being prime to each other.
5. 4, 8, 12, 16, 20, 24 are multiples of 4; and 6, 12, 18, 24, 30 are multiples of 6. What is the least common multiple of 4 and 6?
6. This number is the product of 4 and 6 divided by what? 2 is the common factor of 4 and 6.
7. Are 4 and 10 prime to each other? What is their greatest common factor or divisor?
8. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40 are multiples of 4, and 10, 20, 30, 40 are multiples of 10. What are some common multiples of 4 and 10? What is their least common multiple?
9. Divide the product of 4 and 10 by 2, their G. C. D. What is the result? Compare this result with the last answer to problem 8.

The L. C. M. of two numbers not prime to each other is their product divided by their greatest common divisor (G. C. D.).

10. What is the greatest common factor of 15 and 20? What is the product of 15 and 20? How can you find the L. C. M.?
11. In the same manner find the L. C. M. of 8 and 10; of 12 and 24; of 15 and 25; of 24 and 60; of 40 and 36.

§96. Shorter Process for Three or More Numbers.

ORAL WORK

1. In a certain number there are 8 24's; how many 12's are there in the number? how many 8's? how many 6's?

2. In a certain number, x, there are 15 6's. How many 3's are there in x? how many 2's?

3. If 6 exactly divides a certain number, x, what other numbers also exactly divide x?

4. If any composite number exactly divides a number, y, what other numbers also divide y?

Any multiple of a number is divisible by all the prime factors of that number.

WRITTEN WORK

1. Find the L. C. M. of 150, 504, and 540.

SOLUTION.—
$$150 = 2 \times 3 \times 5 \times 5 = 2 \times 3 \times 5^2.$$
$$504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 2^3 \times 3^2 \times 7.$$
$$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^2 \times 3^3 \times 5.$$

The L. C. M. is a multiple of each of these numbers severally. By the above principle it must then contain the prime factors 2, 3, 5, and 7. But, since $25 = 5^2$ is a factor of 150 the L. C. M. must contain 5 twice as a factor. Since 8, or 2^3, is a factor of 504, the L. C. M. must contain 2 as a factor three times. How many times must the L. C. M. contain 3 as a factor? How many times must the L. C. M. contain 7 as a factor? The number that contains just these factors and no others is $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7 = 2^3 \times 3^3 \times 5^2 \times 7 = 37,800$.

37,800 is the least common multiple, because it contains the necessary factors and no others. Any other common multiple must contain all these factors and some other, and it would therefore be larger than 37,800.

What prime factors must be in the L. C. M. of three numbers?

If a prime factor is repeated in one or more of the numbers, how often must it occur in the L. C. M.?

PRINCIPLE IV.—*The L. C. M. of two or more numbers is the product of all the different prime factors of all the numbers, each factor occurring the greatest number of times it occurs in any one of the numbers.*

2. Find the L. C. M. of these numbers and show your work:

(1) 9, 12, 18; (4) 15, 25, 42, 50;
(2) 6, 24, 30; (5) 7, 28, 24, 42;
(3) 18, 30, 36; (6) 6, 18, 27, 36.

3. The work can be shortened by this arrangement. Let us solve problem 2, (6).

CONVENIENT FORM

$$\begin{array}{r|rrrr} 3) & 6, & 18, & 27, & 36 \\ \hline 2) & 2, & 6, & 9, & 12 \\ \hline 3) & 1, & 3, & 9, & 6 \\ \hline & 1, & 1, & 3, & 2 \end{array}$$

$$\text{L. C. M.} = 3 \times 2 \times 3 \times 3 \times 2 = 108.$$

EXPLANATION.—Select any prime number, as 3, that will divide two or more of the numbers whose L. C. M. is sought. Divide it into all the numbers that are exactly divisible by it, writing in the line below the quotients and any numbers the chosen divisor does not exactly divide.

Select another prime number and do as before. Continue until the numbers last brought down are all prime. The continued product of the divisors and the numbers remaining in the last horizontal line is the L. C. M.

A little study will show this to be merely a more convenient way of finding the factors described in Principle IV.

The least common denominator defined on page 147 is the L. C. M. of all the denominators of the fractions.

4. Find the least common denominator and then add

(1) $\frac{2}{15}$, $\frac{3}{25}$, $\frac{9}{42}$, and $\frac{13}{50}$; (3) $\frac{1}{6}$, $\frac{5}{18}$, $\frac{2}{27}$, and $\frac{1}{36}$;
(2) $\frac{5}{7}$, $\frac{1}{28}$, $\frac{1}{24}$, and $\frac{5}{42}$; (4) $\frac{2}{9}$, $\frac{5}{13}$, $\frac{3}{26}$, and $\frac{7}{39}$.

§97. Definitions and Principles.

Thus far we have had to do with two kinds of numbers: (1) whole numbers, or *Integers*, and (2) fractional numbers, or *Fractions*.

DEFINITIONS.—A proper fraction is a fraction whose numerator is less than its denominator: as $\frac{2}{3}$, $\frac{3}{5}$, $\frac{2}{7}$.

An improper fraction is a fraction whose numerator is equal to, or greater than, its denominator: as $\frac{3}{3}$, $\frac{3}{2}$, $\frac{7}{3}$, $\frac{8}{2}$.

A mixed number is a number, such as $2\frac{1}{2}$, $12\frac{3}{4}$, $7\frac{5}{6}$, that is composed of an integer and a fraction.

1. Name the proper and the improper fractions, the mixed and the whole numbers:

$\frac{4}{3}$, $\frac{3}{8}$, $\frac{1}{16}$, 5, $\frac{12}{6}$, $\frac{8}{2}$, $\frac{2}{8}$, $1\frac{7}{8}$, $3\frac{1}{15}$, 6.75, $\frac{3}{15}$, 2.

2. Make and give an example of each class.

3. How many times is 1 contained in 3? in 18? in 25? in any number?

We may write this in symbols, thus:

$$\frac{3}{1} = 3; \quad \frac{18}{1} = 18; \quad \frac{25}{1} = 25; \quad \frac{a}{1} = a.$$

Any whole number may be written in the fractional form by writing 1 for its denominator.

4. Write the following numbers in fractional form:

$$12; \quad 24; \quad 32; \quad 68; \quad a; \quad x$$

5. How many halves are there in 1? in 2? in 5? in $2\frac{1}{2}$? in $5\frac{1}{2}$?

6. How many 5ths are there in 1? in 2? in 4? in 6? in $6\frac{2}{5}$? in $7\frac{3}{5}$?

7. Change the following integers into 6ths: 2; 5; 16; 29; 120.

8. Change the following numbers into 12ths: 2; 8; 12; 20; 56; 128.

9. How may any whole number be changed into 12ths? into 9ths? into 24ths? into a fraction having any given denominator?

PRINCIPLE V.—*Any integer may be expressed in the form of a fraction having a given denominator by multiplying the integer by the given denominator and writing the product over that denominator.*

10. Express 6, 18, 17, 22, 39, 28 as 4ths; as 7ths; as 10ths; as 100ths.

11. Change into 6ths, 2; $4\frac{1}{6}$; $24\frac{5}{6}$; $37\frac{5}{6}$.

12. Change the following numbers into improper fractions having 7 for a denominator:

$$3\tfrac{1}{7}; \quad 24\tfrac{3}{7}; \quad 106\tfrac{3}{7}; \quad 234\tfrac{6}{7}; \quad 648.$$

13. How can you change any mixed number into an improper fraction whose denominator is the denominator of the given fraction?

PRINCIPLE VI.—*A mixed number may be expressed as an improper fraction by multiplying the whole number by the denominator, adding the numerator to the product, and writing the sum over the given denominator.*

14. Reduce to improper fractions:

$$2\tfrac{1}{5}; \quad 6\tfrac{1}{3}; \quad 12\tfrac{1}{2}; \quad 18\tfrac{3}{4}; \quad 25\tfrac{4}{7}; \quad 328\tfrac{5}{9}; \quad 609\tfrac{11}{13}.$$

15. Express $\frac{12}{6}$ as a whole number. Change $\frac{13}{6}$ to a mixed number.

16. Change to whole, or mixed, numbers the following improper fractions:

$\frac{6}{3}$; $\frac{4}{3}$; $\frac{7}{2}$; $\frac{12}{7}$; $\frac{14}{7}$; $\frac{16}{7}$; $\frac{24}{3}$; $\frac{18}{5}$; $\frac{25}{9}$; $\frac{36}{11}$; $\frac{36}{12}$; $\frac{45}{9}$; $\frac{20}{7}$; $\frac{3}{2}$.

17. How may any improper fraction be changed to a whole, or mixed, number?

PRINCIPLE VII.—*An improper fraction may be changed to a whole, or mixed, number by making the indicated division.*

18. Reduce to whole, or mixed, numbers the following:

$\frac{4}{2}$; $\frac{16}{7}$; $\frac{28}{14}$; $\frac{706}{7}$; $\frac{95}{7}$; $\frac{343}{100}$; $\frac{1682}{13}$.

§98. Addition of Fractions.

1. If the numbers written on the lines of the drawing are the lengths in feet of the inside walls of the house, how many feet of lumber would there be in a baseboard not over 1″ thick, 1 ft. wide extending entirely around the inside of the house, deducting 6 board feet for doors?

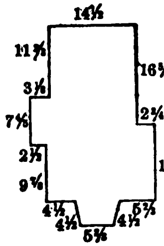
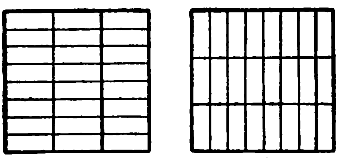


FIGURE 78

To solve this problem we add first all the whole numbers and then all the fractions. We must now learn how to add such common fractions as occur here. This problem will be solved later. (See problem 9 below.)

2. A man owns $\frac{2}{3}$ of an acre in one block and $\frac{7}{8}$ of an acre in another block; how much land does he own?

This problem requires us to add $\frac{2}{3}$ and $\frac{7}{8}$.



(*a*) $\frac{2}{3} = \frac{16}{24}$ (*b*) $\frac{7}{8} = \frac{21}{24}$

FIGURE 79

I. GEOMETRICAL SOLUTION.—Fig. 79 (*a*) is divided by the vertical lines into 3 equal parts and by the horizontal lines into 8 equal parts. Into how many equal parts do the 2 sets of parallel lines divide the square (*a*)?

Show by Fig. 79 (*a*) that $\frac{2}{3} = \frac{16}{24}$.
Show by Fig. 79 (*b*) that $\frac{7}{8} = \frac{21}{24}$.

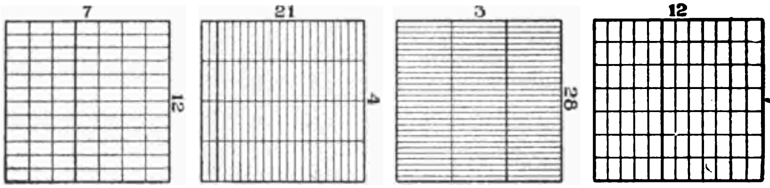
$$\frac{2}{3} + \frac{7}{8} = \frac{16}{24} + \frac{21}{24} = \frac{37}{24} = 1\frac{13}{24}.$$ *Ans.* $1\frac{13}{24}$ acres.

II. ARITHMETICAL SOLUTION.—What is the least common denominator of $\frac{2}{3}$ and $\frac{7}{8}$? What, then, is the largest fractional unit (therefore having the least denominator) in which both $\frac{2}{3}$ and $\frac{7}{8}$ can be exactly expressed?

$\frac{2}{3}$ = how many 24ths? $\frac{7}{8}$ = how many 24ths?

$$\frac{2}{3}+\frac{7}{8}=\frac{16}{24}+\frac{21}{24}=\frac{16+21}{24}=\frac{37}{24}=1\frac{13}{24}.\quad \textit{Ans. } 1\tfrac{13}{24} \text{ acres.}$$

3. Add $\frac{3}{7}$, $\frac{2}{21}$, $\frac{2}{3}$, and $\frac{5}{12}$.



(a) $\frac{3}{7}=\frac{36}{84}$ (b) $\frac{2}{21}=\frac{8}{84}$ (c) $\frac{2}{3}=\frac{56}{84}$ (d) $\frac{5}{12}=\frac{35}{84}$

FIGURE 80

I. GEOMETRICAL SOLUTION.—Show from Fig. 80 (*a*), that $\frac{3}{7}=\frac{36}{84}$; from Fig. 80 (*b*), that $\frac{2}{21}=\frac{8}{84}$; from (*c*) that $\frac{2}{3}=\frac{56}{84}$; from (*d*) that $\frac{5}{12}=\frac{35}{84}$. $\frac{36}{84}+\frac{8}{84}+\frac{56}{84}+\frac{35}{84}=\frac{135}{84}=1\frac{51}{84}=1\frac{17}{28}$.

II. ARITHMETICAL SOLUTION.—The least common denominator of the fractions is 84.

$$\frac{3}{7}+\frac{2}{21}+\frac{2}{3}+\frac{5}{12}=\frac{3\times 12}{7\times 12}+\frac{2\times 4}{21\times 4}+\frac{2\times 28}{3\times 28}+\frac{5\times 7}{12\times 7}=\frac{36+8+56+35}{84}$$
$$=\frac{135}{84}=1\frac{51}{84}=1\frac{17}{28}.$$

The last step consists in reducing the result to its simplest form.

4. $\frac{3}{11}$ of the weight of a quantity of soil was gravel, and $\frac{3}{7}$ was sand; what part of the soil by weight was sand and gravel? Draw a figure and give the geometrical solution. Give also the arithmetical solution.

5. One side of a triangle is $2\frac{3}{4}$ in., another is $2\frac{2}{3}$ in., and the third is $3\frac{5}{8}$ in; how long is the perimeter of (distance around) the triangle? Add first the whole numbers, then the fractions, and finally add the sums.

6. One side of a 4-sided figure is $6\frac{4}{5}$ in. long, a second side is $3\frac{5}{8}$ in., the third $7\frac{3}{8}$ in., and the fourth $7\frac{5}{12}$ in. What is the perimeter of the figure?

7. A coal dealer bought 4 carloads of coal of the following weights: $31\frac{11}{16}$ T., $27\frac{1}{3}$ T., $29\frac{4}{5}$ T., and $30\frac{3}{4}$ T. Find the combined weight, in tons.

8. Solve these problems as rapidly as you can work accurately, using your pencil merely to write the products and the sums:

(1) $\frac{2}{5}+\frac{3}{7}=?$	(4) $\frac{5}{7}+\frac{1}{11}=?$	(7) $\frac{11}{15}+\frac{7}{8}=?$
(2) $\frac{3}{8}+\frac{7}{9}=?$	(5) $\frac{12}{13}+\frac{1}{10}=?$	(8) $\frac{a}{c}+\frac{b}{c}=?$
(3) $\frac{6}{11}+\frac{5}{12}=?$	(6) $\frac{7}{12}+\frac{10}{13}=?$	(9) $\frac{a}{b}+\frac{b}{c}=?$

9. Solve problem 1 of this section.

§99. Subtraction of Fractions.

1. A boy had $\$\frac{4}{5}$ and spent $\$\frac{1}{2}$; what part of a dollar did he have left?

ARITHMETICAL SOLUTION.—The least common denominator is $2\times 5=10$.

$\frac{4}{5}=\frac{8}{10}$; $\frac{1}{2}=\frac{5}{10}$. Then, $\frac{4}{5}-\frac{1}{2}=\frac{8}{10}-\frac{5}{10}=\frac{3}{10}$. *Ans.* $\$\frac{3}{10}$

2. William is $4\frac{2}{3}$ ft. tall and James is $4\frac{5}{12}$ ft. tall; who is the taller and by how much?

3. A man owned $\frac{15}{16}$ A. of land and sold $\frac{3}{7}$ A.; how much land did he then own?

4. From $\frac{5}{6}$ lb. of loam $\frac{2}{7}$ lb. sand was removed; how much of the loam remained?

5. A man having $\$4\frac{1}{5}$ paid a debt of $\$2\frac{3}{4}$; how much money had the man after paying the debt?

SOLUTION.— $\frac{1}{5}=\frac{4}{20}$, $\frac{3}{4}=\frac{15}{20}$. As $\frac{4}{20}$ is less than $\frac{15}{20}$, write $4\frac{4}{20}$ in the form $3\frac{24}{20}$. The problem is then $3\frac{24}{20}-2\frac{15}{20}=1\frac{9}{20}$ for $3-2=1$ and $\frac{24}{20}-\frac{15}{20}=\frac{9}{20}$.

6. Solve these problems:

(1) $13\frac{1}{5}-7\frac{2}{5}=?$	(4) $128\frac{4}{5}-97\frac{15}{16}=?$
(2) $28\frac{3}{7}-19\frac{5}{9}=?$	(5) $639\frac{2}{3}-598\frac{7}{11}=?$
(3) $30\frac{1}{20}-16\frac{7}{8}=?$	(6) $1217\frac{18}{61}-989\frac{19}{25}=?$

7. A tree $72\frac{3}{4}$ ft. high is broken off $28\frac{7}{15}$ ft. from the top; how high is the stump?

8. A 5-cent piece weighs $73\frac{1}{6}$ gr. and a quarter dollar weighs $96\frac{9}{20}$ gr.; how much more does a quarter weigh than a 5-cent piece?

9. The dime weighs $38\frac{7}{12}$ gr. Before 1853 it weighed $41\frac{1}{4}$ gr. By how much was the weight of the dime reduced in 1853?

10. A silver dollar weighs $412\frac{1}{2}$ gr. and a double eagle weighs 516 gr. What is the difference between the weight of a double eagle and that of a silver dollar?

11. What is the difference between the weight of the half dollar ($192\frac{9}{10}$ gr.) and that of the quarter dollar ($96\frac{9}{20}$ gr.)?

12. Write out rapidly the values of these sums and differences:

(1) $\frac{1}{5}+\frac{1}{7}=?$	(6) $\frac{2}{5}+\frac{2}{7}=?$	(11) $\frac{6}{7}+\frac{5}{9}=?$
(2) $\frac{1}{8}-\frac{1}{11}=?$	(7) $\frac{3}{7}-\frac{3}{11}=?$	(12) $\frac{7}{11}-\frac{5}{12}=?$
(3) $\frac{1}{4}-\frac{1}{12}=?$	(8) $\frac{8}{11}+\frac{8}{13}=?$	(13) $\frac{4}{17}+\frac{9}{10}=?$
(4) $\frac{1}{a}+\frac{1}{b}=?$	(9) $\frac{a}{c}+\frac{a}{b}=?$	(14) $\frac{a}{b}+\frac{c}{d}=?$
(5) $\frac{1}{a}-\frac{1}{b}=?$	(10) $\frac{m}{x}-\frac{m}{y}=?$	(15) $\frac{m}{n}-\frac{x}{y}=?$

13. Make a rule for finding quickly the sum, or the difference, of any two fractions.

NOTE.—This latter rule may be shown thus,

$$\text{sum} = \frac{\text{1st num.} \times \text{2d den.} + \text{2d num.} \times \text{1st den.}}{\text{1st den.} \times \text{2d den.}}$$

and

$$\text{difference} = \frac{\text{1st num.} \times \text{2d den.} - \text{2d num.} \times \text{1st den.}}{\text{1st den.} \times \text{2d den.}}.$$

The result must always be reduced to the simplest form.

14. In three successive runs, a train loses $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{2}$ hr. How much time does it lose in all?

15. $\frac{2}{5}$ of a man's salary is spent for clothing, $\frac{1}{3}$ for board and lodging, $\frac{1}{6}$ for books and stationery, $\frac{1}{12}$ for traveling expenses. What part of his salary was spent for these purposes? what part remained?

16. In the six days of one week, a man works $8\frac{1}{2}$ hr., $9\frac{3}{4}$ hr., $8\frac{2}{3}$ hr., $9\frac{1}{4}$ hr., $8\frac{3}{4}$ hr., $10\frac{3}{4}$ hr. What is the whole number of hours of work for the week? What are his wages at \$.30 an hour?

17. Four children are to do a piece of work. Three of them do $\frac{2}{5}$, $\frac{1}{3}$, and $\frac{2}{9}$ of it. What part is left for the fourth?

18. $\frac{2}{5}$, $\frac{1}{6}$, and $\frac{2}{15}$ of a man's money are invested in three different enterprises. What part of his money is invested? What part is free?

19. A merchant sold $\frac{7}{12}$ of a gross of buttons to one customer, and $\frac{11}{24}$ of a gross less to another. What part of a gross did the second customer buy?

20. A owns $\frac{7}{15}$ and B $\frac{5}{21}$ of an estate. How much more of the estate does A own than B?

21. A tank of oil is $\frac{4}{5}$ full. If $\frac{1}{2}$ of the contents of the tank is drawn off and then $\frac{4}{15}$ of the remainder, what part is left?

22. A owns $\frac{23}{54}$ of a mill. B's interest is $\frac{16}{47}$ of the mill less than A's. What part of the whole does B own?

23. At $1 a day how much money does a man earn by working $5\frac{3}{4}$ da., $6\frac{2}{3}$ da., and $8\frac{3}{5}$ days?

24. A boy lives where school is taught 1100 hr. a year. He is compelled by sickness and other causes to lose from month to month the following numbers of hours:

$$115\tfrac{2}{3};\ 39\tfrac{3}{4};\ 15\tfrac{5}{6};\ 6\tfrac{11}{12};\ 4\tfrac{4}{5};\ 3\tfrac{2}{5};\ 5\tfrac{3}{5};\ 18\tfrac{1}{3};\ 89\tfrac{7}{10}.$$

How many hours did he lose in all? What part of the whole school year did he lose?

NOTE.—In adding the fractions of problem 24 group together fractions whose denominators are the same, or are easily made the same.

§100. Multiplying a Fraction by a Whole Number.

1. 6 times 3 mi. = how many miles.
2. 8 × 4 yd. = ? yards.
3. 8 × 4 fifths = ? fifths.
4. $5 \times \frac{4}{7} = \frac{x}{7}$; what is the value of x?
5. $9 \times \frac{3}{19} = \frac{x}{19}$; what is the value of x?
6. A line is divided into 11 equal parts, and 5 of them are taken. What fraction represents the part of the whole which is taken?
7. What fraction would represent 3 times this part?
8. Replace the letter in each of these equations by its correct value; the sign (×) should be read "times" in these problems.

(1) $6 \times \frac{2}{3} = \frac{x}{3}$	(5) $5 \times \frac{1}{6} = \frac{x}{6}$	(9) $2 \times \frac{3}{11} = \frac{x}{11}$
(2) $4 \times \frac{1}{2} = \frac{x}{2}$	(6) $3 \times \frac{5}{6} = \frac{x}{6}$	(10) $6 \times \frac{1}{7} = \frac{x}{7}$
(3) $3 \times \frac{1}{4} = \frac{x}{4}$	(7) $4 \times \frac{1}{9} = \frac{x}{9}$	(11) $5 \times \frac{2}{13} = \frac{x}{13}$
(4) $2 \times \frac{3}{4} = \frac{x}{4}$	(8) $2 \times \frac{4}{7} = \frac{x}{7}$	(12) $2 \times \frac{5}{12} = \frac{x}{12}$

In multiplying these fractions by the whole numbers what change was made in the numerator to get the product?

Make a rule for multiplying a fraction by a whole number.

9. Apply your rule to these problems, reducing to whole or mixed numbers all the products that are improper fractions:

(1) $3 \times \frac{4}{5} =$	(5) $11 \times \frac{3}{11} =$	(9) $6 \times \frac{7}{8} =$
(2) $5 \times \frac{3}{7} =$	(6) $9 \times \frac{1}{12} =$	(10) $8 \times \frac{5}{12} =$
(3) $7 \times \frac{2}{9} =$	(7) $4 \times \frac{5}{6} =$	(11) $10 \times \frac{5}{4} =$
(4) $9 \times \frac{5}{6} =$	(8) $6 \times \frac{4}{9} =$	(12) $9 \times \frac{11}{12} =$

NOTE.—In such as (7) use cancellation, thus: $\overset{2}{\cancel{4}} \times \frac{5}{\underset{3}{\cancel{6}}} = \frac{10}{3} = 3\frac{1}{3}$.

10. Solve these problems and make a rule for multiplying a fraction quickly by its denominator:

(1) $6 \times \frac{1}{6} = ?$	(5) $12 \times \frac{5}{12} = ?$	(9) $75 \times \frac{46}{75} = ?$
(2) $9 \times \frac{1}{9} = ?$	(6) $18 \times \frac{12}{18} = ?$	(10) $b \times \frac{a}{b} = ?$
(3) $5 \times \frac{3}{5} = ?$	(7) $21 \times \frac{13}{21} = ?$	(11) $m \times \frac{c}{m} = ?$
(4) $7 \times \frac{5}{7} = ?$	(8) $48 \times \frac{135}{48} = ?$	(12) $x \times \frac{y}{x} = ?$

NOTE.—In all such problems use cancellation, thus: $\cancel{7} \times \frac{5}{\cancel{7}} = 5$.

11. Give the values of the letter in each of these problems and make a rule for multiplying a fraction quickly by some factor of its denominator; (use cancellation thus: $\cancel{6} \times \frac{7}{\underset{4}{\cancel{24}}} = \frac{7}{4} = 1\frac{3}{7}$).

(1) $3 \times \frac{5}{6} = \frac{x}{2}$;	(3) $12 \times \frac{7}{24} = \frac{x}{2}$;	(5) $25 \times \frac{19}{125} = \frac{x}{5}$;
(2) $5 \times \frac{7}{15} = \frac{x}{3}$;	(4) $9 \times \frac{5}{36} = \frac{x}{4}$;	(6) $13 \times \frac{16}{91} = \frac{x}{7}$.

State the rule.

PRINCIPLE VIII.—*A fraction is multiplied by*

1. Multiplying its numerator by the multiplier; or,

2. Dividing its denominator by the multiplier.

NOTE.—*The usefulness of the method of cancellation consists in making these two processes undo* (balance) *each other.* (See problem II.)

QUERY.—When will the second method (Principle VIII, 2), be the more convenient?

A fraction is multiplied by its denominator by dropping its denominator.

§101. Multiplying a Mixed Number by a Whole Number.

1. What will be the cost of 12 yd. of muslin at $8\frac{2}{3}$ cents?

SOLUTION.—If 1 yd. costs $8\frac{2}{3}$ ¢ what will 12 yd. cost?
$12 \times 8\frac{2}{3}$ means $12 \times 8 + 12 \times \frac{2}{3} = 96 + 8 = 104$. *Ans.* \$1.04.

NOTE.—*In dealing with such expressions as* $12 \times 8 + 12 \times \frac{2}{3}$, *which contain both signs,* ($\times$) *and* ($+$), *the indicated multiplications should be performed first, and then the additions.*

2. $13 \times 4\frac{5}{9} = ?$

$$\begin{array}{rl} 4\frac{5}{9} & \\ 13 & \\ \hline 52 & = 13 \times 4 \\ 7\frac{2}{9} & = 13 \times \frac{5}{9} \\ \hline 59\frac{2}{9} & = 13 \times 4\frac{5}{9} \end{array}$$

Multiply the whole numbers in the usual way.
$13 \times \frac{5}{9} = \frac{65}{9} = 7\frac{2}{9}$, as in problem 8, §100.

PROBLEMS

Solve the following, using the more convenient method in each case, employing cancellation whenever it shortens the work:

1. $5 \times \frac{11}{15} =$	5. $9 \times \frac{17}{36} =$	9. $2 \times \frac{13}{4} =$
2. $3 \times \frac{5}{9} =$	6. $9 \times \frac{8}{9} =$	10. $3 \times 6\frac{7}{8} =$
3. $5 \times 4\frac{3}{4} =$	7. $9 \times \frac{7}{12} =$	11. $9 \times \frac{5}{12} =$
4. $2 \times 2\frac{1}{2} =$	8. $9 \times \frac{4}{5} =$	12. $8 \times \frac{9}{11} =$

13. Find the cost of 25 yd. of cloth at \$$\frac{2}{5}$ per yard.

14. A earns \$$1\frac{3}{4}$, and B earns 4 times as much. B earns \$$x$. Find x.

15. A grocer sold $\frac{3}{4}$ bu. of apples to one customer, and 5 times that quantity to another. He sold y bu. to the second customer. Find y. If apples were \$.40 a peck what amount of money did the grocer receive from the first customer? from the second?

16. If a man can cut $\frac{7}{8}$ of a cord of wood in one day, how much can he cut in 6 da., working at the same rate?

17. One boy lives $\frac{5}{16}$ mi. from school and another 4 times as far. What is the distance from school to the second boy's home?

18. Solve problems based upon the following facts:

An iron tube weighs $\frac{7}{8}$ lb. per foot.

$\frac{8}{9}$ of the weight of water is oxygen.

The circumference of a bicycle wheel is $6\frac{17}{21}$ feet.

A west wind moved $47\frac{4}{5}$ mi. per hour.

§102. Multiplying a Whole Number by a Fraction.

The whole number is here the multiplicand.

DEFINITION.—To *multiply* a whole number by a *fraction* means to divide the multiplicand into as many equal parts as there are units in the denominator, and to take as many of these equal parts as there are units in the numerator of the multiplier.

ILLUSTRATION.—12 multiplied by $\frac{5}{6}$ means that 5 of the 6 equal parts of 12 are wanted. $\frac{1}{6}$ times $12 = 2$, and $\frac{5}{6}$ times $12 = 5 \times 2 = 10$.

$\frac{5}{6}$ times 12 and $\frac{5}{6}$ of 12 mean the same thing.

1. Solve these problems, reading the sign (×) "times":

(1) $\frac{3}{5} \times 10 = ?$	(4) $\frac{2}{3} \times 15 = ?$	(7) $\frac{4}{9} \times 45 = ?$
(2) $\frac{4}{5} \times 25 = ?$	(5) $\frac{3}{7} \times 28 = ?$	(8) $\frac{8}{13} \times 26 = ?$
(3) $\frac{7}{8} \times 32 = ?$	(6) $\frac{6}{11} \times 33 = ?$	(9) $\frac{7}{11} \times 47 = ?$

SUGGESTION.—In (9), $\frac{1}{11}$ of $47 = 4\frac{3}{11}$, and $7 \times 4\frac{3}{11} = 28\frac{21}{11} = 28 + 1\frac{10}{11} = 29\frac{10}{11}$. *Ans.* $29\frac{10}{11}$.

2. Solve the following:

(1) $\frac{9}{16} \times 50 = ?$	(3) $\frac{17}{20} \times 105 = ?$	(5) $\frac{5}{23} \times 110 = ?$
(2) $\frac{11}{18} \times 39 = ?$	(4) $\frac{25}{27} \times 220 = ?$	(6) $6\frac{7}{9} \times 28 = ?$

SUGGESTION.—$6\frac{7}{9} \times 28$ means $6 \times 28 + \frac{7}{9} \times 28 = 168 + 21\frac{7}{9} = 189\frac{7}{9}$.

NOTE.—See note to problem 1, page 159.

§103. Factors May Be Interchanged.

In Fig. 81. (*a*) the rectangle is supposed to be $\frac{1}{2}$ in. wide and 5 in. high. The area is then 5 times $\frac{1}{2}$ sq. in. = $2\frac{1}{2}$ square inches.

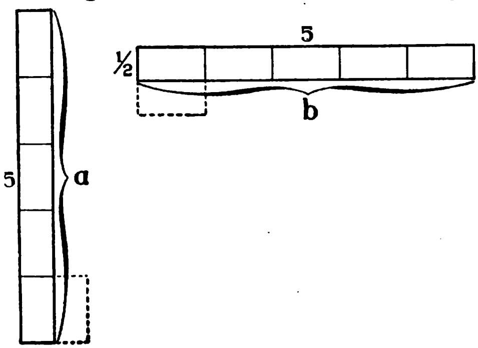


FIGURE 81

In (*b*) the rectangle is 5 in. long and $\frac{1}{2}$ in. high. Its area is $\frac{1}{2}$ times 5 sq. in., or $\frac{1}{2}$ of 5 sq. in. = $2\frac{1}{2}$ sq. in., as is plain from Fig. 81.

This shows the truth that when a fraction and a whole number are to be multiplied it makes no difference in the numerical value of the product which of the factors we regard as the multiplicand.

The *name* of the result is learned from the nature of the problem.

Letting $\frac{x}{y}$ (read "x divided by y") stand for any fraction, and a stand for any whole number, state the following principle in symbols:

The product of a whole number by a fraction, or of a fraction by a whole number, equals the product of the whole number by the numerator, divided by the denominator.

PROBLEMS

1. Find the cost of $2\frac{1}{3}$ yd. of cloth at \$3.

2. A man had \$75; he spent $\frac{2}{3}$ of it for a bicycle, and $\frac{4}{5}$ of the remainder for clothing. Find the amount spent, and the amount he had left.

3. A gallon contains 231 cu. in. How many cubic inches in $\frac{2}{3}$ gal.? in $\frac{5}{6}$ gallon?

4. A cubic foot of granite weighs 170 lb. How many pounds in $5\frac{3}{4}$ cubic feet?

5. $\frac{7}{18}$ of a plot of ground containing 918 sq. rd. was fenced for a garden. The garden contained how many square rods?

6. Make and solve problems, based upon facts obtained by yourself, or upon those given below.

An avoirdupois pound of gold is worth about \$$348\frac{2}{5}$.

A bale of cotton ordinarily weighs 450 lb. Price $17\frac{1}{3}$¢ per pound.

A tank contains 126 gal. linseed oil. Price \$.62 per gallon.

Corn is quoted and sold @ $50\frac{7}{8}$¢ a bu. Wheat is quoted and sold @ $77\frac{5}{8}$¢ a bushel. Oats are quoted and sold @ $32\frac{5}{8}$¢ a bushel.

To plow an acre with a plow cutting a furrow 10″ wide (a 10″ plow) a horse must walk $9\frac{9}{10}$ mi.; with a 12″-plow, a horse must walk $8\frac{1}{4}$ mi.; with a 15″-plow, $6\frac{3}{5}$ miles.

For a team drawing a plow, a distance of 16 mi. is a fair day's work and a distance of 18 mi. is a large day's work. The following table shows to what fractional part of an acre a furrow $\frac{1}{2}$ mi. long and of the stated widths is equivalent:

WIDTH OF FURROW	EQUIVALENT	WIDTH OF FURROW	EQUIVALENT
12″	$\frac{2}{33}$ A.	16″	$\frac{8}{99}$ A.
13″	$\frac{13}{198}$ A.	17″	$\frac{17}{198}$ A.
14″	$\frac{7}{99}$ A.	18″	$\frac{1}{11}$ A.
15″	$\frac{5}{66}$ A.	20″	$\frac{10}{99}$ A

§104. Multiplying a Fraction by a Fraction.

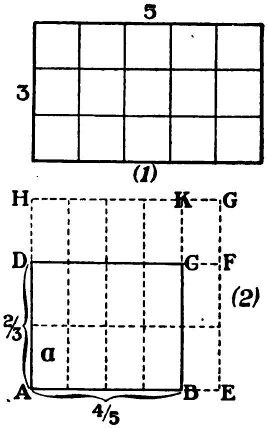


FIGURE 82

1. What is the area of a rectangle 3"×5", Fig. 82 (1)? 6"×8"? 7'×8'? 2 rd. × 80 rd.? 16×20? $a \times b$?

2. How many square feet in the area of a rectangle $\frac{1}{2}$'×8'? $\frac{3}{4}$'×12'? $\frac{7}{8}$'×16'? $\frac{9}{10}$'×30'? (Use cancellation.)

3. What is the area of a rectangle $\frac{2}{3}$"×$\frac{4}{5}$"? [See Fig. 82 (2).]

SOLUTION.—*ABCD* is a rectangle $\frac{4}{5}$" long and $\frac{2}{3}$" wide.

AEGH is a square inch.

What part of a square inch is *ABKH*? What part of *ABKH* is *ABCD*? Point out on the figure the part that represents $\frac{2}{3}$ of $\frac{4}{5}$ of 1 square inch?

Into how many small rectangles, such as *a*, do the dotted lines divide the square inch? What part of the square inch is one of the small rectangles, *a*? How many of these are there in the given rectangle *ABCD*? The area of *ABCD* is then what part of a square inch?

We may write the two expressions for the area equal, thus:

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}.$$

We see then that, if our law for finding the area of the rectangle, which we have found to hold for integers, is to hold for fractions also, we must define both $\frac{2}{3} \times \frac{4}{5}$ and $\frac{4}{5} \times \frac{2}{3}$ to be the same as $\frac{4}{5}$ of $\frac{2}{3}$. Each is equal to $\frac{8}{15}$.

It is clear that, whatever the fractions to be multiplied may be, the square unit [sq. in. in Fig. 82 (2)] may be divided by the cross lines into as many equal parts as there are units in the product of the denominators, and that there will be as many of these equal parts in the given rectangle as there are units in the product of the numerators.

Hence our fractions may be multiplied by merely writing the product of the numerators over the product of the denominators with the dividing line between. Cancellation should be used whenever possible.

4. Draw a square inch and find these products as above:

(1) $\frac{2}{3}'' \times \frac{3}{4}'' = ?$	(4) $\frac{5}{6}'' \times \frac{3}{8}'' = ?$	(7) $\frac{7}{8}'' \times \frac{3}{4}'' = ?$
(2) $\frac{3}{7}'' \times \frac{1}{2}'' = ?$	(5) $\frac{1}{9}'' \times \frac{5}{8}'' = ?$	(8) $\frac{4}{7}'' \times \frac{3}{5}'' = ?$
(3) $\frac{3}{4}'' \times \frac{2}{3}'' = ?$	(6) $\frac{3}{8}'' \times \frac{5}{6}'' = ?$	(9) $\frac{3}{4}'' \times \frac{7}{8}'' = ?$

5. By drawing and dividing a square inch, as in Fig. 81, show that the following equations are true:

(1) $\frac{6}{7} \times \frac{5}{11} = \frac{6}{7}$ of $\frac{5}{11}$; (3) $\frac{6}{7} \times \frac{5}{11} = \frac{5}{11}$ of $\frac{6}{7}$; (5) $\frac{6}{7} \times \frac{5}{11} = \frac{30}{77}$

(2) $\frac{5}{11} \times \frac{6}{7} = \frac{5}{11}$ of $\frac{6}{7}$; (4) $\frac{5}{11} \times \frac{6}{7} = \frac{6}{7}$ of $\frac{5}{11}$; (6) $\frac{5}{11} \times \frac{6}{7} = \frac{30}{77}$

To find $\frac{1}{7}$ of $\frac{5}{11}$, we take a fractional unit which is $\frac{1}{7}$ of $\frac{1}{11}$ (the fractional unit of $\frac{5}{11}$), and use the same number (5) of them, thus obtaining $\frac{5}{77}$. But $\frac{6}{7}$ of $\frac{5}{11} = 6 \times \frac{1}{7}$ of $\frac{5}{11} = 6 \times \frac{5}{77} = \frac{30}{77}$. As $\frac{6}{7} \times \frac{5}{11} = \frac{6}{7}$ of $\frac{5}{11}$, $\frac{6}{7} \times \frac{5}{11} = \frac{30}{77}$. Similar reasoning would show also that $\frac{5}{11} \times \frac{6}{7} = \frac{30}{77}$. Now we recall that the numerator (30) of the product ($\frac{5}{11} \times \frac{6}{7}$) was obtained by multiplying the numerators (6 and 5) of the factors ($\frac{5}{11}$ and $\frac{6}{7}$). How was the denominator (77) of the product obtained?

6. State a rule for quickly multiplying a fraction by a fraction.

7. Solve* these problems *as rapidly as you can work accurately:*

(1) $\frac{3}{8} \times \frac{1}{2}$	(5) $\frac{2}{5} \times \frac{1}{2}$	(9) $\frac{1}{8} \times \frac{2}{3}$	(13) $\frac{2}{9} \times \frac{2}{3}$
(2) $\frac{1}{2} \times \frac{3}{4}$	(6) $\frac{1}{4} \times \frac{2}{3}$	(10) $\frac{2}{3} \times \frac{5}{8}$	(14) $\frac{3}{5} \times \frac{5}{9}$
(3) $\frac{2}{7} \times \frac{2}{5}$	(7) $\frac{3}{7} \times \frac{2}{3}$	(11) $\frac{2}{7} \times \frac{3}{5}$	(15) $\frac{5}{6} \times \frac{2}{3}$
(4) $\frac{3}{5} \times \frac{5}{9}$	(8) $\frac{6}{7} \times \frac{3}{4}$	(12) $\frac{5}{7} \times \frac{2}{9}$	(16) $\frac{5}{8} \times \frac{5}{9}$

8. Letting $\frac{p}{x}$ and $\frac{q}{y}$ denote any two fractions, state the following principle in symbols:

The product of any two fractions is a fraction whose numerator is the product of the numerators and whose denominator is the product of the denominators.

§105. Multiplying a Mixed Number by a Mixed Number.

1. How many square feet in the area of a rectangle $4\frac{3}{4}'' \times 5\frac{1}{2}''$?

(1) How long is *a* (Fig. 83)? how wide? What is its area?

(2) Answer similar questions for *b*, *c*, and *d*.

(3) How long is the entire rectangle? how wide? What is its area?

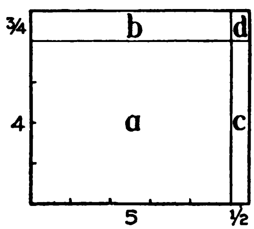


FIGURE 83

Since the area of the entire rectangle equals the sum of the areas of its parts we may write (see note, problem 1, page 159):

$$4\tfrac{3}{4} \times 5\tfrac{1}{2} = 4 \times 5 + \tfrac{3}{4} \times 5 + 4 \times \tfrac{1}{2} + \tfrac{3}{4} \times \tfrac{1}{2} = 20 + 3\tfrac{3}{4} + 2 + \tfrac{3}{8} = 26\tfrac{1}{8}.$$

Ans. $26\frac{1}{8}$ sq. ft.

* Cancel whenever it is possible.

Point out on figure the areas that represent all the parts of the product.

2. Find the cost of $9\frac{1}{2}$ T. hard coal at \$$7\frac{3}{4}$.

First Solution.—If 1 T. costs \$$7\frac{3}{4}$, $9\frac{1}{2}$ T. will cost $9\frac{1}{2} \times$ \$$7\frac{3}{4}$.

Convenient Form

$$\begin{array}{rl}
 & 7\frac{3}{4} \\
 & 9\frac{1}{2} \\
\hline
63 & = 9 \times 7 \\
6\frac{3}{4} & = 9 \times \frac{3}{4} = \frac{27}{4} \\
3\frac{1}{2} & = \frac{1}{2} \times 7 \\
\frac{3}{8} & = \frac{1}{2} \times \frac{3}{4} \\
\hline
73\frac{5}{8} &
\end{array}$$

Explanation

$9\frac{1}{2} \times 7\frac{3}{4}$ means $9 \times 7\frac{3}{4} + \frac{1}{2} \times 7\frac{3}{4}$.
$\frac{1}{2} \times 7\frac{3}{4}$ means $\frac{1}{2}$ of $7\frac{3}{4}$.

$$\begin{array}{r}
9 \times 7\frac{3}{4} = 69\frac{3}{4} \\
\frac{1}{2} \times 7\frac{3}{4} = 3\frac{7}{8} \\
\hline
9\frac{1}{2} \times 7\frac{3}{4} = 73\frac{5}{8}
\end{array}$$

Ans. \$$73\frac{5}{8}$.

Second Solution.—$9\frac{1}{2}$ T. $= \frac{19}{2}$ T.; \$$7\frac{3}{4}$ = \$$\frac{31}{4}$
$\frac{19}{2} \times \frac{31}{4} = \frac{589}{8}$, or $73\frac{5}{8}$. *Ans.* \$$73\frac{5}{8}$.

Find areas of surfaces having the following dimensions:

3. $44\frac{3}{7}$ ft. $\times$ $28\frac{3}{4}$ feet.
4. $36\frac{5}{8}$ ft. $\times$ $27\frac{2}{7}$ feet.
5. $24\frac{3}{4}$ ft. $\times$ $18\frac{2}{3}$ feet.
6. $45\frac{5}{6}$ ft. $\times$ $30\frac{1}{3}$ feet.
7. $27\frac{1}{4}$ ft. $\times$ $18\frac{5}{6}$ feet.

Find the cost of the following items:

8. $24\frac{3}{4}$ doz. eggs @ \$.$16\frac{2}{3}$.
9. $46\frac{3}{4}$ gal. of oil @ \$.$12\frac{1}{2}$.
10. $15\frac{1}{3}$ yd. of cloth @ \$.$66\frac{2}{3}$.
11. $52\frac{3}{4}$ lb. of sugar @ \$.$05\frac{5}{9}$.
12. $265\frac{3}{8}$ M. of pine flooring @ \$$35\frac{1}{4}$.

Make original problems from the following items:

A cubic inch of water weighs $252\frac{24}{25}$ grains.

A water tower is $2\frac{47}{63}$ ft. higher than the mound upon which it is built. The mound is $81\frac{1}{2}$ ft. high.

$69\frac{1}{6}$ statute miles = 1 degree of longitude at the equator.

A pine tree 87 ft. high has no branches for $\frac{13}{29}$ of its height.

$\frac{31}{89}$ of a library containing 55,447 volumes was destroyed by fire.

Flaxseed cost \$$1\frac{3}{5}$ per pound when a certain linseed oil factory bought supplies.

A spring furnishes $28\frac{7}{16}$ bbl. of water daily.

A certain vessel sails $11\frac{3}{4}$ mi. per hour, on an average.

A room is 32 ft. long and $24\frac{5}{6}$ ft. wide. Painting costs \$$\frac{1}{50}$ per square foot.

A cubic foot of water weighs 62.5 pounds.

Gold is $19\frac{1}{4}$ times as heavy as water.

$30\frac{1}{4}$ sq. yd. = 1 square rod.

§106. Dividing a Fraction by a Whole Number.

ORAL WORK

1. What is $\frac{1}{8}$ of 40A.? 40A. divided by 8 equals what?
2. $\frac{1}{7}$ of 35 lb. = ? 35 lb. ÷ 7 = ?
3. $\frac{1}{10}$ of 80 = ? 80 ÷ 10 = ?
4. $\frac{1}{3}$ of 3 fourths = ? 3 fourths ÷ 3 = ?
5. $\frac{1}{5}$ of $\frac{5}{6}$A. = ? $\frac{5}{6}$A. ÷ 5 = ?
6. $\frac{1}{3}$ of $\frac{6}{7}$ ft. = ? $\frac{6}{7}$ ft. ÷ 3 = ?
7. $\frac{1}{9}$ of $\frac{27}{32}$ in. = ? $\frac{27}{32}$ in. ÷ 9 = ?
8. Compare these fractional units, or unit fractions:

(1) $\frac{1}{4}$ is what part of $\frac{1}{2}$?
(2) $\frac{1}{6}$ is what part of $\frac{1}{3}$?
(3) $\frac{1}{8}$ is what part of $\frac{1}{4}$?
(4) $\frac{1}{10}$ is what part of $\frac{1}{5}$?
(5) $\frac{1}{6}$ is what part of $\frac{1}{2}$?
(6) $\frac{1}{9}$ is what part of $\frac{1}{3}$?
(7) $\frac{1}{12}$ is what part of $\frac{1}{4}$?
(8) $\frac{1}{15}$ is what part of $\frac{1}{5}$?
(9) $\frac{1}{8}$ is what part of $\frac{1}{2}$?
(10) $\frac{1}{12}$ is what part of $\frac{1}{3}$?
(11) $\frac{1}{16}$ is what part of $\frac{1}{4}$?
(12) $\frac{1}{20}$ is what part of $\frac{1}{5}$?

9. How is the size of a unit fraction changed by multiplying its denominator by 2? by 3? by 4? by 10? by 100? by a? by m?

10. By what must you multiply the first fraction in each of these pairs to get the second?

(1) $\frac{2}{9}$ and $\frac{2}{3}$?
(2) $\frac{3}{16}$ and $\frac{3}{4}$?
(3) $\frac{6}{14}$ and $\frac{6}{7}$?
(4) $\frac{8}{45}$ and $\frac{8}{9}$?
(5) $\frac{13}{75}$ and $\frac{13}{15}$?
(6) $\frac{25}{96}$ and $\frac{25}{32}$?
(7) $\frac{37}{200}$ and $\frac{37}{50}$?
(8) $\frac{a}{12b}$ and $\frac{x}{b}$?
(9) $\frac{a}{12b}$ and $\frac{x}{12}$?

11. Make a rule for dividing a fraction by 3 without changing its numerator; by 5; by 28; by a.

WRITTEN WORK

1. What number should stand in place of x in these examples?

(1) $\frac{12}{17} \div 3 = \frac{x}{17}$; (4) $\frac{256}{19} \div 16 = \frac{x}{19}$; (7) $\frac{72}{17} \div 9 = \frac{x}{17}$;
(2) $\frac{75}{7} \div 15 = \frac{x}{7}$; (5) $\frac{125}{36} \div 25 = \frac{x}{36}$; (8) $\frac{ac}{b} \div a = \frac{x}{b}$;
(3) $\frac{196}{15} \div 14 = \frac{x}{15}$; (6) $\frac{1728}{13} \div 144 = \frac{x}{13}$; (9) $\frac{mc}{n} \div c = \frac{x}{n}$.

2. Solve these problems by finding what number should stand in place of x:

(1) $\frac{5}{6} \div 3 = \frac{5}{x}$; (3) $\frac{15}{16} \div 8 = \frac{15}{x}$; (5) $\frac{241}{175} \div 3 = \frac{241}{x}$;
(2) $\frac{7}{8} \div 4 = \frac{7}{x}$; (4) $\frac{13}{8} \div 25 = \frac{13}{x}$; (6) $\frac{971}{341} \div 15 = \frac{971}{x}$.

3. In what two ways may a fraction be divided by a whole number? When would you use the method of problem 1? of problem 2?

PRINCIPLE IX.—*A fraction may be divided by a whole number, (1) by dividing its numerator by the whole number, without changing its denominator, or (2) by multiplying its denominator by the whole number, without changing its numerator.*

NOTE.—The note after Principle VIII, page 158, applies here also.

PROBLEMS

1. Divide $\frac{4}{3}$ lb. of shot equally among 5 boys, and give the weight of one share.

2. $\frac{7}{9}$ lb. of maple sugar is to be equally divided among 3 children. What part of a pound will each receive?

3. $\frac{4}{5}$ of a piece of work can be done in 8 da. What part of it can be done in 1 day?

4. Make and solve problems based upon the following items, showing work in each case:

$\$\frac{21}{25}$ to buy lace costing \$5 a yard.
$\$\frac{11}{20}$ to buy apples costing \$3 per barrel.
$\frac{3}{7}$ of an estate divided among 4 heirs.

5. If $\frac{4}{5}$ lb. of coffee is divided into 4 equal portions, how much is there for each portion?

6. 3 lb. of butter cost $\$\frac{3}{4}$; 1 lb. cost how much?

7. A strip of tin-foil $\frac{8}{9}$ ft. long is to be cut across into 4 equal pieces. What is the length of one strip?

8. 4 children have a garden containing $\frac{8}{9}$ A. If the children receive equal portions, what part of an acre does each receive?

9. 4 men own $\frac{5}{9}$ of an estate, equally. What part belongs to each man? If the estate is valued at \$36,000, how much money is represented by each part?

10. If $\frac{15}{8}$ A. is divided equally among 7 men, how many acres are there for each man?

§107. Dividing a Mixed Number by a Whole Number.

1. 7 strips of carpeting cover a room $5\frac{1}{4}$ yd. wide. What is the width of one strip?

$5\frac{1}{4}$ yd. = $\frac{21}{4}$ yard.
$\frac{21}{4}$ yd. $\div$ 7 = $\frac{3}{4}$ yd., the width of one strip.

$5\frac{1}{4}$ is what kind of number?
Before dividing what change was made? How was the division performed?

2. A boy earned \$$4\frac{1}{2}$ in 6 da. What did his earnings average per day?

$$4\frac{1}{2} = \frac{9}{2};\ \frac{9}{2} \div 6 = \frac{\cancel{9}}{2 \times \underset{2}{\cancel{6}}} = \frac{3}{4}. \quad \textit{Ans. } \$\tfrac{3}{4}.$$

How was the dividing done in this case?

3. Make a rule for dividing a mixed by a whole number.

To divide a mixed number by a whole number first change the mixed number to an improper fraction and then proceed as in the division of a fraction by a whole number.

PROBLEMS

1. $36\frac{4}{7}$ yd. of cloth were made up into 6 ladies' suits. If the suits contain the same number of yards, how many yards are there in each suit?

2. A ceiling 16 ft. wide contains $233\frac{1}{3}$ sq. ft. Find the length.

$233\frac{1}{3}$ ft. = $\frac{700}{3}$ feet.

$$\frac{700}{3} \div 16 = \frac{\overset{175}{\cancel{700}}}{3 \times \underset{4}{\cancel{16}}} = \frac{175}{12} = 14\tfrac{7}{12}.$$ $14\frac{7}{12} = 14'\ 7''$ which is the length of the room.

3. Solve the following problems:

(1) $14\frac{3}{4}$ bu. $\div$ 8 =	(6) \$$12\frac{5}{8} \div 5 =$
(2) $9\frac{7}{12}$ yd. $\div$ 6 =	(7) $45\frac{9}{16} \div 5 =$
(3) $16\frac{2}{3}$ hr. $\div$ 12 =	(8) $\frac{224}{5} \div 4 =$
(4) $44\frac{9}{16}$ lb. $\div$ 9 =	(9) $14\frac{7}{21} \div 7 =$
(5) $39\frac{5}{8}$ mi. $\div$ 7 =	(10) $66\frac{2}{3} \div 8 =$

Note.—In problems like (7) and (9), in which the integral part of the dividend exactly contains the divisor, divide the whole number and the fraction separately and add the quotients.

4. Find the lengths of the surfaces whose areas and widths are given here:

	Areas	Widths	Lengths
(1)	$212\frac{1}{2}$ sq. ft.	17 ft.	
(2)	$261\frac{1}{2}$ "	14 "	
(3)	$406\frac{1}{4}$ "	25 "	
(4)	$164\frac{3}{4}$ "	13 "	

Make problems based on these items:

5. A ship sails $246\frac{3}{10}$ mi. in 36 days.

6. A man paid \$$195\frac{1}{4}$ for 72 days' board.

7. A field contains $2172\frac{1}{6}$ sq. rd. One side is $40\frac{3}{4}$ rd. long.

8. 15 lb. of fresh salmon cost \$$2.73\frac{3}{4}$.

§108. Dividing Any Number by a Fraction.

(*A*) Dividend a whole number.

1. How many $\frac{3}{4}$ lb. packages can be filled from 3 lb. tea?

(1) How many $\frac{1}{4}$ lb. packages can be made from 1 lb.? How many $\frac{3}{4}$ lb. packages can be made from 1 pound?

(2) How many $\frac{3}{4}$ lb. packages can then be made from 3 lb. tea?

SOLUTION.—$\frac{1}{4}$ is contained in 1, 4 times. $\frac{3}{4}$ is contained in 1, $\frac{1}{3}$ this number of times. $\frac{3}{4}$ is then contained in 1, $\frac{4}{3}$ times. How many times is it contained in 3?

$$1 \div \tfrac{3}{4} = \tfrac{4}{3};\ \frac{4}{\cancel{3}} \times \cancel{3} = 4.$$

Or, since $\frac{3}{4} \times \frac{4}{3} = 1$, $\frac{3}{4}$ is contained in 1, $\frac{4}{3}$ times.

2. A boy earns at the average rate of $\$\frac{4}{5}$ per day selling papers. How many days did it take him to earn \$12?

(1) What effect does it have on a fraction to multiply its numerator? to multiply its denominator?

(2) What effect does it have on a fraction to multiply both its numerator and its denominator by the same number?

(*B*) Dividend a fraction.

3. Divide $\frac{4}{5}$ by $\frac{3}{7}$.

SOLUTION.—

$$\frac{\frac{4}{5}}{\frac{3}{7}} = \frac{\frac{4 \times 7}{5 \times 7}}{\frac{3 \times 5}{7 \times 5}};\ \text{why? [See prob. 2 (2) above.]}\quad \frac{\frac{4 \times 7}{5 \times 7}}{\frac{3 \times 5}{7 \times 5}} = \frac{28}{15};\ \text{why?}$$

(*C*) Dividend and divisor both mixed numbers.

4. I paid $\$49\frac{3}{5}$ for coal @ $\$7\frac{3}{4}$ per ton. How many tons did I buy?

SOLUTION.—

$$49\tfrac{3}{5} = \frac{248}{5};\quad 7\tfrac{3}{4} = \frac{31}{4};\quad 49\tfrac{3}{5} \div 7\tfrac{3}{4} = \frac{\frac{248}{5}}{\frac{31}{4}}$$

$$\frac{\frac{248}{5}}{\frac{31}{4}} = \frac{\frac{\overset{8}{\cancel{248}}}{\cancel{5}} \times \cancel{5} \times 4}{\frac{\cancel{31}}{\cancel{4}} \times 5 \times \cancel{4}} = \tfrac{32}{5} = 6\tfrac{2}{5}$$

Ans. $6\frac{2}{5}$ T.

Let us now seek a general method of dividing *any number* by a fraction.

5. How many times is each of these fractions contained in 1?

$\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{5}$; $\frac{1}{7}$; $\frac{1}{9}$; $\frac{1}{10}$; $\frac{1}{12}$; $\frac{1}{15}$; $\frac{1}{25}$; $\frac{1}{50}$; $\frac{1}{100}$; $\frac{1}{a}$; $\frac{1}{x}$.

6. How many times is each of these fractions contained in 1?

$\frac{2}{3}$; $\frac{3}{5}$; $\frac{6}{7}$; $\frac{3}{4}$; $\frac{5}{9}$; $\frac{9}{10}$; $\frac{7}{12}$; $\frac{13}{15}$; $\frac{21}{25}$; $\frac{33}{50}$; $\frac{37}{100}$; $\frac{b}{x}$.

7. What number should stand in place of the letter x in each of these products?

(1) $\frac{2}{3} \times \frac{3}{2} = x$;	(4) $\frac{7}{8} \times \frac{8}{7} = x$;	(7) $\frac{9}{13} \times \frac{13}{9} = x$;
(2) $\frac{3}{4} \times \frac{4}{3} = x$;	(5) $\frac{9}{10} \times \frac{10}{9} = x$;	(8) $\frac{a}{b} \times \frac{b}{a} = x$;
(3) $\frac{5}{6} \times \frac{6}{5} = x$;	(6) $\frac{12}{17} \times \frac{17}{12} = x$;	(9) $\frac{c}{d} \times \frac{d}{c} = x$.

8. What number should stand in place of the letter y in these products?

(1) $\frac{2}{3} \times y = 1$;	(3) $\frac{7}{12} \times y = 1$;	(5) $\frac{25}{12} \times y = 1$;
(2) $\frac{3}{4} \times y = 1$;	(4) $\frac{12}{25} \times y = 1$;	(6) $\frac{a}{b} \times y = 1$;

1 divided by $\frac{5}{6}$ equals $\frac{6}{5}$, and $1 \div \frac{6}{5} = \frac{5}{6}$; $1 \div 8 = \frac{1}{8}$ and $1 \div \frac{1}{8} = 8$ and so with other numbers.

DEFINITION.—Dividing 1 by any number, whole or fractional, is called *inverting* the number.

The *reciprocal* of any number is the number inverted.

9. To what number is the product of any number and its reciprocal equal?

10. Examine your answers to problem 7 and state how to find quickly the number of times any fraction is contained in 1.

11. If a fraction is contained in one $\frac{5}{3}$ times, how many times is it contained in 2? in 6? in 12? in $\frac{1}{2}$? in $\frac{3}{4}$? in $\frac{7}{8}$? in any number? in a?

NOTE.—7 may be written $\frac{7}{1}$, 12 may be written $\frac{12}{1}$, and so on. Consequently 7 and $\frac{1}{7}$ are reciprocals, as are also 12 and $\frac{1}{12}$; 25 and $\frac{1}{25}$, and so on.

12. What is actually done by inverting the divisor in a problem in division of fractions? (See Definition above.)

13. If, then, the reciprocal of a fraction is multiplied by 6, what operation is performed?

14. Can you make a rule for quickly dividing any whole number by a fraction?

PRINCIPLE X.—*The reciprocal of any fraction shows how many times the fraction is contained in* 1.

PRINCIPLE XI.—*Any number, mixed, integral, or fractional, may be divided by a fraction by multiplying the number by the reciprocal of the divisor.*

15. Why do we invert the divisor? Why do we multiply the reciprocal of the divisor by the dividend?

PROBLEMS

1. $\frac{2}{9}$ yd. of silk costs \$$\frac{3}{4}$. Find the cost of 1 yard.

2. At \$$1\frac{1}{4}$ a yd. how much cloth can be bought for \$$\frac{3}{4}$?

3. At \$$6\frac{3}{10}$ per hundred how much beef can be bought for \$$2\frac{1}{2}$?

4. How many meals will $4\frac{1}{5}$ bu. of oats supply, if $\frac{3}{14}$ bu. are fed at a meal?

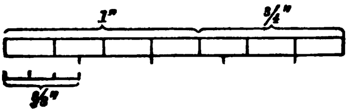


FIGURE 84

5. How many pieces of wire $\frac{3}{4}$ ft. long can be cut from a piece $49\frac{1}{2}$ ft. long?

6. Measure $1\frac{3}{4}$ ft. by $\frac{3}{8}$ ft. (See Fig. 84.)

Draw lines and illustrate the following:

7. $2\frac{1}{3}$ ft. + $\frac{2}{3}$ ft. =	10. $3\frac{2}{3}$ ft. + $\frac{5}{6}$ ft. =
8. $4\frac{5}{6}$ ft. + $\frac{5}{12}$ ft. =	11. $1\frac{3}{4}$ ft. + $\frac{1}{2}$ ft. =
9. $2\frac{7}{8}$ ft. + $\frac{3}{4}$ ft. =	12. Divide $3\frac{1}{3}$ yd. by $\frac{1}{3}$ yard.

13. Divide $16\frac{1}{2}$ yd. by $2\frac{1}{2}$ yd.; by $1\frac{3}{4}$ yd.; by $4\frac{1}{2}$ yd.; by $1\frac{7}{8}$ yd.; by $\frac{5}{6}$ yard.

14. Divide \$$6\frac{2}{5}$ by \$$\frac{3}{10}$; by \$$\frac{9}{20}$; by \$$\frac{2}{3}$; by \$$\frac{3}{4}$.

15. Find the number of strips of carpeting running the long way of the room:

WIDTH OF ROOMS	CARPETING
$22\frac{1}{2}$ ft.	$2\frac{1}{4}$ ft. wide.
$7\frac{2}{3}$ yd.	$\frac{5}{6}$ yd. "
$18\frac{3}{4}$ ft.	$2\frac{1}{2}$ ft. "
$8\frac{1}{4}$ yd.	$\frac{5}{6}$ yd. "
$15\frac{1}{3}$ ft.	$2\frac{1}{2}$ ft. "

Make and solve problems based upon the following items:

16. Portland cement \$$2\frac{3}{4}$ per bbl.; \$9 purchasing fund.

17. Sidewalk having an area of $607\frac{3}{4}$ sq. ft.; flagstones $5\frac{1}{4}$ ft. square.

18. $15\frac{3}{8}$ gal. maple syrup; $\frac{3}{4}$ gal. measures.

19. $29\frac{5}{6}$ doz. oranges; $\frac{9}{12}$ doz. oranges.

20. The ratio of two numbers is $\frac{3}{5}$; one of the numbers is $\frac{6}{7}$.

21. A man bought a house for \$1860. The house cost him $\frac{3}{5}$ as much as he paid for a mill. What was the cost of the mill?

22. My library contains 1075 volumes, which are $\frac{5}{9}$ as many as a friend's contains. How many volumes are in my friend's library?

23. Of a number of cars of grain inspected on a certain day 36 were rejected. This number was $\frac{12}{35}$ of the entire number inspected. Find the number.

24. Let $\dfrac{a}{b}$ and $\dfrac{x}{y}$ denote two fractions, and state the following principle in symbols: The quotient of one fraction by another is equal to the product of the dividend by the inverted divisor. (Compare Principle XI.)

§109. Complex Fractions.

Any problem in division of fractions may be written in fractional form, the dividend being written above and the divisor below the line.

EXAMPLE.—The problem, to divide $\frac{6}{7}$ by $\frac{5}{11}$ may be written:

$$\frac{\frac{6}{7}}{\frac{5}{11}} = ?$$

Or, the problem, to divide $18\frac{2}{3}$ by $6\frac{2}{5}$, may be written:

$$\frac{18\frac{2}{3}}{6\frac{2}{5}} = ?$$

DEFINITION.—Fractions that contain fractions in one or both terms, are called *complex fractions.*

Principle XI, §108, will enable us to solve all such problems. If either the divisor or the dividend, or both, are mixed numbers, they should first be reduced to improper fractions.

Following this principle:

$$\frac{\frac{6}{7}}{\frac{5}{11}} = \frac{6}{7} \times \frac{11}{5} = \frac{66}{35}.$$

The second step becomes unnecessary if we notice that the product of the outside terms (6 and 11), called *the extremes*, gives the numerator, and the product of the inside terms (7 and 5), called *the means*, gives the denominator of the result.

A complex fraction is simplified when its terms are freed of fractions, and the resulting fraction is expressed in its simplest form.

In many cases factors may be cancelled from both numerator and denominator before multiplying the fractions. Cancel whenever possible.

Show that the method explained is the same as dividing the product of *the extremes* by the product of *the means* in the given complex fractions.

PROBLEMS

Simplify the following complex fractions, cancelling when possible:

1. $\dfrac{\frac{3}{4}}{\frac{1}{2}}$	4. $\dfrac{\frac{5}{16}}{\frac{4}{3}}$	7. $\dfrac{\frac{2}{7}}{3}$	10. $\dfrac{20\frac{2}{5}}{12\frac{3}{4}}$	13. $\dfrac{84\frac{2}{7}}{62\frac{4}{5}}$
2. $\dfrac{\frac{5}{6}}{23}$	5. $\dfrac{\frac{7}{12}}{\frac{3}{8}}$	8. $\dfrac{15\frac{2}{3}}{12\frac{1}{2}}$	11. $\dfrac{25\frac{2}{3}}{6\frac{3}{10}}$	14. $\dfrac{123\frac{5}{9}}{116\frac{2}{3}}$
3. $\dfrac{\frac{7}{9}}{\frac{4}{15}}$	6. $\dfrac{4}{6\frac{2}{3}}$	9. $\dfrac{1\frac{5}{9}}{\frac{3}{7}}$	12. $\dfrac{44\frac{4}{9}}{32\frac{2}{5}}$	15. $\dfrac{\frac{a}{b}}{\frac{c}{d}}$

§110. Joint Effects of Forces.

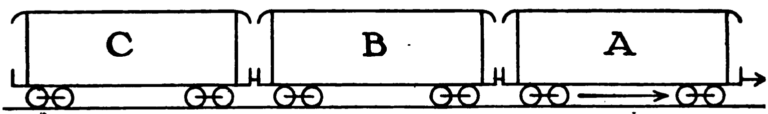


FIGURE 85.

1. The rear car, C, pulls back on the middle car, B, with a force of 100 lb., and the front car, A, pulls it forward with a force of 250 lb. If car, B, were taken out of the train what single forward force would draw it the same as it is being drawn in the train?

2. A force of 23 lb. is pulling the car (Fig. 96) toward the right and another of 12 lb. is pulling it toward the left. If the car is free to move, in which direction will it move? What single force would move the car in the same manner as do both of these forces pulling at the same time?

FIGURE 96

FIGURE 97

FIGURE 98

3. What single force will produce the same motion of the car (Fig. 97) as $8x$ lb. acting toward the right and $5x$ lb. acting at the same time toward the left?

4. What single force will move the car (Fig. 98) in the same manner as $6m$ lb. pulling against $2m$ lb. will move it?

When a force of 18 lb. is supposed to be pulling a car toward the *right*, it will be written thus: R 18 lb. When the same force pulls toward the *left*, it will be written thus: L 18 lb. When two forces, R 16 lb. and L 12 lb., act at the same time, their joint effect is the same as the effect of the single force, R 4 lb.

5. What would be the joint effect of R $28\frac{1}{2}$ lb. and L $21\frac{3}{4}$ lb.?

6. What would be the joint effect of each of these pairs of forces:

(1) R $48\frac{1}{4}$ lb. and L $12\frac{5}{8}$ lb.? (4) R $23\frac{1}{8}$ lb. and L $68\frac{7}{16}$ lb.?
(2) R $75\frac{3}{16}$ lb. and L $69\frac{5}{16}$ lb.? (5) R $16\frac{5}{16}$ lb. and L $81\frac{1}{8}$ lb.?
(3) R $181\frac{1}{8}$ lb. and L $129\frac{7}{10}$ lb.? (6) R $87\frac{3}{8}$ lb. and L $684\frac{3}{4}$ lb.?

A force of 25 T. (tons), pulling *forward*, may be written F 25 T., and a force of 18 T., pulling *backward*, may be written B 18 T. The joint effect of both would be written thus: F 7 T.

7. What is the joint effect of the three forces: F 17 T., F 12 T. and B 25 T.?

8. What is the joint effect of each of the following sets of forces:

(1) F $80\frac{1}{2}$ T., B $28\frac{5}{12}$ T., and B $96\frac{5}{8}$ T.?
(2) F $49\frac{1}{2}$ T., B $84\frac{1}{8}$ T., and F $248\frac{7}{12}$ T.?
(3) B $68\frac{9}{10}$ T., F $73\frac{5}{8}$ T., and B $12\frac{5}{8}$ T.?
(4) F $28\frac{3}{20}$ T., F $18\frac{1}{8}$ T., and B $37\frac{3}{16}$ T.?
(5) F $692\frac{1}{2}$ T., B $386\frac{5}{8}$ T., and B $307\frac{7}{10}$ T.?
(6) F $28x$ T., F $3x$ T., and B $36x$ T.?

9. A force of 182 lb. pulling *upward* may be written U 182 lb. How would you write a force of 78 lb. pulling *downward?*

10. What is the joint effect of each of the following sets of forces:

(1) U $8\frac{5}{8}$ lb., D $16\frac{1}{2}$ lb., and U $3\frac{3}{8}$ lb.?
(2) U $16\frac{1}{8}$ lb., U 28 lb., and D $29\frac{1}{2}$ lb.?

(3) D 29 oz., D $32\frac{1}{2}$ oz., and D $16\frac{5}{8}$ oz.?

(4) D $29\frac{3}{8}$ oz., U $65\frac{4}{5}$ oz., and D $42\frac{1}{2}$ oz.?

11. If we call a force of 16 lb. pulling forward, or upward, or eastward, or northward, or to the right, a *positive* force, and write it $+16$ lb., we should call the same force pulling backward, or downward, or westward, or southward, or to the left, respectively, a *negative* force. How should we write it?

12. Give the joint effect, with proper sign (+ or −) of each of these sets of forces:

(1) 2 forces, each $+8\frac{1}{4}$ lb., and one force, $-12\frac{7}{8}$ lb.

(2) 3 forces, each $+12\frac{1}{8}$ oz., and 4 forces, each $-8\frac{7}{8}$ oz.

(3) 5 forces, each $-16\frac{2}{3}$ T., and 12 forces, each $+9\frac{3}{4}$ T.

(4) 18 forces of $-16\frac{9}{16}$ lb. each, and 20 forces of $+25\frac{1}{5}$ lb. each.

§111. Exercises for Practice.

1. Find the following sums and differences as rapidly as you can work correctly.

(1) $\frac{7}{8}+\frac{6}{11}=?$

(2) $\frac{12}{13}-\frac{7}{8}=?$

(3) $\frac{11}{15}-\frac{5}{12}=?$

(4) $\frac{16}{27}+\frac{15}{32}=?$

(5) $\frac{25}{32}-\frac{16}{25}=?$

(6) $\frac{32}{45}+\frac{27}{64}=?$

(7) $8\frac{8}{9}+4\frac{2}{27}=?$

(8) $9\frac{4}{9}-3\frac{14}{15}=?$

(9) $16\frac{2}{9}-10\frac{8}{15}=?$

(10) $210\frac{5}{12}+49\frac{9}{10}=?$

(11) $37\frac{1}{2}-18\frac{5}{21}=?$

(12) $101\frac{1}{8}-98\frac{7}{12}=?$

2. Find the products and quotients as rapidly as you can work correctly, using cancellation when it makes the work easier.

(1) $\frac{7}{8}\times\frac{16}{21}=?$

(2) $\frac{6}{11}\times\frac{22}{27}=?$

(3) $\frac{14}{17}\times\frac{17}{21}=?$

(4) $\frac{16}{21}\div\frac{8}{21}=?$

(5) $\frac{18}{23}\div\frac{9}{46}=?$

(6) $\frac{12}{17}\div\frac{5}{18}=?$

(7) $\frac{15}{28}\times\frac{7}{40}=?$

(8) $8\frac{4}{7}\div3\frac{2}{9}=?$

(9) $18\frac{2}{7}\times9\frac{7}{8}=?$

(10) $18\frac{9}{10}\div16\frac{2}{3}=?$

(11) $21\frac{3}{8}\div14\frac{5}{16}=?$

(12) $6\div15=?$

(13) $8\frac{15}{28}\div12\frac{5}{7}=?$

(14) $15\frac{6}{7}\div3\frac{7}{8}=?$

(15) $\frac{35}{54}\div6\frac{5}{9}=?$

(16) $26\frac{11}{12}\div80\frac{2}{3}=?$

3. Find the simplest results for these exercises:

(1) $\frac{3}{4}\times\frac{2}{5}\times\frac{5}{14}=?$

(2) $\frac{5}{6}\times\frac{9}{15}\times\frac{11}{12}=?$

(3) $(12\frac{1}{2}\times8\frac{1}{5})\div6\frac{1}{4}=?$

(4) $(9\frac{4}{7}\times4\frac{2}{5})\div3\frac{7}{8}=?$

(5) $\frac{5}{12} + \frac{9}{10} + \frac{11}{15} = ?$

(6) $\frac{4}{9} + \frac{7}{8} - \frac{19}{24} = ?$

(7) $\frac{3}{14} \times \frac{7}{12} \times \frac{16}{21} = ?$

(8) $\frac{3}{14} + \frac{7}{12} + \frac{16}{21} = ?$

(9) $\frac{6}{11} - \frac{2}{5} + \frac{2}{7} = ?$

(10) $12\frac{1}{2} - 3\frac{7}{8} + 6\frac{1}{4} = ?$

(11) $16\frac{2}{3} + 70\frac{5}{6} - 87\frac{1}{2} = ?$

(12) $16\frac{2}{3} \times 3\frac{1}{5} \times 1\frac{8}{9} = ?$

(13) $(9\frac{3}{8} \div 6\frac{1}{4}) \times 6\frac{2}{3} = ?$

NOTE—First divide $9\frac{3}{8}$ by $6\frac{1}{4}$ and then multiply the quotient by $6\frac{2}{3}$.

(14) $(22\frac{6}{7} \div 11\frac{1}{4}) \times 19\frac{1}{16} = ?$

(15) $(18\frac{3}{4} + 27\frac{3}{7}) + 24\frac{5}{6} = ?$

§112. Dividing Lines, and Angles.

PROBLEM I.—Divide the line AB into 2 equal parts, see Problem VI, p. 101.

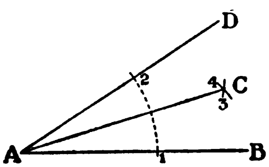
EXERCISES

1. Draw a straight line with a ruler on the blackboard and bisect it with crayon, string and ruler.

2. Bisect each of the 3 sides of a triangle and connect each mid-point with the opposite corner of the triangle. These lines are the *medians* of the triangle. How do they cross each other?

3. How might a line be divided into 4 equal parts by repeating this method? Divide a line into 4 equal parts by this method.

PROBLEM II.—Divide the angle BAD into 2 equal parts.



Angle Bisected
FIGURE 99

EXPLANATION. — With any convenient radius and with the pin foot on A (*vertex*) draw arcs 1 and 2 across AB and AD.

Place the pin foot on the crossing point at 2, and with a radius longer than half way from 2 to 1 draw an arc 3.

Now place the pin foot on 1 and with the radius used for arc 3 draw arc 4 across arc 3. Call the point of crossing C.

With the ruler draw the bisector AC. Then angle BAC = angle CAD.

EXERCISES

1. Draw an angle on paper, or on the blackboard, and with pencil, or crayon and string, bisect the angle.

2. Draw a triangle and bisect each of its 3 angles. How do the bisectors of the angles cross each other?

3. The line CD of Fig. 35, p. 101, drawn from C to D, is called the *perpendicular bisector* of the line AB. Draw a triangle of 3 unequal sides, and then draw the 3 perpendicular bisectors of its sides. How do these lines cross each other?

§113. **The Parallel Ruler.**—A very good *parallel ruler* may be made by cutting out two strips of cardboard or of very thin wood, exactly alike, as shown at (*a*), Fig. 100. The numbers show the widths and the lengths. To use the ruler place the two parts with their long sides together, the thin ends being in opposite directions as in the cut.

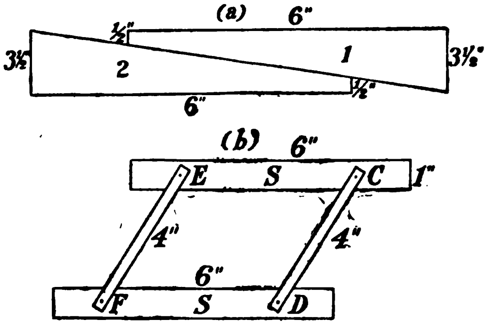


FIGURE 100

Another sort, which the pupil may make for himself or purchase for a few cents, is shown at (*b*), Fig. 100. The strips *S* may be of light cardboard or of thin wood of the lengths and the widths shown in the cut. The outside edges of these strips should be made as smooth and as straight as possible. The cross strips, which may be narrower than the strips *S*, should be of exactly the same length between the pins at *C*, *D*, and at *E*, *F*. The distances *EC* and *FD* between the pins, should also be made equal. Common pins may be stuck through and bent over to hold the strips together at *C*, *D*, *E*, and *F*.

The pupil should provide himself with a parallel ruler for the following problems:

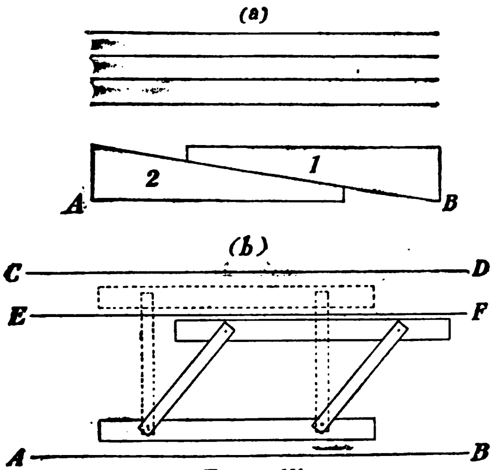


FIGURE 101

PROBLEM I.—Draw a line parallel to a given line.

EXPLANATION. — Let *AB* be the given line.

Fig. 101 (*a*) shows how this is done with the first kind of ruler, by holding the part 2 on the line, and sliding part 1 along, drawing the lines along the upper edge of part 1.

Fig. 101 (*b*) shows how to solve the problem with the second kind of ruler.

Problem II.—Draw a line parallel to a given line and through a given point.

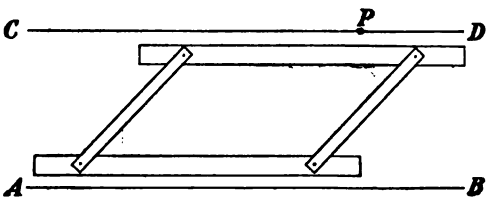


Figure 102

Explanation.—Suppose *AB* (Fig. 102) is the given line, and *P* the point.

Hold one edge of the parallel ruler along *AB*, raise the other strip until its edge goes through the point *P*, and draw a line *CD* along this edge. *CD* is the desired line

Problem III.—Solve Problem II with ruler and compass.

Explanation.—First Step: Place the pin foot on the given point *P* and spread the feet until the pencil foot reaches some point, as *C*, on the line *AB*, Fig. 103. Draw the arc *C*2.

Second Step: Place the pin foot on *C* and using the same radius as before, draw arc *P1*, cutting *AB* at *D*.

Third Step: Spread the feet of the compass apart as far as from *P* to *D*, and placing the pin foot on *C*, draw the short arc 3. Connect the crossing point *E* with *P*. *EP* is the desired parallel to *AB* through *P*.

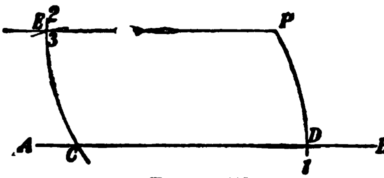


Figure 103

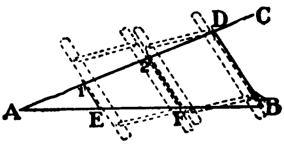
EXERCISES

1. Draw a line on paper, mark a point not in the line and with a parallel ruler draw a line through the point and parallel to the first line.

2. Solve Exercise 1 on the blackboard with chalk, string, and ruler.

3. Draw a triangle on the blackboard. Draw a line through each corner of this triangle and parallel to the opposite side.

Problem IV.—Divide a line *AB* into 3 equal parts (or trisect *AB*).



Line Trisected
Figure 104

Explanation.—Draw an indefinite line *AC*, making any convenient angle with *AB*.

Measure off 3 equal spaces from *A* toward *C*. Connect *D* with *B*.

Draw through the points 2 and 1 lines parallel to *DB* (see Problem II). Then $AE = EF = FB$, and *E* and *F* are the trisection points.

EXERCISES

1. Draw a line on the blackboard and trisect it.

Note.—Draw the parallels by the method of Exercise 2 under Problem III above.

2. From the suggestion of Fig. 105 draw a line on the blackboard and divide it into 5 equal parts.

FIGURE 105

3. How may a line be divided into 7, or 9, or 13 equal parts? Draw a line on the blackboard and divide it into 7 equal parts.

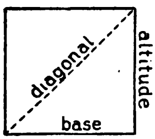


FIGURE 106

4. How does the line connecting the opposite corners of a square divide the square?

5. Answer a question like 4 for the rectangle; for the parallelogram.

6. If, then, the area of a square equals the product of its base and its altitude, what is the area of one of the 2 equal triangles into which a diagonal divides the square?

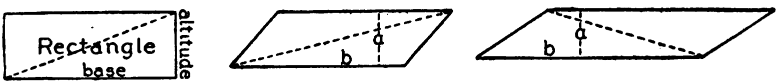


FIGURE 107

7. Answer similar questions for the rectangle; for the parallelogram.

8. The area (A) of a rectangle of base b ft. and altitude a ft. is how many square feet?

9. What is the area of a right-angled triangle (*a right triangle*) of base b in. and altitude a inches?

10. The base of a parallelogram is b in. and the altitude is a in.; what is the area?

11. The base of a triangle is b and the altitude is a; what is the area?

12. The bases of a rectangle, of a parallelogram and of a triangle are b in. and their altitudes are a in. Find the ratio of the area of the rectangle to the area of the parallelogram; the ratio of the area of the parallelogram to the area of the triangle.

§114. Uses of the 30° and the 45° Triangles.

PROBLEM V.—To make the triangles for use in drawing.

EXPLANATION.—(a) Fold a piece of smooth heavy paper, having one straight edge (like the piece shown in Fig. 108), over a line near the middle. Bring the straight edges carefully together as shown in Fig. 109.

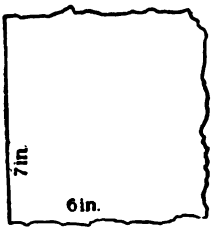


FIGURE 108

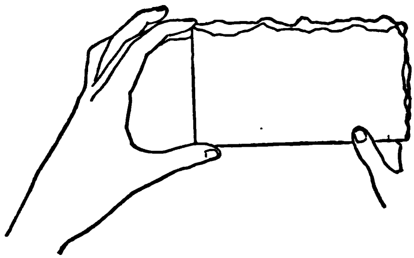


FIGURE 109

Crease the paper smoothly with a ruler or a paper knife and paste or glue the two pieces together. When the paper is dry, mark off distances of 4″ from the square corner on the crease and on the straight side. Connect the 4 in. marks and cut the paper smoothly along the connecting line. This will give a triangle of the form T, Fig. 110.

(b) In the same way, fold, crease, and paste another piece of paper a little larger than before. On the straight side mark off a distance CA equal to 3″. With compasses, or with a string or ruler, mark a point, B, on the crease so that AB equals 6″. Draw AB and cut out the triangle S, Fig. 111.

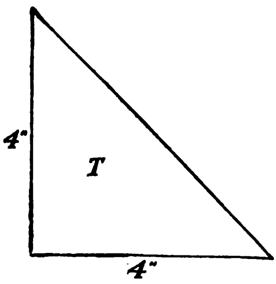


FIGURE 110

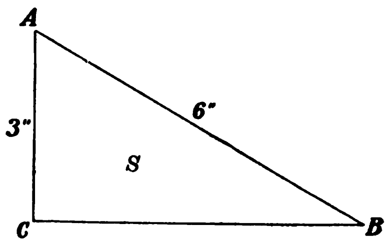


FIGURE 111

If preferred the triangles T and S may be made of thin wood.

EXERCISE.—Place the side, AC, of S against the long side of T, and holding the triangles with the left hand, draw a line along AB. Now hold T, slide S a little (say $\frac{1}{4}''$) and draw another line along AB. Similarly, draw a third line. Lines in such positions are called *parallel lines*.

Problem VI.—Through a given point with a triangle draw a line parallel to a given line.

Explanation.—AB is the given line and P is the point the parallel is to pass through.

Place a ruler CD, Fig 112, in such a position that when the triangle S is placed against it, one of the sides of S will lie along AB. Press the ruler against the paper and hold it with the left hand; with the right, slide the triangle along the ruler until its side just touches the point P. Draw line FE through P and along the edge of the triangle. FE is the desired parallel line.

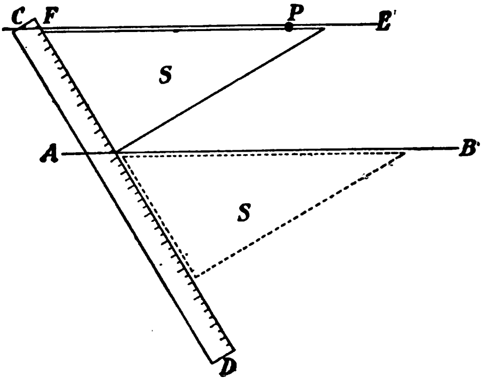


Figure 112

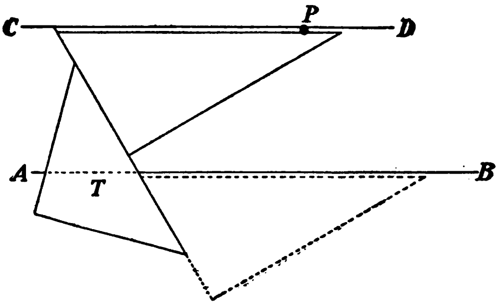


Figure 113

Fig. 113 shows how this problem is solved with the triangles alone.

EXERCISES

1. Solve the first exercise of Problem III with the triangles S and T as shown in Fig. 113.

2. Draw a triangle with sides of 1″, $1\frac{1}{2}$″, and 2″, and through each corner draw a line parallel to the opposite side. Use the ruler and the triangles. To draw the triangle see Problem IX, p. 103.

PROBLEM VII.—At a given point on a line with the triangles draw a perpendicular to the line.

EXPLANATION.—Let *AB* be the line and let *P* be the given point.

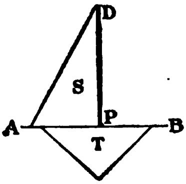


FIGURE 114

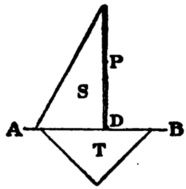


FIGURE 115

Hold one of the triangles, as *T*, in the position shown in Fig. 114, and placing one side of the other triangle, *S*, against the upper side of *T*, slip *S* along until the square corner comes to the point *P*. Hold *S* firmly and draw the line *PD* along the side of *S*.

PD is the required perpendicular.

PROBLEM VIII.—Through a given point not in a line, with the triangles draw a perpendicular to the line.

EXPLANATION.—Let *AB* be the line and let *P* (Fig. 115) be the given point.

Hold one of the triangles, as *T*, so that its side lies along *AB* and slide the other triangle, *S*, with its side against the upper side of *T* until it comes up to *P*. Then draw *PD*.

PD is the required perpendicular.

EXERCISES

1. Draw a line, mark two points on it 1″ apart and draw a perpendicular to the line at each of the two points (by Problem VII). Mark a point on one of the perpendiculars 1″ above the given line and at this point draw a third perpendicular completing a 1″ square.

2. Draw a triangle and through each corner draw a perpendicular to the opposite side (by Problem VIII). How do these perpendiculars cross?

3. Draw any circle, also a diameter, marking its ends *A* and *B*. Through its center draw a perpendicular radius and prolong it, making a diameter, *CD*. Connect *AC*, *CB*, *BD* and *DA*, forming an *inscribed square*.

§115. Scale Drawings of Familiar Objects.

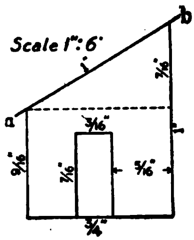


FIGURE 116

1. Notice the scale of the drawing of the playhouse (Fig. 116), and compute the lengths of the following dimensions:

(1) The width; (2) the height of the lower side; of the higher side; (3) the rise (difference of higher and lower sides); (4) the height of the door; the width; (5) the distance from the right side of the door to the right corner of the house.

2. From the dimensions in Fig. 116, with ruler and triangles, make a drawing of the playhouse to a scale 4 times as large as that of the drawing (Fig. 116).

3. From the scale of the drawing (Fig. 117) find the following dimensions of the house:

(1) The width; (2) the height of the eaves; (3) the rise (*ab*); (4) the width and the height of the door; (5) the width and the height of the window.

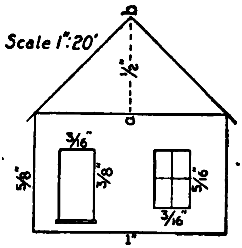


FIGURE 117

4. Find the area of the end of the house, including the gable and excluding the areas of the door and the window.

5. Make an enlarged drawing of the house to a scale 8 times as large as the scale of the drawing (Fig. 117).

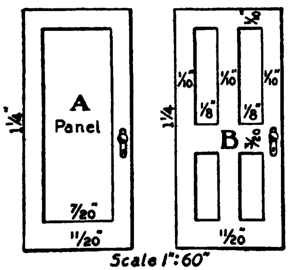


FIGURE 118

6. From the scale of Fig. 118, give the length and width of the door; the length and the width of the panel. (Find the length by measurement).

7. Similarly, give the length and the width of the door B, also the length and the width of the upper panels, and the widths of the strips enclosing the panels.

8. Make an enlarged drawing of each of the doors to a scale 5 times as large as the scale of the drawings (Fig. 118).

9. How long are the cross-pieces and the strings of the kite represented by the drawing (Fig. 119)?

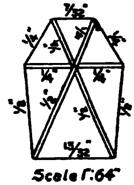


FIGURE 119

10. Make an enlarged drawing of the kite to a scale 8 times as large as that of Fig. 119.

11. Notice that the horizontal piece divides the surface of the kite into two trapezoids. The altitude of the upper trapezoid in the drawing (Fig. 119) is $\frac{11}{64}''$, and that of the lower trapezoid is $\frac{31}{64}''$. How many square feet of paper are needed to cover the kite? (Allow 144 sq. in. for folding and pasting over strings.)

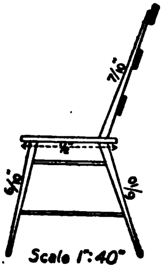


FIGURE 120

12. What is the length of each leg of the chair shown in the drawing (Fig. 120)? What is the height of the back? the depth of the seat to the extreme rear?

13. Make a drawing of the chair to a scale 5 times as large as that of Fig. 120.

14. Find from the drawing of the gate (Fig. 121), (1) the height of the high end of the gate; (2) of the lower end; (3) the length of the long brace; (4) the length of the gate; (5) the width of the strips.

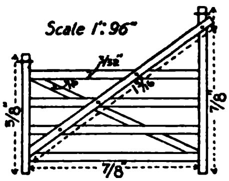


FIGURE 121

15. Make a drawing of the gate to a scale 8 times as large as that of Fig. 121.

16. Fig. 122 is a scale drawing of the side and the end views of a large book. How long is the book? how wide? how thick?

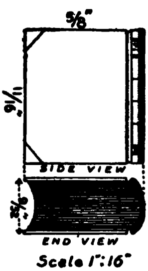


FIGURE 122

17. Make an enlarged drawing of the book to a scale 4 times as large as that of Fig. 122.

18. From your own measurements make a drawing, to any convenient scale, of a thick object, as a block, a brick, a crayon-box, showing two views as in Fig. 122.

19. Make a scale drawing from your own measures of a desk, table, bookcase, or other object in your schoolroom, showing three different views (top, side, and edge views) of it.

§116. Schoolhouse and Grounds.

1. Using a foot rule, graduated to 16ths of an inch, and regarding the scale of the drawing (Fig. 123), find the width of the grounds; the length; the area in square rods. ($30\frac{1}{4}$ sq. yd. = 1 square rod.)

2. Find the length of the field; the width; the area in square rods.

3. Find the length and the width of the school yard; the area in square rods.

4. Find the length and the width of the schoolhouse; the area, in square yards, covered by it.

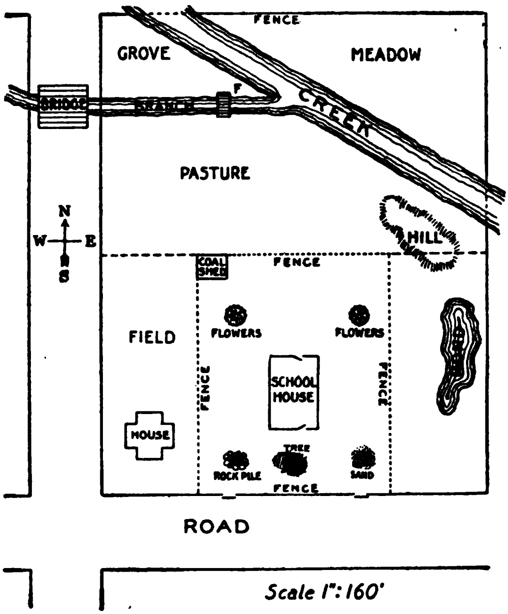


FIGURE 123

5. How far is the front door of the schoolhouse from the front fence? from the west front gate? from the sand pile? from the tree?

6. How far is it from the back door to the east flower bed? to the back fence? to the west fence? to the coal shed? to the northeast corner of the school yard? to the south end of the pond? to the hill? to the nearest point on the creek bank? to the foot bridge (F)?

7. How wide is the south road? the creek? the branch?

8. How many square rods in the south road in front of the grounds? in the crossing of the roads?

9. How many square rods in the meadow? in the grove? in the pasture?

10. How many square rods are covered by the creek and the branch together, within the fence lines?

11. How many rods of fence will be needed to enclose the grounds and to run along the lines indicated?

12. From your own measurements make a similar drawing, to a convenient scale, of some tract of ground (school yard or field) near your schoolhouse. Locate any fixed objects on your tract in their proper places on the drawing by measuring their shortest distance from a fence and from a corner.

§117. **Proportion.** ORAL WORK

1. Compare the ratio 3 : 2 with the ratio 6 : 4. What do you find?

2. Compare the ratio 8 d. : 2d. with the ratio 16 men: 4 men.

3. Compare the ratio 10 ft. : 800 ft. with the ratio 100 mi.: 8000 miles.

4. A proportion is an equation of ratios. Thus 6 : 12 = 3 : 6 and $\frac{6}{12} = \frac{3}{6}$ are two different ways of writing the proportion.

DEFINITIONS.—The first, second, third, and fourth numbers of the proportion are called the *first, second, third,* and *fourth terms* of the proportion. The first and fourth terms are called the *extremes*, and the second and third terms are the *means*. The first two terms are the *first couplet*, the third and fourth terms are the *second couplet*.

5. In 6 : 12 = 3 : 6, to what is the product of the extremes equal? the product of the means?

6. Answer the same questions for 3 : 5 = 12 : 20.

7. Which of these pairs of ratios may form proportions:

4 : 9 and 8 : 18?	6 : 11 and 18 : 33?	1 : 3 and 8 : 21?
3 : 7 and 12 : 28?	6 : 11 and 12 : 22?	$\frac{3}{4}$ and $\frac{75}{100}$?
$\frac{a}{2a}$ and $\frac{b}{2b}$?	$\frac{a}{ax}$ and $\frac{c}{cx}$?	$\frac{m}{n}$ and $\frac{6m}{6n}$?

8. Is $\frac{5}{7} = \frac{15}{21}$ a proportion?

Multiply both sides by 21 and we have $\frac{5}{7} \times 21 = 15$.

Now multiply both sides of this equation by 7 and we have $5 \times 21 = 7 \times 15$.

What were the terms 5 and 21 in the proportion called? What were the 7 and 15 in the proportion called?

9. Compare the product of the 1st and the 4th numbers in these proportions with the product of the 2d and the 3d terms:

(1) $\frac{3}{7} = \frac{18}{42}$; (5) $\frac{7}{11} = \frac{63}{99}$;
(2) $\frac{6}{13} = \frac{42}{91}$; (6) $\frac{2}{a} = \frac{24}{12a}$;
(3) $\frac{12}{15} = \frac{36}{45}$; (7) $\frac{a}{b} = \frac{18a}{18b}$;
(4) $\frac{9}{1} = \frac{108}{12}$; (8) $\frac{x}{1} = \frac{20x}{20}$.

Can you state the principle problems 8 and 9 illustrate?

PRINCIPLE.—*In a proportion the product of the means equals the product of the extremes.*

WRITTEN WORK

1. What must x be in each of these expressions to give a proportion? Solution of first equation: $2x = 15$, or $x = 7.5$.

1. $2:3 = 5:x$
2. $4:3 = 8:x$
3. $6:x = 9:27$
4. $7:2 = x:14$
5. $x:6 = 8:12$
6. $a:b = 2a:x$
7. $18:x = 54:18$
8. $4:6 = x:9$
9. $\frac{a}{2a} = \frac{x}{4ab}$
10. $\frac{ab}{bc} = \frac{x}{c}$

2. What does the letter stand for in each of these proportions?

(1) $\frac{3}{8} = \frac{12}{a}$; (9) $\frac{x}{18} = \frac{36}{324}$;
(2) $\frac{3}{7} = \frac{b}{35}$; (10) $\frac{3}{y} = \frac{1}{5}$;
(3) $\frac{8}{7} = \frac{x}{63}$; (11) $\frac{5}{z} = \frac{13}{65}$;
(4) $\frac{2}{11} = \frac{16}{m}$; (12) $\frac{11}{44} = \frac{d}{8}$;
(5) $\frac{3}{c} = \frac{21}{30}$; (13) $\frac{23}{161} = \frac{a}{14}$;
(6) $\frac{14}{a} = \frac{7}{12}$; (14) $\frac{32}{4} = \frac{1}{n}$;
(7) $\frac{a}{10} = \frac{16}{45}$; (15) $\frac{12}{3} = \frac{2}{m}$;
(8) $\frac{2a}{16} = \frac{9}{36}$; (16) $\frac{15}{18} = \frac{x}{4}$;

§118. Practical Applications.

1. At the rate of 20 lb. for $1.00, what will 15 lb. of sugar cost?

2. A boy buys 36 oranges at the rate of 3 for 5 cents. What do they all cost him?

NOTE.—Write the problem in the form $\frac{3}{5} = \frac{36}{c}$ and find c, the cost of the oranges. Use Principle §117.

3. The boy sells his 36 oranges at the rate of 2 for 5 cents. How much does he receive for them?

4. A man walks 54 miles at the rate of 9 mi. in 2 hr. How long does it take him to walk the 54 miles?

5. A number of boys buy 72 marbles at the rate of 6 marbles for 5 cents. How much do the 72 marbles cost?

6. If it takes 5 yd. of ribbon to trim 4 hats, how many hats can be trimmed with $37\frac{1}{2}$ yards?

7. At the rate of $4.00 for 5 dozen, what will 3 doz. cocoanuts cost?

8. In a class there are 3 girls to every 4 boys. If there are 24 boys in the class, how many girls are there?

9. At the rate of $7.00 for 3 days, how much money does a man earn in 21 days? in 27 days? in $16\frac{1}{2}$ days?

10. If it takes 7 yards of gingham to make 4 aprons, how many aprons will 42 yd. gingham make? 56 yd.? $59\frac{1}{2}$ yd.?

11. If $\frac{7}{8}$ of a ton of coal is worth $4.81, what are 12 T. worth at the same rate?

NOTE.—Call the unknown term x. Write the proportion, using x, and then use the Principle, §117, to find x.

12. A spelling class of 52 pupils writes a total of 1352 words; at the same rate how large a class will write a total of 728 words?

13. A girl jumping a rope makes 441 skips in 3 min. How long will it take her to make 392 skips at the same rate?

14. If the average column in a newspaper contains 1600 words, how much should a writer receive for 700 words at the rate of $5 per column?

15. In going 10,725 ft. the front wheel of a bicycle revolves 1430 times. How far would it go in making 1001 revolutions?

16. A clock ticks 7 times in 5 sec.; how many times does it tick in 2 da. 6 hours?

17. If 60 A. cost $3000, how many acres will cost $2450?

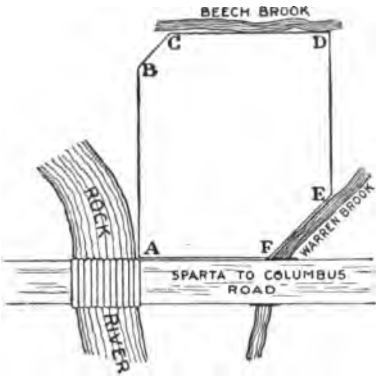


FIGURE 124

18. The line BC (Fig. 124) represents 20 rods. FE is twice as long, AF four times as long, CD and ED five times as long, and AB six times as long. Find the length of each side.

19. Find the value of x in the following proportions:

$$BC : FE = AB : (x);$$
$$BC : (x) = FE : CD;$$
$$AF : FE = (x) : BC.$$

20. When a foot rule is held $2\frac{1}{2}$ ft. (arm's length) in front of the face as shown in Figure 125, 7 in. on the ruler seems just to cover the edge of a door 7 ft. high. If the ruler is held parallel to the edge of the door, how far is the eye from the door?

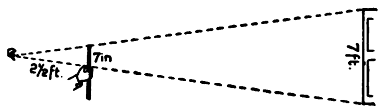


FIGURE 125

21. If the shadow cast by a 4-ft. stake is 5 ft. long, how high is a tree which casts a shadow 50 ft. long on the same day and hour?

22. The shadow cast by a $3\frac{1}{2}$ ft. stake is $10\frac{1}{2}$ ft. long. How high is a flagpole that casts a shadow 225 ft. long at the same time?

23. How high is a house that casts a shadow 120 ft. long at the moment when a stake 6 ft. high casts a shadow 20 ft. long?

24. A road runs 2 miles south, then 1 mile west, then $\frac{3}{4}$ mi. south, then $1\frac{1}{2}$ mi. southeast, and then $2\frac{1}{2}$ mi. south. In a scale drawing of the road the two-mile part is represented by a line 18 in. long. How long a line will represent each of the other parts of the road in the same drawing?

25. What is the scale of the drawing just mentioned?

26. What is the scale of a drawing in which a distance of 20 ft.

is represented by a line $2\frac{2}{5}$ in. long? By a line $\frac{4}{5}$ in. long? By a line $2\frac{1}{2}$ ft. long?

27. The food parts of beef are water, fat, and protein. In 1 lb. of loin steak there are $3\frac{2}{5}$ oz. of water, $2\frac{4}{5}$ oz. fat, $2\frac{3}{5}$ oz. protein, and $2\frac{1}{5}$ oz. of waste material. How much of each of these parts are there in 3 lb. of loin steak? In $5\frac{1}{2}$ lb.? In a 12-pound loin steak?

28. In 4 lb. sirloin steak there are $34\frac{2}{5}$ oz. water, $10\frac{2}{5}$ oz. fat, $10\frac{2}{3}$ oz. protein, and $8\frac{8}{15}$ oz. waste matter. How much of each is there in an 8-lb. sirloin steak? In 24 lb. sirloin steak? In 30 pounds?

29 In a 10-lb. porterhouse steak there are $5\frac{1}{4}$ lb. water, $1\frac{3}{4}$ lb. fat, $1\frac{2}{3}$ lb. protein, and $1\frac{1}{3}$ lb. waste. How much of each is there in a 30-lb. porterhouse steak? In 15 lb.? In 18 lb.? In 48 pounds?

30. In 15 lb. of beef ribs there are $6\frac{1}{2}$ lb. water, $3\frac{1}{5}$ lb. fat, $2\frac{1}{20}$ lb. protein, $3\frac{3}{4}$ lb. waste. How much of each is there in 10 lb. beef ribs? In 25 lb.? In 16 lb.? In 75 pounds?

DECIMAL FRACTIONS

§119. Notation of Decimals. ORAL WORK

1. In \$1111 what does the 1st 1 on the right stand for? the 2d? the 3d? the 4th?

2. In \$6666 what does the 1st 6 on the left denote? the 2d? the 3d? the 4th?

3. In \$372.68 what is the unit of the 3 (*Ans.* \$100)? of the 7? of the 2? the 6? the 8?

4. In \$5555 how does the number denoted by each 5 compare with the number denoted by the 5 to its left? to its right?

5. How do the units of the places of a number change as we pass through the number from left to right?

6. Which unit is the fundamental unit out of which the other units are made? How is the place of this fundamental unit indicated in \$675.28?

DEFINITION.—A dot, called the *decimal point*, or *point*, is used to show the units' digit. The point always stands just to the right of the units' digit or place.

7. If the law of problem 5 holds in 444.444, the unit of the 1st 4 to the right of the point equals what part of the unit of the 1st 4 to the left of the point? The unit of the 2d 4 to the right equals what part of the unit of the 1st 4 to the right?

8. In any number what part of the unit of the 1st place to the left of the point equals the unit of the 1st place to the right? of the 2d place to the right? of the 3d? of the 4th?

DEFINITION.—The unit of the 1st place, or digit, to the right is called the *tenth;* of the 2d place, or digit, the *hundredth;* of the 3d, the *thousandth;* of the 4th, the *ten-thousandth* and so on.

§120. Numeration of Decimals.

1. The number 444.444 might be read, "4 hundreds, 4 tens, 4 units and 4 tenths, 4 hundredths, 4 thousandths"; but it is simpler to read it, "Four hundred forty-four *and* four hundred forty-four *thousandths.*" Read 234.234 both ways. Which is the shorter?

2. The latter mode of reading is the one used in practice. It may be stated thus: Read the integral (whole) part of the number, then read the part of the number to the right of the point just as though it were a whole number standing to the left of the point, then pronounce the name of the unit of the last digit on the extreme right. The word "and" must be pronounced only at the decimal point. Read 675.328; 236.89; 7.65; 43.6587.

3. Read the following

(1)	.1;	.01;	.001;	.0101;	1.1;	10.1;
(2)	.6;	6.06;	66.6;	.0606;	600.06;	6000.006;
(3)	.60;	7.070;	8.50;	.0600;	87.087;	10.0008.

4. What is the ratio of 5 to .5; of .5 to .05; of .5 to .005; of 55 to 5.5; of 55 to .55?

5. Find the ratio of 875 to 87.5; of 87.5 to 8.75; of 875 to 8.75; of 875 to .875.

6. How is a number affected by moving the decimal point 1 place toward the left (see problems 4 and 5)? 2 places? 3 places? 6 places?

7. What is a quick way of dividing any number by 10? 100? 1000?

8. Express the ratio of 2.358 to 23.58; to 235.8; to 2358; to 23,580; to .2358; to .02358.

QUERY.—Where is the point supposed to be in 2358? When the decimal point is not written, where is it supposed to be?

9. How is a number changed by moving the decimal point 1 place to the right? 2 places? 3 places? 6 places?

10. Make a rule for quickly multiplying any number by 10; by 100; by 1000; by 1 followed by any number of zeros.

11. Referring to problem 4, can you tell what effect is produced in such a number as .683 or .492 by writing a zero between the decimal point and its first digit? 2 zeros? any number of zeros?

12. Compare .5 with each of these numbers: .50; .500; .50000. What is the effect of writing any number of zeros to the right of the last digit of a decimal?

DEFINITIONS.—A *decimal fraction*, or *decimal*, is a fraction whose denominator is 10, 100, 1000, or some power of ten, in which the denominator is not written but is indicated by the position of the decimal point.

A *power* of 10 is a number obtained by using 10 *as a factor* any number of times.

§121. To Reduce a Decimal to a Common Fraction.

The fractions we have studied whose denominators are actually written, are called Common Fractions.

1. Express the following decimals as common fractions:

.5; .50; .500; .4; .40; .400; .6; .60; .25; .250; .125; .375; .625; .875.

2. Reduce the results of problem 1 to their lowest terms.

3. Write these mixed decimals as improper fractions:

1.5; 2.50; 2.75; 10.4; 12.5; 6.75; 18.25.

4. Reduce these improper fractions to their lowest terms.

5. After dropping the decimal point from the following decimals, what numbers must be written beneath them to express them as common fractions:

.6; .67; .625; .875; .1275; 12.75; 25.786; 33.333; 6.6666?

6. Make a rule for expressing any decimal as a common fraction in its lowest terms.

7. How may $2\frac{1}{2}$ tenths be expressed as a decimal? $62\frac{1}{2}$ hundredths? $33\frac{1}{3}$ hundredths? 14 and $666\frac{2}{3}$ thousandths?

A *pure decimal* is a decimal whose value is less than 1; as, 38 thousandths, $66\frac{2}{3}$ hundredths. A *mixed decimal* is a decimal whose value is greater than 1: as, 3.58 or $2.87\frac{1}{2}$.

8. Read and give the meaning of these mixed decimals:

$.12\frac{1}{2}$; $3.33\frac{1}{3}$; $18.66\frac{2}{3}$; $2.1\frac{1}{2}$; $.0\frac{3}{4}$; $.04\frac{2}{9}$; $.00\frac{3}{5}$; $36.000\frac{5}{6}$.

NOTE.—Numbers expressed in both decimals and common fractions are called *complex decimals*. A *simple decimal* is expressed without the use of common fractions.

To reduce such an expression as $3.44\frac{5}{7}$ to a mixed number proceed thus:

SOLUTION.—$3.44\frac{5}{7} = \frac{344\frac{5}{7}}{100} = \frac{\frac{2413}{7}}{100} = \frac{2413}{700} = 3\frac{313}{700}$.

Or, thus: $3.44\frac{5}{7} = 3\frac{44\frac{5}{7}}{100} = 3\frac{\frac{313}{7}}{100} = 3\frac{\frac{313}{\cancel{7}} \times \cancel{7}}{100 \times 7} = 3\frac{313}{100}$.

9. Reduce the decimals of problem 8 to mixed numbers or common fractions.

10. Make a rule for expressing any mixed decimal as a common fraction, or a mixed number.

PRINCIPLE I.—*Any decimal may be expressed as a common fraction in its lowest terms or as a mixed number, by dropping the decimal point, writing the denominator, and reducing the resulting common fraction to its lowest terms.*

§122. Rain and Snowfall (ADDITION).

DEFINITION.—1 in. of rainfall means a fall of 1 cu. in. of water on each square inch of surface of the ground. (Review §45, pp. 56-7).

1. The following quantities of rain fell from week to week during May, 1902, in Chicago; find the total rainfall for the month: First week, 1.09 in.; second week, 1.11 in.; third week, .45 in., and fourth week, 2.43 inches.

SOLUTION.—

CONVENIENT FORM

	1.09 in.
	1.11 in.
	.45 in.
	2.43 in.
Ans.	5.08 in.

EXPLANATION.—It is convenient to write the addends in a column so that the units digits are all in the same vertical column. Then begin on the right and add as with whole numbers. In the sum the point should stand directly under the points in the addends.

DEFINITION.—Finding the sum of decimal numbers is called *addition of decimals.*

2. The following numbers denote the monthly rain or snowfall for 1902; what was the total rain or snowfall for the year?

.66 1.53 4.16 2.26 5.08 6.45 4.25 1.44 4.83 1.45 2.03 1.90

3. Without rewriting the numbers, find the total yearly rain or snowfall for the years 1891–1901 from these recorded data:

JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.	TOTAL
1.99	1.95	2.13	2.14	2.09	2.42	2.47	4.52	.32	.36	3.83	1.32	
1.99	1.57	2.21	2.17	6.77	10.58	2.23	1.85	1.34	1.54	2.68	1.63	
2.08	2.44	1.69	4.16	1.93	3.59	3.08	.18	1.98	1.75	2.45	2.14	
1.55	2.13	2.66	2.65	3.35	1.96	.60	.60	8.28	.85	1.18	1.66	
2.15	1.60	1.32	36	1.99	1.79	2.43	6.49	1.89	.51	5.60	6.76	
1.12	3.48	1.26	2.79	4.16	2.82	3.61	3.52	6.70	1.36	2.16	.16	
4.53	2.22	3.56	2.23	.84	3.60	1.47	1.70	.84	.18	3.06	1.62	
3.54	2.59	4.60	.76	2.23	5.30	1.94	3.03	3.16	3.26	2.25	1.11	
.58	1.60	2.11	.14	4.35	2.71	6.66	.91	2.39	2.09	1.14	6.81	
1.21	3.52	1.58	1.02	3.59	2.06	4.64	4.24	1.56	1.35	3.30	.58	
1.15	2.05	3.38	.33	2.18	2.42	4.25	2.00	2.92	1.29	.85	1.70	

4. Foot and average the vertical columns and tell what the footings and averages mean.

§123. Other Applications.

1. A coal dealer received 8 carloads of coal of the following tonnages: 24.6, 28.785, 31.25, 24.95, 31.8, 25.125, 28, and 29.25. What was the total tonnage (number of tons)?

2. A farm was divided by its owner iuto lots of the following acreages (number of acres): $32.87\frac{1}{2}$, $7.12\frac{1}{2}$, $68.33\frac{1}{3}$, $11.66\frac{2}{3}$, $16.28\frac{3}{8}$, and $21.13\frac{7}{8}$. What was the total area of the farm?

3. The following numbers represent in thousands of feet a lumber dealer's sales in 1 da.: 6.865, 24.245, 16.398. 12.28, 18.2, 6.395, 24, and 18.967. What were the total sales for the day?

4. A quantity of soil contained .125 lb. gravel; .268 lb. coarse sand; .175 lb. fine sand; $.037\frac{1}{2}$ lb. organic matter; $.214\frac{3}{4}$ lb. clay, and .275 lb. water. What was the total weight of the soil?

5. During 8 hr. a freight train made the following mileages: 32.15, 28.375, 15.687, 20.2, 15.63, 17.5, 8.95, and 21.3. How far did the train run during the 8 hours?

6. Following are the weights in grains of the U. S. coins: 1¢-piece, 48; 5¢-piece, $73.166\frac{2}{3}$; dime, $38.583\frac{1}{3}$; quarter dollar,

96.45; half dollar, 192.9; dollar, 412½; quarter eagle, 64½; half eagle, 129; eagle, 258; double eagle, 516. Find the total weight of all.

§124. **Nature Study**—(SUBTRACTION).

1. 54 cu. in. of soil in its natural state weighed 1.94 lb. After being thoroughly dried it weighed 1.459 lb. How much moisture passed off in drying?

SOLUTION.—
CONVENIENT FORM

1.940 lb.
1.459 lb.

Ans. .481 lb.

EXPLANATION.—We have seen that 1.94 may be written 1.940. For convenience write the numbers so that units digits stand in the same column. Beginning on the right subtract as though the numbers were whole numbers. In the result the point should stand directly under the points in the minuend and subtrahend.

DEFINITION.—Finding the difference of decimal numbers is called *subtraction of decimals.*

2. After drying, the same soil occupied only 37.125 cu. in. How much did it shrink in bulk in drying?

3. 100 green oak leaves weighed .22 lb. After thorough drying they weighed .087 lb. What was the weight of water contained in the green leaves?

4. The organic matter in the leaves was then driven off by burning the dry leaves. The ash weighed .0053 lb. How much organic matter did the 100 leaves contain?

5. The corresponding numbers for 100 green elm leaves were: Weight of green leaves, .132 lb.; weight of dry leaves, .0345 lb.; weight of ash, .0035 lb. Answer questions like 3 and 4 for these leaves.

6. Answer similar questions for these leaves:

KIND OF LEAVES	WEIGHT IN POUNDS		
	FRESH, GREEN	DRY	ASH
50 Poplar leaves........	.132	.043	.0043
35 Compound ash leaves.	.099	.044	.0026

7. 1.135 lb. of dry beans, soaked for 24 hr., weighed 2.212 lb. What was the amount of water taken up by the beans?

8. The dry beans occupied 34.875 cu. in. and the soaked beans 85.5 cu. in. How much did the beans increase in bulk?

§125. Stature and Weight of Persons.

The following table contains the average heights and weights of boys, girls, men, and women for the ages indicated by the numbers in the first column. Heights are given in feet and weights in pounds.

Age Yr.	Heights		Diff.	Growth in Height		Weights		Diff.	Growth in Weight	
	Males	Females		Males	Females	Males	Females		Males	Females
2	2.60	2.56				25.01	23.53			
4	3.04	3.00				31.38	28.67			
6	3.44	3.38				38.80	35.29			
9	4.00	3.92				49.95	47.10			
11	4.36	4.26				59.77	56.57			
13	4.72	4.60				75.81	72.65			
15	5.07	4.92				96.40	89.04			
17	5.36	5.10				116.56	104.43			
18	5.44	5.13				127.59	112.55			
20	5.49	5.16				132.46	115.30			
30	5.52	5.18				140.38	119.82			
40	5.52	5.18				140.42	121.81			
50	5.49	5.04				139.96	123.86			
60	5.38	4.97				136.07	119.76			
70	5.32	4.97				131.27	113.60			
80	5.29	4.94				127.54	108.80			
90	5.29	4.94				127.54	108.81			

1. Fill out on a separate slip the columns headed Difference (Diff.); the first by subtracting the height of the females from that of the males of the same age, and the second by subtracting the weights of females from those of males of the same age. Subtract without rewriting the numbers.

2. There are 4 vacant columns headed Growth; two for growth in height and two for growth in weight. Fill out on a separate slip columns like the first of these from the column of heights of males by subtracting each number of this column from the one

next below it. Tell what the difference means. Fill out a column like the second similarly from the column of heights of females.

3. In a similar way fill out the last two columns from the columns of weights of males and weights of females. Tell what these differences mean.

4. At what age are boys growing most rapidly in height? in weight? At what age do men begin to decrease in height? in weight?

5. Answer questions similar to 4 for females.

6. Compare your own height and weight with the numbers of this table, for your age.

NOTE.—If your age is not in the table, it will be between two ages given there. Use the mean of the numbers for these two ages for your comparison.

7. The following table contains the height in feet of children of Manchester and of Stockport, (1) who are working in factories, and (2) who are not working in factories. Without rewriting the numbers, fill out the vacant columns of differences between the heights of the two classes for both boys and girls. What effect, if any, of such work can you detect (find out) on the growth of the boy or girl?

NOTE.—When the boy or girl *not* working in factories is taller than the one working in factories, mark the difference with a plus (+) sign before it. In the opposite case mark the difference with the minus (−) sign before it.

AGES	BOYS		DIFF.	GIRLS		DIFF.
	Working in Factories	Not Working in Factories		Working in Factories	Not Working in Factories	
9 years..	4.009	4.045		3.996	4.036	
10 "	4.167	4.219		4.134	4.114	
11 "	4.272	4.252		4.261	4.341	
12 "	4.446	4.413		4.475	4.472	
13 "	4.537	4.580		4.636	4.590	
14 "	4.715	4.725		4.813	4.852	
15 "	4.971	4.836		4.875	4.928	
16 "	5.134	5.266		4.990	5.003	
17 "	5 223	5.338		5.039	5.059	
18 "	5.276	5.825		5.226	5.397	

§126. Pointing the Product of Decimals—(MULTIPLICATION).

ORAL WORK

1. What is the relation between the following pairs of numbers?

(1) 25 and 2.5	(4) 1.28 and 12.8	(7) 2847 and 28.47
(2) 25 and .25	(5) 1.28 and 128	(8) 284.7 and 2.847
(3) 75 and 7.5	(6) 47.8 and 4.78	(9) 28.47 and .2847

2. The product 37×25 equals how many times the product 37×2.5?

3. The product 684×7.5 equals what part of the product 684×75?

4. What is the product 684×75? What, then, is the product 684×7.5? What is the product $684 \times .75$? $684 \times .075$? $68.4 \times .075$? $6.84 \times .075$? $.684 \times .075$?

DEFINITION.—By the number of decimal places of a number is meant the number of digits (zero included) on the right of the decimal point.

ILLUSTRATION.—In 2.005 there are 3 decimal places.

WRITTEN WORK

1. Find the product 4862×784, and from it write the following products:

486.2×784; 48.62×784; 48.62×78.4; 4.862×7.84; $4862 \times .784$.

2. How many decimal places are there in 486.2? in 48.62? 4.862? 10.03? 3.0060? .0600? 20.0806?

3. How many decimal places are there in each of the products of problem 1?

4. Compare the number of decimal places in each of the products of problem 1 with the sum of the numbers of decimal places in both the multiplicand and the multiplier. What do you find?

5. Make a rule for finding how many decimal places there must be in the product of two decimals.

6. How, then, can you find where the decimal point belongs in the product of any two decimals?

PRINCIPLE II.—*The number of decimal places in the product equals the sum of the numbers of decimal places in the factors.*

7. A field containing 38.75 A. yielded 23.9 bu. of wheat per acre; what was the total yield?

SOLUTION.—Since each acre yielded 23.9 bu., 38.75 A. must yield 38.75 × 23.9 bushels.

CONVENIENT FORM	
38.75	3875
23.9	239
1195	1195
1673	1673
1912	1912
717	717
926.125	926125

Ans. 926.125 bu.

EXPLANATION.—
239 × 38.75 = how many times 23.9 × 38.75?
239 × 3875 = " " 239 × 38.75?
239 × 3875 = " " 23.9 × 38.75?
What is $\frac{1}{1000}$ of 926,125?
Does the rule of problem 5 hold true here?

8. A steer weighing 16.22 cwt. sold at $8.85 per cwt.; what price did he bring?

9. A passenger train ran for 12.27 hr. at the rate of 65.75 mi. per hour; how far did it run during the time?

§127. Force Needed to Draw Loads on Road Wagon.

1. To draw a load in a road wagon at a slow walk over hard, level country roads a horizontal pull of .075 of the total weight of the wagon and load is required. What horizontal pulls will be required to draw the following:

	WEIGHT OF WAGON	WEIGHT OF LOAD	TOTAL WEIGHT	HORIZONTAL PULL IN LB.
(1)	1068 lb.	2168 lb.	....	....
(2)	969.5	2408.3	....	....
(3)	1580.75	3675.6	....	....
(4)	2368	3890	....	....

2. Over fresh earth .125 of the total load as a horizontal pull will draw it on a road wagon. Fill out the vacant columns, for these conditions:

	WEIGHT OF WAGON	WEIGHT OF LOAD	TOTAL WEIGHT	HORIZONTAL PULL IN LB.
(1)	980 lb.	1260 lb.	....	....
(2)	940.8	768.78	....	....
(3)	1164	3890.85	....	....
(4)	1675	4060.75	....	......

3. Over loose sand .258 of the total load will draw it. Find the forces needed to draw these loads:

	Weight of Wagon	Weight of Load	Total Weight	Horizontal Pull in Lb.
(1)	375 lb.	685.9 lb.	....	....
(2)	1860	2586.38	....	....
(3)	2800	3869.25	....	....

4. On good, broken stone pavement the pull is about .0285 of the total load; on wood pavement it is .019 of the total load; and on Macadam pavement it is .0333 of the load. Find the pull in pounds needed for each of these three kinds of pavement for the following:

	Weight of Wagon	Weight of Load	Total Weight	Horizontal Pull in Lb.
(1)	4060 lb.	3795.65 lb.	....	....
(2)	5190	6340.86	....	....
(3)	4960	7640.65	....	....

§128. Division by an Integer.

1. I paid $941.25 for 15 A. of land; what was the price per acre?

Convenient Form

With Decimals	With Integers
62.75	6275
15)941.25	15)94125
90	90
41	41
30	30
11.2	112
10.5	105
.75	75
.75	75

Explanation.—Compare the steps in the work with decimals with the corresponding steps in the work with integers.

How do the decimal points stand through the problem? How does the point stand in the quotient? How may the dividend be found from the divisor and the quotient? How, then, may you check division?

Ans. $62.75.

2. I paid $2823.75 for 45 A. of land; what was the price paid per acre?

3. The 36 members of a society were assessed (made to pay) equally to meet a debt of $1341 against the society. What was the amount of the assessment against each member?

SOLUTION. —

```
     37.25
36)1341.00
   108
    261
    252
      9.0
      7.2
      1.80
      1.80
```

Notice particularly how by writing zeros after the decimal point the quotient may be carried out in a decimal form.

What effect does writing zeros after the point have on a number?

Ans. $37.25.

4. The number of states and territories and the total areas in square miles of the land surface of the principal geographic divisions of continental United States according to the Twelfth Census, are given in the following table. Find the average land surface of a state for each division and for the whole United States. Carry the division to three decimal places.*

DIVISION	NUMBER	LAND SURFACE	AVERAGE
North Atlantic	9	162,103	
South Atlantic	9	168,620	
North Central	12	753,550	
South Central	9	610,215	
Western	11	1,175,742	
Continental United States			

5. The 10 loads given in pounds in column 2 of the following table required the number of pounds of force given in the third column, to draw them on a common road wagon over a good, level country road. For example, 1,400 lb. required 98 lb. to pull it;

*If the next decimal place after the last one required is less than 5, write the given quotient as the required decimal. If it is greater than 5, add 1 to the last figure of the decimal required. This rule applies to all cases of division of decimals.

1616 lb. required 112 lb., and so on. Divide each load by the number of pounds of force needed to draw it, and put the quotient to two decimal places in the column headed "Ratio."

EXPERIMENT	LOAD	PULL	RATIO
1	1400	98	....
2	1616	112	....
3	1825	126	....
4	2236	155	....
5	2440	170	....
6	2650	185	....
7	2863	198	....
8	3072	216	....
9	3281	229	....
10	3662	244	....

§129. Division by a Decimal.

1. What is $\frac{1}{11}$ of 55?

2. When no decimal point is written with a number where is it understood to be?

3. Where is the decimal point in 55?

4. How is a number changed by moving its decimal point 1 place toward the left? 3 places toward the left? 1 place toward the right? 2 places toward the right?

5. 55 equals how many times 5.5? .55? .055?

6. If $55 \div 11 = 5$, to what is $5.5 \div 11$ equal? $.55 \div 11$?

7. Name these quotients: $250 \div 25$; $25 \div 25$; $2.5 \div 25$; $.25 \div 25$.

8. Name these quotients: $250 \div 2.5$; $25 \div 2.5$; $25 \div .25$; $250 \div .25$.

9. $2176 \div 32 = 68$; to what number is x equal in each of the following equations:

(1) $217.6 \div 32 = x$; (4) $217.6 \div 3.2 = x$; (7) $2176 \div 3.2 = x$;
(2) $21.76 \div 32 = x$; (5) $217.6 \div .32 = x$; (8) $21.76 \div .32 = x$;
(3) $2.176 \div 32 = x$; (6) $21.76 \div 3.2 = x$; (9) $2.176 \div .032 = x$?

10. In each case of problem 9 compare the number of decimal places in the quotient with the number of decimal places in the dividend minus the number in the divisor.

11. 101,388 ÷ 426 = 238; without dividing, write out the numbers to which x is equal in the following equations:

(1) $10138.8 \div 426 = x$; (4) $101388 \div 42.6 = x$; (7) $101.388 \div 42.6 = x$;
(2) $1013.88 \div 426 = x$; (5) $101388 \div 4.26 = x$; (8) $10.1388 \div .426 = x$;
(3) $1.01388 \div 426 = x$; (6) $1013.88 \div 4.26 = x$; (9) $1.01388 \div .0426 = x$.

12. From the results of problems 10 and 11 make a rule for finding the number of decimal places in the quotient.

PRINCIPLE III.—*The number of decimal places in the quotient equals the number of decimal places in the dividend minus the number of decimal places in the divisor.*

§130. Problems.

1. In 1891 the total imports of tea into U. S. were 82,395,924 lb. and of coffee 511,041,459 lb. If the average cost of tea was 37¢ per lb., and of coffee 18¢, what was the total cost of both?

2. On Dec. 6, 1901, a carload of 34 Angus show cattle of weights given in table annexed sold at the prices per cwt. set beside the weights. What was the total weight of the carload? the average weight of the animals?

3. What was the average price per hundredweight?

4. For how much did the entire load sell?

5. What was the average price per head which the owner received for the carload?

6. The previous year on the same occasion, a carload of 26 prize-winning Angus cattle, averaging 1492 lb., sold at $15.50 per hundredweight. What average price did the cattle bring the owner?

7. How much did he receive for the carload, problem 6?

No.	WEIGHT	PRICE
1	1503	$ 9.00
2	1622	8.85
3	1606	8.60
4	1524	8.50
5	1524	8.50
6	1504	8.10
7	936	8.70
8	1327	6.85
9	1273	8.05
10	1130	8.75
11	1326	7.65
12	1141	8.50
13	1318	8.75
14	1327	8.70
15	1190	7.70
16	1446	7.85
17	1449	7.85
18	1542	7.70
19	980	6.80
20	1450	8.10
21	852	8.30
22	1376	8.30
23	1540	25.00
24	1631	9.30
25	1468	8.65
26	1297	8.50
27	1529	8.20
28	1100	7.60
29	1073	7.60
30	1110	8.10
31	1095	8.15
32	1456	8.75
33	1298	8.00
34	2130	10.75

8. A dairyman finds that during November one of his cows furnished this record:

	MORNING MILKINGS	EVENING MILKINGS
1st week	47.2 lb.	40.4 lb.
2d "	58.6 "	48.0 "
3d "	53.8 "	49.8 "
4th "	62.7 "	47.3 "
29th and 30th days	17.8 "	16.4 "

What was the total number of pounds of milk given by this cow during the month at the morning milkings? at the evening milkings? at both milkings?

9. If milk was worth 6.25¢ a quart (8.6 lb. per gallon), what was the milk of this one cow worth to the owner during November?

§131. Ratio of Circumference of Circle to Diameter.

1. The distance around the rung of a chair was measured and found to be 3.625 in.; the diameter was found to be 1.153 in. Divide the distance around the rung by the diameter and find the quotient to 3 decimal places.

DEFINITION.—The distance around a circle is the *circumference* of the circle.

2. The circumference of a circular rod, 1.875 in. in diameter, was measured and found to be 5.884 in. Find the ratio to 3 decimal places, of the circumference to the diameter.

3. Measure the diameters and the circumferences of any circles in your schoolroom and find to 3 decimal places the ratio of their circumferences to their diameters. If no circles are at hand use the measures of this table:

OBJECT	CIRCUMFERENCE	DIAMETER	RATIO
Ink bottle	5.5 in.	1.75 in.	
Tin box	6.154	1.953	
Globe of lamp	26.538	8.444	
Terrestrial globe	57.080	18.062	
Barrel-head	71.458	22.750	
Iron ring	18.125	5.675	
Average			

4. Measure the circumferences and the diameters of the following objects and find the ratios to three decimal places. If you are unable to make your own measures use those of the table:

OBJECT	CIRCUMFERENCE	DIAMETER	RATIO
Bicycle wheel	81.177	26.025	
Bicycle wheel	88.055	28.012	
Front carriage wheel	151.189	48.125	
Rear " "	176.333	56.125	
Locomotive driver..	174.762	55.625	
Average	....	....	

5. Find the ratio of the circumferences to the diameters of the coins of problem 27, p. 108.

6. It is proved in Geometry that the ratio of the circumference of any circle to its diameter is about $3\frac{1}{7}$, or more accurately 3.1416. If then, the diameter of a circle is known, how may the circumference be found without measurement?

7. Find the average of all the values of the ratios found in problems 1 to 4 and compare this last average with $3\frac{1}{7}$; with 3.1416. What is the difference in each case?

NOTE.—For most practical purposes this ratio, denoted by the Greek letter π, and called pī, may be taken as $3\frac{1}{7}$. For greater accuracy use $\pi =$ 3.1416.

§132. Original Problems.

Make and solve problems based on the following facts:

1. 1 cu. ft. is about .8 of a bushel of small grain. A grain bin is $8' \times 12' \times 22'$.

2. 1 bu. of ear corn is about 2.25 cu. ft. A wagon box is $2.67' \times 2.85' \times 9.6'$.

3. Well settled timothy hay runs about 355.25 cu. ft. to the ton. A hay shed is $24' \times 40'$ and is filled with hay to a height of 18.5′.

4. Loose timothy hay runs about 460 cu. ft. to the ton. A load of hay is $8' \times 18' \times 22.7'$.

5. Stove coal runs about 35.1 cu. ft. to the ton. A coal bin is $6' \times 12.5' \times 15.35'$.

6. 1 perch of stone is 24.75 cu. ft. A stone wall is $2.75' \times 4.385' \times 126.8'$.

7. A man walks about 3.5 mi. per hour. It is 85 mi. from Chicago to Milwaukee.

8. A horse trots about 7.5 mi. per hour.

9. A horse runs about 18 mi. per hour for short distances.

10. A steamboat runs 18 mi. per hour.

11. A slow river flows 3 mi. per hour.

12. A rapid river flows 7 mi. per hour.

13. A crow flies 25 mi. per hour; a falcon, 75 mi.; a wild duck, 90 mi.; a sparrow, 92 mi.; and a hawk, 150 mi. per hour.

14. A carrier pigeon flies 80 mi. per hour for long distances.

15. Sound travels through air 1134 ft. per second; through water, 5000 ft. per second; and through iron or steel, 17,000 ft. per second.

16. A rifle ball travels 1460 ft. per second at starting; and a 20-lb. cannon ball, 16,000 ft. per second at starting.

17. Light travels 186,600 mi. per second; electricity, 288,000 mi. per second. The sun is 93,000,000 mi. from the earth; the moon, 240,000 miles.

18. One horse-power raises 33,000 lb. through a height of 1 ft. in 1 minute.

19. The equatorial diameter of the earth is 7925.6 mi.; the polar diameter, 7899.1 miles.

§133. Physical Measurements.

The following table gives, for men, of ages from 18 to 26 years, the average lung capacity in cubic inches, the height in inches, and the weight in pounds.

AGE	LUNG CAPACITY	HEIGHT	WEIGHT
18	251.4	68.2	134.25
19	251.8	68.2	135.40
20	258.2	68.1	138.65
21	260.4	68.1	140.60
22	264.8	68.2	141.15
23	263.7	68.1	138.60
24	267.1	68.2	143.90
25	267.2	68.2	143.15
26	267.1	68.9	142.30

1. Find the number of cubic inches of lung capacity per inch of height, for each age, by computing (finding) the ratio to 2 decimal places of each number of column 2 to the corresponding number of column 3. Is this ratio the same for all ages?

2. Similarly find the number of cubic inches, to 2 decimal places, of lung capacity per pound of weight, for each age.

3. For each age find the number of pounds of weight per inch of height to 2 decimal places. Are these numbers the same for all ages?

§134. Specific Gravity.

The *specific gravity* of any solid or liquid substance is the ratio of its weight to the weight of an equal bulk of water. The weight of a cubic foot of water is 62.5 pounds.

1. The following table contains the weight in pounds of 1 cu. ft. of the substances mentioned. Find to 3 decimal places the specific gravities of these substances:

Metal	Weight of 1 Cu. Ft.	Specific Gravity	Wood	Weight of 1 Cu. Ft.	Specific Gravity	Liquid	Weight of 1 Cu. Ft.	Specific Gravity
Aluminum..	166.5		Cork.........	15.00		Alcohol	50.0	
Zinc........	436.5		Spruce	31.25		Turpentine ..	54.4	
Cast iron...	450.0		Pine (yellow)	34.60		Petroleum ...	55.7	
Tin.........	458.3		Cedar..	35.06		Olive oil	57.0	
Wr't iron...	480.0		Pine (white)	28.00		Linseed oil ...	59.5	
Steel	490.0		Walnut	41.90		Sea water	64.1	
Brass	523.8		Maple.	46.88		Milk	64.5	
Copper	552.0		Ash	52.80		Acetic acid ..	66.5	
Silver.......	655.1		Beech	53.25		Muriatic acid.	75.0	
Lead	709.4		Oak	65.00		Nitric acid...	95.0	
Gold	1200.9		Ebony.......	76.00		Sulphuric acid	115.1	
Platinum ..	1347.0		Lignum vitæ	83.30		Mercury	880.0	

2. The specific gravity of any gas or vapor, is the ratio of its weight to the weight of an equal volume of air, the gas or vapor and the air being at the same temperature and under the same

pressure. Find to 3 decimal places the specific gravities of the gases and vapors in the following table:

Gas	Weight, in Pounds, of 1 Cubic Foot	Specific Gravity
Hydrogen	.00559	
Smoke (wood)	.00727	
Smoke (soft coal)	.00815	
Steam at 212° F.	.03790	
Carbonic oxide	.07810	
Nitrogen	.07860	
Air	.08073	
Oxygen	.08925	
Carbonic acid	.12344	
Chlorine	.19700	

3. The following familiar substances may be compared with water as to weight. Find their specific gravities to 2 decimal places:

Substance	Weight, in Pounds, of 1 Cubic Foot	Specific Gravity
Glass (average)	175.8	
Chalk	174.5	
Marble	169.2	
Granite	166.4	
Stone (common)	158.2	
Salt "	133.4	
Soil "	124.5	
Clay	121.8	
Brick	118.3	
Sand	113.9	

§135. To Reduce a Common Fraction to a Decimal.

1. Express $\$\frac{3}{8}$ decimally.

Solution

$$\begin{array}{r} .375 \\ 8\overline{)3.000} \\ 2.4 \\ \hline .60 \\ .56 \\ \hline .040 \\ .040 \\ \hline \end{array}$$

Explanation.—Annex zeros to the right of the decimal point after the numerator, and then divide by the denominator.

Ans. $\$\frac{3}{8} = \$.375$

2. Express the following common fractions decimally (to 3 decimal places):

$$\tfrac{3}{4};\ \tfrac{2}{5};\ \tfrac{4}{5};\ \tfrac{7}{8};\ \tfrac{5}{8};\ \tfrac{3}{16};\ \tfrac{5}{32};\ \tfrac{13}{64}.$$

3. Express the following mixed numbers decimally (to 3 places):

$$1\tfrac{1}{2};\ 3\tfrac{1}{5};\ 2\tfrac{3}{4};\ 6\tfrac{5}{8};\ 13\tfrac{5}{16};\ 18\tfrac{13}{32}.$$

4. Express the following decimally to 5 decimal places:

$$\tfrac{1}{7};\ \tfrac{3}{13};\ \tfrac{8}{17};\ \tfrac{9}{23};\ \tfrac{16}{37};\ \tfrac{5}{19}.$$

Such decimals as these fractions give rise to, that do not terminate, are called *non-terminating decimals.*

5. Express the following numbers decimally to 6 decimal places:

$$\tfrac{1}{3};\ \tfrac{2}{3};\ \tfrac{1}{6};\ \tfrac{5}{6};\ \tfrac{1}{9};\ \tfrac{2}{9};\ \tfrac{7}{9};\ \tfrac{4}{11};\ \tfrac{6}{11};\ \tfrac{9}{11};\ \tfrac{10}{27};\ \tfrac{11}{27};\ \tfrac{20}{27}.$$

DEFINITION.—Such non-terminating decimals as these that repeat the same digit or group of digits indefinitely, are called *repetends,* or *circulating decimals,* or *circulates.*

6. Express the values of these numbers by the use of integers and decimals only:

$1.3\tfrac{1}{2}$; $16.87\tfrac{1}{2}$; $5.28\tfrac{3}{5}$; $17.9\tfrac{5}{8}$; $20.0\tfrac{2}{5}$; $1.00\tfrac{4}{5}$; $.00\tfrac{7}{8}$; $.0\tfrac{7}{8}$; $.000\tfrac{1}{16}$; $30.060\tfrac{3}{5}$.

7. Express the following fractions decimally to 4 places:

$$\tfrac{11}{12};\ \tfrac{10}{13};\ \tfrac{7}{19};\ \tfrac{3}{11};\ 4\tfrac{2}{9};\ \frac{3\frac{1}{2}}{7\frac{1}{2}};\ \frac{6.7}{7.8};\ \frac{8.65}{12.17};\ \frac{7\frac{3}{5}}{12\frac{7}{9}}.$$

8. A man sold $37\tfrac{1}{2}$ A. of his farm of $79\tfrac{3}{4}$ A.; how many hundredths of his farm did he sell?

9. A man owned a piece of city land 450′ square. A strip $37\tfrac{1}{2}'$ wide was cut from each of its four sides for streets. How many hundredths of the square were cut away?

10. How many hundredths of the area of a page of this book are in the margins? (Measure to the nearest 16th of an inch).

11. How many hundredths of the area of the surface of your desk is the area of the surface of your book?

12. How many hundredths of the area of the surface of the floor is the area of the surface of your desk?

§136. Area of a Circle.

Draw a circle and diameter AB (see Fig. 132). With an opening of the compasses equal to the radius of the circle and with

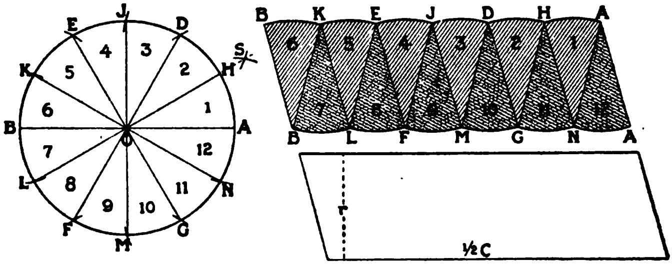


FIGURE 132

A as center draw short arcs at D and G, also, with same radius and with B as center, draw short arcs at E and F. Draw radii OD, OE, OF, and OG.

Bisect angle AOD, as in Problem II, p. 176, and draw OS.

With an opening of the compasses equal to the distance between D and H and with center D draw an arc at J; with center B draw arcs at K and L; also, with center G draw arcs at M and N. Draw radii OJ, OK, OL, OM, and ON.

Cut the circle into the twelve equal sectors thus formed and place these sectors as shown in the second part of the figure. How long is the base AB of the *approximate* parallelogram thus formed? How wide is the approximate parallelogram?

If each of the twelve sectors were split into halves and the resulting twenty-four sectors were fitted together as are the twelve sectors, would the wavy base line become more nearly straight? How long and how wide would the new *approximate* parallelogram be? What would be its area?

Having the circumference and the radius of a circle how can you find the area of the circle?

Having the radius of any circle how can you find the area of the circle (See §131)?

PROBLEMS

In problems 1 to 3, inclusive, use $\pi = 3\frac{1}{7}$. Let r denote the length of the radius of a circle, let c denote its circumference, and A, its area.

1. Find the areas, A, of these circles

(1) $r = 12.5'$, $c = 78.54'$; (3) $r = 20''$, $c = 125.664''$;
(2) $r = 6.25'$, $c = 39.27'$; (4) $r = 60''$, $c = 377.143''$.

2. The diameter of a drum-head is 2.5′ and the circumference is 7.854′, how many square inches of skin are in the two heads?

3. The diameter of a circular window is 18.5″ and its circumference is 58.12″, what is its area?

In the following problems π is taken as 3.1416. Results should be correct to 4 decimal places.

4. The wind is blowing squarely against a circular signboard, whose diameter is 32.75 ft., with a pressure of 25.5 lb. to the square foot. Find the total wind pressure against the board.

5. The circumference of a cylindrical chair rung is 5.5″; find the diameter and area of the right section of the rung.

NOTE.—A right section is the section that would be made by sawing the rung square across.

6. Steam passes from the boiler of an engine through the passage, P, into the cylinder, and pushes against the circular piston, C, with a pressure of 65 lb. to the square inch. If the diameter of the circular head, C, is 12.5″, what is the total pressure against the left side of the piston?

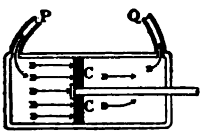


FIGURE 133

7. Answer similar questions for pistons having these diameters with steam pressures per square inch as indicated:

	DIAMETER OF PISTON	PRESSURE PER SQUARE INCH
(1)	10.85″	86
(2)	22.35″	120
(3)	14.85″	180
(4)	16.45″	175
(5)	20.75″	125
(6)	15.85″	160
(7)	21.35″	205

8. Find the area, A, of a circle whose radius is r ft. long and whose circumference is c ft. long.

9. Find the circumference, c, of a circle whose radius is r rods long. Find the area.

10. A cow is tied to the corner of a corn crib, 18′ × 18′, with a rope 18′ long. Over how many square feet can she graze?

§137. Exercises for Practice.

1. Add the following:

(1)	(2)	(3)	(4)	(5)	(6)
37.8	248.001	.96	7.68	862.01	328.
8.65	16.25	3.004	86.043	2.8106	.032
12.08	9.018	10.	190.608	843.7	7.001½
.75	24.	7.	9.	1200.	29.080¾
16.	3.	.206	.9	.16	109.
14.01	.03	200.	8.3	9.	9000.
.6	.4	860.03	60.5	.0068	.08
10.	17.003	.2	3.7	.0309	.0007

2. Subtract the following:

(1)	(2)	(3)	(4)	(5)	(6)
84.96	98.01	100.01	106.038	67.083	800.00
6.38	1.97	87.08	69.879	29.187	98.68

(7)	(8)	(9)	(10)	(11)	(12)
810.02	3.786	26.	3.	68.	1.0101
86.983	.989	.748	2.983	9.8789	.9098

3. Find the following products:

(1) 89.3 × 42.1	(7) 8.08 × .008	(13) 31.416 × 2.518
(2) 68.01 × 9.82	(8) 16.8 × .072	(14) 314.16 × 251.8
(3) 86.3 × 8.761	(9) 9.001 × 1.801	(15) 785.4 × 2.781
(4) 100.001 × 86.8	(10) 1.68 × 7.2	(16) .7854 × .2781
(5) 76.9 × .93	(11) 168 × .072	(17) 3 1416 × 16.84
(6) 7.69 × .093	(12) 3.1416 × 25.18	(18) 314.16 × .1684

4. Find the following quotients:

(1) 120.716 ÷ 26.82	(5) 109.624 ÷ 3.86	(9) 443.52 ÷ .96
(2) 123.1498 ÷ 26.82	(6) 548.12 ÷ 14.2	(10) 14.784 ÷ .032
(3) 276.496 ÷ 3.142	(7) 1774.08 ÷ .384	(11) .3696 ÷ .0008
(4) 268.928 ÷ 8.4	(8) 8.8704 ÷ 192	(12) 9.24 ÷ .00004

5. Find the following quotients to 3 decimal places:

(1) 8 ÷ .3	(7) 2 ÷ 3	(13) 4 ÷ .011	(19) 1.74 ÷ 9.09
(2) 631 ÷ .072	(8) 4 ÷ 9	(14) 2.5 ÷ .033	(20) .77 ÷ 1.21
(3) 9 ÷ .007	(9) 44 ÷ .09	(15) 260 ÷ 3.3	(21) 1.111 ÷ .099
(4) 11 ÷ .0009	(10) 13 ÷ 3	(16) 2.6 ÷ 33	(22) 60.6 ÷ 1.818
(5) 12 ÷ 36	(11) 7.7 ÷ 9	(17) 11.5 ÷ 3.33	(23) .625 ÷ 3.36
(6) 1 ÷ 3	(12) 123 ÷ 9.99	(18) 10.4 ÷ .909	(24) .1 ÷ .09

COMPOUND DENOMINATE NUMBERS

§138. Definitions.

A *denominate number* is a number whose unit is concrete; as, 13 mi., 8 hr., 40 A., $125, etc.

A *concrete unit* is a unit having a specific name; as, 1 yd., 1 lb., $1, 1 hat, 1 horse, etc.

A *compound denominate number* is a number expressed in two or more units of the same kind; as, 11 hr. 25 min. 15 sec.; 12 gal. 3 qt. 1 pt. 3 gills.

TABLES OF MEASURES

NOTE.—Read carefully the tables that follow and fix them in mind by solving the problems beginning on page 219.

§139. Measures of Value.

The *standards of value* of the United States and of some of the European countries are here given with both their rough and their accurate equivalents in U. S. money.

COUNTRY	STANDARD	SYMBOL	ROUGH EQUIVALENT	ACCURATE EQUIVALENT
United States	Dollar	$	$1.	$1.
Great Britain	Pound (Sterling)	£	$5.00	$4.8665
Germany	Mark (Reichsmark)	M.	$ .25	$ .2385
France	Franc	F.	$ .20	$ .193
Russia	Ruble	R.	$ .75	$ .772
Austria-Hungary	Crown, or Filler	C. or F.	$ .20	$ .203
Italy	Lira	L.	$ .20	$ .193

TABLE OF U. S. MONEY

10 mills (m.)	=	1 cent (ct. or ¢)
10 cents	=	1 dime (d.)
10 dimes	=	1 dollar
10 dollars	=	1 eagle
5 dollars	=	$\frac{1}{2}$ eagle
$2\frac{1}{2}$ dollars	=	$\frac{1}{4}$ eagle
20 dollars	=	1 double eagle

The coins of the United States are bronze, nickel, silver, and gold.

TABLE OF ENGLISH MONEY

4 farthings (far.)	=	1 penny (d.)
12 pence	=	1 shilling (s.)
20 shillings	=	1 pound (£)
21 shillings	=	1 guinea

TABLE OF MONETARY UNITS OF OTHER NATIONS

Germany,	1 mark (M.)	= 100 pfennige (pf.)
France,	1 franc (fr.)	= 100 centimes (c.)
Russia,	1 ruble (r.)	= 100 copecks (c.)
Austria-Hungary,	1 crown, or filler	= 100 heller
Italy,	1 lira	= 100 centessimi.

§140. Measures of Weight.

Three systems of weight units are used in the United States, viz.: troy weight, avoirdupois weight, and apothecaries' weight.

Troy weight is used in weighing gold, silver, and jewels. Avoirdupois weight is used for weighing all ordinary articles, and apothecaries' weight is used by druggists in mixing medicines.

The standard of weight in the U. S. is the *troy pound.* The grain is the same in all three systems; the pound the same in troy and in apothecaries' weight and different in avoirdupois.

TABLE OF TROY WEIGHT

24 grains (gr.)	= 1 pennyweight (dwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

TABLE OF EQUIVALENTS

$$\left.\begin{array}{r} 5760 \text{ gr.} \\ 240 \text{ dwt.} \\ 12 \text{ oz.} \end{array}\right\} = 1 \text{ lb.}$$

TABLE OF AVOIRDUPOIS WEIGHT

7000 grains (gr.)	= 1 pound (lb.)
16 ounces	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)
2240 pounds	= 1 long ton (L.T.)

TABLE OF APOTHECARIES' WEIGHT

20 grains (gr.)	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ)
8 drams	= 1 ounce (℥)
12 ounces	= 1 pound (lb.)

§141. Measures of Length, or Distance (Linear Measure).

The standard unit for measures of length, or distance, is the *yard.*

TABLE OF COMMON LINEAR MEASURE

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ ft.	= 1 rod (rd.)
320 rods	= 1 mile (mi.)

TABLE OF EQUIVALENTS

$$\left.\begin{array}{r} 63360 \text{ in.} \\ 5280 \text{ ft.} \\ 1760 \text{ yd.} \\ 320 \text{ rd.} \end{array}\right\} = 1 \text{ mi.}$$

TABLE OF SURVEYORS' LINEAR MEASURE

7.92 inches	=	1 link (li.)
100 links	=	1 chain (ch.)
80 chains	=	1 mile (mi.)

§142. Measures of Surface.

The unit upon which surface measure is based is the *square yard*, which is a square each of whose sides equals 1 yard.

TABLE OF COMMON SURFACE MEASURE

144 square inches (sq. in.)	=	1 square foot (sq. ft.)
9 square feet	=	1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	=	1 square rod (sq. rd.)
160 square rods	=	1 acre (A.)

TABLE OF SURVEYORS' SURFACE MEASURE

625 square links (sq. li.)	=	1 square rod (sq. rd.)
16 square rods	=	1 square chain (sq. ch.)
10 square chains	=	1 acre (A.)
640 acres (a section)	=	1 square mile (sq. mi.)
36 square miles	=	1 township (Tp.)

TABLE OF EQUIVALENTS

$$\left.\begin{array}{r}3686400 \text{ sq. rd.}\\ 230400 \text{ sq. ch.}\\ 36 \text{ sq. mi.}\end{array}\right\} = 1 \text{ Tp.}$$

§143. Measures of Volume.

TABLE OF CUBIC MEASURE

1728 cubic inches (cu. in.)	=	1 cubic foot (cu. ft.)
27 cubic feet	=	1 cubic yard (cu. yd.)

TABLE OF EQUIVALENTS

$$\left.\begin{array}{r}46656 \text{ cu. in.}\\ 27 \text{ cu. ft.}\end{array}\right\} = 1 \text{ cu. yd.}$$

A *perch* of stone is a square-cornered mass, $1' \times 1\frac{1}{2}' \times 16\frac{1}{2}'$, or $24\frac{3}{4}$ cubic feet.

Fire wood is measured by the *cord*. A cord of wood is a straight pile, $4' \times 4' \times 8'$, or 128 cubic feet. A *cord foot* is a straight pile of wood, $4' \times 4' \times 1'$. How many cubic feet are there in a cord foot?

§144. Measures of Capacity.

Liquid measure is used in measuring liquids, the capacity of cisterns, tanks, etc.

TABLE OF LIQUID MEASURE		TABLE OF EQUIVALENTS	
4 gills (gi.)	= 1 pint (pt.)	32 gi.	= 1 gal.
2 pints	= 1 quart (qt.)	8 pt.	
4 quarts	= 1 gallon (gal.)	4 qt.	

NOTE.—A liquid gallon contains 231 cubic inches.

Dry measure is used in measuring grain, fruit, vegetables, etc. The standard of dry measure is the *Winchester bushel*, which is a cylinder 18½ in. in diameter and 8 in. deep, containing 2150.42 cubic inches.

TABLE OF DRY MEASURE		TABLE OF EQUIVALENTS	
2 pints (pt.)	= 1 quart (qt.)	64 pt.	= 1 bu.
8 quarts	= 1 peck (pk.)	32 qt.	
4 pecks	= 1 bushel (bu.)	4 pk.	

NOTE.—A dry gallon = 4 qt. or ½ pk., or 268.8 cubic inches.

AVOIRDUPOIS POUNDS IN A BUSHEL PRESCRIBED BY VARIOUS STATES

COMMODITIES.	California.	Connecticut.	Illinois.	Indiana.	Iowa.	Kansas.	Kentucky.	Louisiana.	Maine.	Massachusetts.	Michigan.	Minnesota.	Missouri.	New Hampshire.	New Jersey.	New York.	Ohio.	Pennsylvania.	Tennessee.	Texas.	Vermont.	Virginia.	Washington.	Wisconsin.
Barley	50	48	48	48	48	48	47	48	48	48	48	48	48	..	48	48	48	47	48	48	48	48	48	48
Beans	..	60	60	60	60	60	60	60	62	..	60	..	60	60	60	62	60	..	60	60	60	60	60	60
Blue Grass Seed	..	..	14	14	14	14	14	14	..	..	14	..	14	..	..	..	..	..	14	..	..	14	..	..
Buckwheat	40	48	52	50	52	50	56	..	48	48	48	50	52	..	50	48	50	48	50	42	48	52	42	48
Castor Beans	..	..	46	46	46	46	..	..	..	..	..	..	46	..	..	..	..	..	..	..	..	..	..	..
Clover Seed	..	..	60	60	60	60	60	60	..	..	60	60	60	..	64	60	60	62	..	60	60	60	60	60
Coal (Anthracite)	..	..	80	..	80	80	76	..	..	..	..	..	..	..	..	..	80	..	..	80	..	80	..	..
Corn on the Cob	..	..	70	68	70	70	70	..	..	..	70	..	..	..	..	..	70	..	70	70	..	70	..	70
Corn, Shelled	52	56	56	56	56	56	56	56	56	56	56	56	56	56	56	58	56	56	56	56	56	56	56	56
Cornmeal	..	50	48	50	..	50	50	..	50	50	50	..	..	50	..	..	..	..	50	..	..	50	..	..
Dried Apples	..	..	24	25	24	24	24	..	..	..	22	28	24	..	25	..	24	..	24	28	..	28	28	25
Dried Peaches	..	..	33	33	33	33	39	..	..	..	28	28	33	..	33	..	33	..	..	28	..	32	28	28
Flax Seed	..	..	56	..	56	56	56	..	..	..	56	..	56	..	55	55	56	..	56	56	..	56	..	56
Hemp Seed	..	..	44	44	44	44	44	..	..	..	..	..	44	..	..	..	..	..	..	44	..	..	..	..
Millet	..	..	50	..	48	50	50	—	..	..	..	..	..	..	..	..	50	..	..	50	..	50	..	55
Oats	32	32	32	32	32	32	32	32	32	32	32	32	32	32	30	32	33	30	32	32	32	32	32	32
Onions	..	50	57	48	37	57	57	..	52	52	54	..	57	..	57	..	55	..	56	57	52	57	50	57
Peas	..	60	..	..	..	..	60	60	60	..	60	..	..	60	60	60	60	..	60	..	60	60	60	..
Potatoes	..	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
Rye	54	56	56	56	56	56	56	56	50	56	56	56	56	56	56	56	56	56	56	56	56	56	56	56
Sweet Potatoes	..	..	50	..	46	50	55	..	..	54	56	..	..	..	54	..	50	..	50	55	..	56	..	..
Timothy Seed	..	..	45	45	45	45	45	45	..	..	45	..	45	..	..	44	45	..	45	45	45	45	40	45
Turnips	..	50	55	..	..	55	60	..	60	..	58	..	..	..	..	..	..	..	..	55	60	55	50	42
Wheat	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60

§145. Measures of Time.

The standard unit for measuring time is the *mean solar day*. The mean solar day is the average time interval from the instant when the sun crosses the meridian of a place (noon) to the next instant of crossing the meridian (the next noon).

TABLE OF TIME MEASURE

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
30 days	= 1 calendar month (mo.)
12 calendar months	= 1 calendar year (yr.)
365 days	= 1 common year
366 days	= 1 leap year
100 years	= 1 century

NOTE.—One mean solar year = 365 da. 5 hr. 48 min. 46 sec. = $365\frac{1}{4}$ da. (nearly).

TABLE OF EQUIVALENTS

$$\left.\begin{array}{r} 31536000 \text{ sec.} \\ 525600 \text{ min.} \\ 8760 \text{ hr.} \\ 365 \text{ da.} \\ 52 \text{ wk. } 1 \text{ da.} \\ 12 \text{ mo.} \end{array}\right\} = 1 \text{ common yr.}$$

According to the *Julian calendar*, adopted by Julius Caesar (whence the name), every year whose date number (as 1896, 1904) is divisible by 4 must contain 366 da. and all other years contain 365 da. Years containing 366 da. are called *leap years;* those containing 365 da. are called *common years.*

According to the *Gregorian calendar*, which is now used by nearly all civilized nations, every year whose date number is divisible by 4 is a leap year, unless the date number ends in two zeros (as 1600, 1900), in which case the date number must be divisible by 400 to be a leap year.

The extra day of the leap year is added to February, giving this month 29 da. in leap years.

§146. Measurement by Counting.

TABLE FOR COUNTING

12 things	= 1 dozen (doz.)
20 things	= 1 score
12 dozen	= 1 gross
12 gross	= 1 great gross

TABLE FOR MEASURING PAPER

24 sheets	= 1 quire
20 quires	= 1 ream
2 reams	= 1 bundle
5 bundles	= 1 bale

The operations to be performed in compound denominate numbers are the following:

(1) To change from higher to lower units, or denominations;
(2) To change from lower to higher units, or denominations;
(3) To add compound denominate numbers;
(4) To subtract such numbers;
(5) To multiply such numbers;
(6) To divide such numbers.

These processes will now be illustrated in order.

1. Express 16 bu. 3 pk. 3 qt. 1 pt. as pints.

CONVENIENT FORM

	16 bu. 3 pk. 3 qt. 1 pt.	
	4	= No. pk. in 1 bu.
	64 pk.	= No. pk. in 16 bu.
	3 pk.	
	67 pk.	= No. pk. in 16 bu. 3 pk.
	8	= No. qt. in 1 pk.
	536 qt.	= No. qt. in 67 pk.
	3 qt.	
	539 qt.	= No. qt. in 67 pk. 3 qt.
	2	= No. pt. in 1 qt.
	1078 pt.	= No. pt. in 539 qt.
	1 pt.	
Ans.	1079 pt.	= No. pt. in 16 bu. 3 pk. 3 qt. 1 pt.

2. Express (1) 1079 pt. in higher units; (2) 2 pk. 3 qt. 1 pt. in bushels.

CONVENIENT FORM

(1)

2) 1079 pt.
8) 539 qt. + 1 pt. remaining
4) 67 pk. + 3 qt. remaining
16 bu. + 3 pk. remaining
Ans. 16 bu. 3 pk. 3 qt. 1 pt.

(2)

$$3 \text{ qt. } 1 \text{ pt.} = 3.5 \text{ qt.} = \frac{3.5}{8} \text{ pk.} = .4375 \text{ pk.}$$

$$2 \text{ pk. } 3 \text{ qt. } 1 \text{ pt.} = 2.4375 \text{ pk.} = \frac{2.4375}{4} \text{ bu.} = .609375 \text{ bu.} \quad \textit{Ans.}$$

3. The three sides of a triangular grass plot were: 8 yd. 2 ft. 10 in.; 12 yd. 1 ft. 9 in.; and 9 yd. 2 ft. 7 in.; how far is it around the plot?

CONVENIENT FORM

	8 yd.	2 ft.	10 in.
	12	1	9
	9	2	7
Ans.	31 yd.	1 ft.	2 in.

EXPLANATION.—First, adding the inches, we obtain 26 in. = 2 ft. 2 in. Write the 2 in. in "inches" column, and add the 2 ft. to the numbers in the "feet" column, giving 7 ft. = 2 yd. 1 ft. Write 1 ft. in "feet" column and add 2 yd. to the numbers in "yards" column, giving 31 yd.

4. From a vessel containing 25 gal. 3 qt. 1 pt. 2 gi. of oil, 8 gal. 3 qt. 1 pt. 3 gi. were drawn out; how much oil remained in the vessel?

CONVENIENT FORM

25 gal. 2 qt. 1 pt. 2 gi.
8 gal. 3 qt. 1 pt. 3 gi.

16 gal. 2 qt. 1 pt. 3 gi.

Ans. 16 gal. 2 qt. 1 pt. 3 gi.

EXPLANATION.—3 gi. can not be taken from 2 gi. But the 1 pt. of the minuend equals 4 gi. and this added to 2 gi. gives 6 gi. 6 gi. — 3 gi. = 3 gi. 1 pt. from 0 pt. can not be taken. But 1 qt. is taken from 2 qt., and changed to pints, giving 2 pt. 2 pt. — 1 pt. = 1 pt. 1 gal. = 4 qt. 4 qt. + 1 qt. = 5 qt. 5 qt. — 3 qt. = 2 qt. Finally, 24 gal. — 8 gal. = 16 gal. Prove the work by adding the remainder to the subtrahend and comparing the result with the minuend.

5. A man built an average of 45 ft. 8 in. of fence a day for 16 da.; how much fence did he build in 16 days?

CONVENIENT FORM

	45 ft.	8 in.
		16
	10 ft.	8 in.
	720	
Ans.	730 ft.	8 in.

EXPLANATION.—
8 in. × 16 = 128 in. = 10 ft. 8 in.
45 ft. × 16 = 720 ft.

6. An iron rod, 4 yd. 2 ft. 8 in. long, was cut into 5 equal pieces; how long was each piece?

CONVENIENT FORM

5) 4 yd. 2 ft. 8 in.

Ans. 2 ft. 11⅕ in.

EXPLANATION.—4 yd. =12 ft. 12 ft. + 2 ft. = 14 ft. 14 ft. + 5 = 2, with a remainder of 4 ft. 4 ft. = 48 in. 48 in. + 8 in. = 56 in.

§147. Exercises on Denominate Number Tables.

STANDARDS OF VALUE

1. How many cents are there in 23 dimes? in $23.00? in 23 eagles?

2. How many dollars are there in 248 cents? in 248 dimes? in 248 eagles?

3. How many half eagles are there in $245.00? in 240 dimes?

4. About what is the value of $50.00 in pounds sterling (English money)? in Marks? in Francs? in Rubles? in Crowns? in Lira?

5. What are the more accurate equivalents of $100.00 in these same units?

6. I owe a debt of 32 M. 50 Pf. (pfennig = $\frac{1}{100}$ of a Mark). How much U. S. money will it take to pay this debt if I have to pay 24.5 cents for a Mark?

7. If I have to pay $4.94 for a pound sterling, how much U. S. money will it take to pay a debt in London of £65 8s.?

WEIGHT

1. How many grains are there in

(1) 12 dwt.; (3) $8\frac{1}{2}$ oz.; (5) 9 oz. 12 dwt.
(2) 3 oz.; (4) 8 oz. 10 dwt.; (6) 4 lb. 8 oz.?

2. A carat (of weight) usually means 32 troy grains. How many carats are there in a diamond weighing 13 dwt. 8 gr.?

3. How many carats weight are there in a diamond weighing 2 troy ounces?

4. The avoirdupois table is used for weighing iron, coal, and all coarser (less costly) articles. How many grains heavier is a pound of coal than a pound of silver? How many troy ounces?

5. Express 8 lb. (av.) as ounces. Express 8 lb. (troy) as ounces.

6. How many ounces are there in 24 lb. of iron? in 48 lb. 6 oz. of iron?

7. How many ounces are there in 3 cwt.? in 1 cwt. 40 lb.? in 5 cwt. 8 oz.?

8. Express as avoirdupois ounces the following avoirdupois weights:

(1) 3 lb. 8 oz.; (3) 86 lb. 15 oz.; (5) 8 cwt. 13 lb.;
(2) 18 lb. 12 oz.; (4) 1 cwt. 8 lb.; (6) $9\frac{1}{2}$ cwt.

9. Express as tons the following:

(1) 40 cwt.; (3) 16000 lb. (5) 12500 lb.;
(2) 30 cwt.; (4) 9000 lb. (6) 13500 lb.

10. Express the following as long tons:

(1) 6720 lb.; (3) 112 cwt.; (5) 56 tons;
(2) 19040 lb.; (4) 952 cwt.; (6) 39 cwt. 20 lb.

11. How many pounds are there in $\frac{1}{4}$ of a ton of coal? in $\frac{1}{2}$ of a long ton of pig iron?

LINEAR MEASURE

1. How many inches make a rod? in 1 chain?

2. How many feet are there in 1 chain?

3. Express 3 rd. $2\frac{1}{3}$ yd. in inches. 2 rd. 1 yd. 2 ft. in inches.

4. Express 6408 inches in rods and yards.

5. A toy railroad track is 10 yd. long. How many feet of track are needed for it?

6. A 12-rod fence is made of 10-inch boards set vertically with no cracks between them. How many boards are needed?

7. The four sides of a garden are 10 rods, 1020 ft., $162\frac{1}{2}$ ft., and 1019 ft. 4 in. How many 10-inch boards will be needed to enclose it with a fence like that described in problem 6?

8. The tire of a bicycle is 88 inches long. How many times will the wheel turn in going 40 rods?

9. The tire of a wagon wheel is 15 ft. 9 in. long. How many turns will the wheel make in going 5 miles?

SURFACE MEASURE

1. Express as square inches:

(1) 3 sq. ft.;	(4) 2 sq. yd. 3 sq. ft.;	(7) $3\frac{1}{2}$ sq. yd.;
(2) 3 sq. ft. 8 sq. in.;	(5) 6 sq. yd. 48 sq. in.;	(8) $45\frac{1}{4}$ sq. ft.;
(3) 1 sq. yd.	(6) 45 sq. ft. 108 sq. in.;	(9) 1 sq. rd.

2 Express:

(1) 121 sq. yd. as sq. rd.; (4) 80 sq. rd. as A.;
(2) 640 sq. rd. as A.; (5) 1 sq. rd. as sq. ft.;
(3) 40 A. as sq. rd.; (6) 1 A. as sq. yd.

3. Express:

(1) 10 sq. ch. as A.; (4) 1 Tp. as A.;
(2) 20 A. as sq. ch.; (5) 1 A. as sq. rd.;
(3) 1 sq. mi. as sq. ch.; (6) A. section as sq. rd.

4. How wide must a rectangle 80 rd. long be to contain 1 A.? 5 A.? 20 A.? 80 A.?

5. How wide must a rectangle 16 rd. long be to contain 1 A.? How many sq. rds. more than 1 A. are there in a 14-rd. square?

6. How many square yards more than 1 A. are there in a 70-yd. square?

7. How many square yards less than 1 A. are there in a 69-yd. square?

8. How many square feet are there in 1 A.?

9. How many square yards more than 1 A. are there in a 213-ft. square?

CAPACITY

1. Express:

(1) 2 qt. 1 pt. as pt.; (4) 2 gal. as pt.;
(2) 1 qt. $\frac{1}{2}$ pt. as gi.; (5) 12 gal. as qt.;
(3) 3 qt. 1 pt. 2 gi. as gi.; (6) 11 gal. 3 qt. as pt.

2. Express:

(1) 24 pt. as qt. (4) 64 gi. as gal.
(2) 48 gi. as qt. (5) 17 pt. as qt. and pt.
(3) 76 pt. as gal. (6) 84 pt. as gal. and qt.

3. There are 231 cu. in. in a liquid gallon. How many cubic inches make a quart? a pint?

4. There are 268.8 cu. in. in a dry gallon. How many more cubic inches are there in a dry than in a liquid quart?

5. Express:

(1) 3 bu. as pk.; (4) 4 bu. as dry gallons;
(2) $2\frac{1}{4}$ bu. as pk.; (5) 18 dry gallons as bu.;
(3) $2\frac{1}{2}$ bu. as pk.; (6) 20 pk. as pt.

6. What is the difference between the weight of a quart of barley in California and in Illinois?

7. How many bushels are there in a long ton (2240 lb.) of anthracite in Illinois? in a short ton?

8. My coal bin is 7′x12′x12′. How many bushels of anthracite will it hold? how many tons? (Count $1\frac{1}{4}$ cu. ft. per bushel.)

9. About what part of a Winchester bushel is 1 cubic foot?

10. Express 3.5 bu. as pints; 160 pt. as bushels.

11. A grocer buys beans at $1.80 per bu. and sells them at 9¢ a qt. Compare the buying and selling prices per bushel; per quart.

12. Find the error in the adage "A pint's a pound" if there are $7\frac{1}{2}$ gal. in a cubic foot of water weighing 1000 ounces.

TIME

1. Express:

(1) 1 hr. as. sec.; (4) 30 sec. as hr.; (7) 1209600 sec. as wk.;
(2) 1 da. as sec.; (5) 7200 sec. as hr.; (8) 1000000 sec. as wk. da. and hr.
(3) 1 wk. as sec.; (6) 604800 sec. as da.; (9) 1 wk. as minutes.

2. How many weeks in a common year? July 4 falls on Tuesday in 1905. On what day of the week will July 4, 1906, fall? Why?

3. How many months old are you this month?

4. How many days old are you to-day? How many hours in this many days?

5. A clerk earned $15 a week for 28 calendar months. How much did he earn in all?

6. A clerk saved $6 a week for 7 yr. How much did he save in all?

7. Illinois was admitted as a state in December, 1818. How many years and months old is the state of Illinois this month?

8. How many years and months old is your native state. (See Table, page 39.)

9. On the Saturdays of April, 1905, the sun rose and set at Chicago at the times given here:

Day	Date	Sun Rose	Sun Set
Saturday	April 1	5:46 a. m.	6:23 p. m.
Saturday	April 8	5:34 a. m.	6:30 p. m.
Saturday	April 15	5:24 a. m.	6:37 p. m.
Saturday	April 22	5:14 a. m.	6:43 p. m.
Saturday	April 29	5:05 a. m.	6:50 p. m.

Give the length in hours and minutes of each of these 5 days "from sun to sun," and also give in minutes the change per week of the days' length during April, 1905.

10. It is noon at 12:00. How long in hours and minutes was the forenoon of April 1, 1905? the afternoon? How many minutes longer was the afternoon than the forenoon?

11. Answer similar questions for Apr. 8th; 15th; 22d; 29th.

12. From the numbers of a common almanac compare the days' length of April with the same dates for October.

DATE	MORNING TWILIGHT		EVENING TWILIGHT	
1905	BEGINS	ENDS	BEGINS	ENDS
Jan. 1st	5:46 a.m.	7:25 a.m.	4:42 p.m.	6:21 p.m.
April 1st	4:10 a.m.	5:45 a.m.	6:24 p.m.	7:58 p.m.
July 1st	2:27 a.m.	4:32 a.m.	7:35 p.m.	9:40 p.m.
Oct. 1st	4:26 a.m.	5:56 a.m.	5:44 p.m.	7:14 p.m.

13. From the table find how long morning twilight lasted on Jan. 1, 1905; on Apr. 1; July 1; Oct. 1.

14. Answer similar questions for evening twilight.

COUNTING THINGS

1. A stationer pays 25¢ per dozen for lead pencils and sells them at 5¢ apiece. What is his profit on a gross of pencils?

2. How many are "three score and ten"? How many years are "a score of centuries"? How many months?

3. Lincoln's Gettysburg address was spoken in November, 1863. The address begins "Four score and seven years ago," etc. To what date did Lincoln refer?

4. What is the value of 1 bale of paper at 12¢ per quire?

5. What is the value of 3 great gross of steel pens at the rate of 2 pens for 5¢?

6. What is the value of 10 great gross boxes matches at the rate of 2 boxes for 5¢?

§148. General Exercises.

1. Reduce 9 great gross to units.

2. How many great gross are there in 79,636 units?

3. A dealer bought 3600 lead pencils at $2.00 per gross. He sold them at 25 cents per dozen. What was his profit?

4. Bought 2 gr. foot-rules at $.12 a doz.; 7 gr. Spencerian pens No. 1 at $.07 a doz.; 8 gr. Eagle pencils No. 3 at $.30 a dozen. Find the amount of my bill.

5. A stationer bought 3 gr. boxes of paper, each box containing 6 reams. How many sheets did he buy?

6. A stationer sold 3 quires, 20 sheets of paper to one man, 18 quires to another, 4 reams, 15 quires to another. How many sheets did he sell in all?

7. There are 2 reams, 9 quires of paper in one package, 3 times as much in a second package, and 7 times as much in a third package as in the second package. How much paper is there in the second and third packages each?

8. In £28 there are how many shillings? how many pence? how many farthings?

9. In 18s. there are how many pence? how many farthings?

10. In £28 18s. 9d. 3 far., there are how many farthings?

11. In £342 15s. 6d. there are how many pence?

12. How many pounds, shillings, and pence are there in 28,643 pence?

13. How many feet are there in 78 yd.? how many inches?

14. How many inches are there in 78 yd. 2 ft. 6 inches?

15. How many yards, feet, and inches are there in 2838 inches?

16. How many ounces are there in 12 cwt.? in 10 pounds?

17. How many ounces are there in 12 cwt. 10 lb. 11 ounces?

18. How many hundredweight, pounds, and ounces in 19,360 ounces?

There are two classes of problems to be solved in reducing denominate numbers to their equivalents in different units.

In one class numbers expressed in larger units are to be expressed in smaller units. This is called *reduction descending*, or reduction from *higher to lower denominations*.

In the other class numbers expressed in smaller units are to be expressed in larger units. This is *reduction ascending*, or reduction from *lower to higher denominations*.

Problems 15 and 18 are examples of the second class, and problems 13, 16, and 17 are examples of the first class.

19. Reduce 160 yd. 1 ft. 9 in. to inches.
20. Reduce 5781 in. to yards, feet, and inches.
21. Reduce 8 bu. 2 pk. 5 qt. to quarts.
22. Reduce 27 T. 18 cwt. 15 lb. to pounds.
23. Reduce 23 cu. yd. 14 cu. ft. to cubic feet.
24. Change £395 15s. sterling to dollars and cents.

United States gold and silver coins are .9 pure gold or silver and .1 copper.

25. What weight of pure silver is there in a silver dollar weighing 412½ grains?

26. The U. S. standard 5 dollar gold piece weighs 129 gr. What is the number of grains of pure gold in the standard five dollar gold piece?

27. The 5-cent piece weighs 73.16 gr. .75 of the weight of the coin is copper and .25 is nickel. What is the weight of copper in the 5-cent piece? of the nickel in the 5-cent piece?

28. The eagle weighs 258 gr. What is the weight of pure gold in the eagle?

29. How many standard gold dollars can be coined from 1 oz. of pure gold?

30. Express the following ratios:

1 franc : \$.25;	\$.25 : 1 franc;
1 mark : \$.25;	\$.25 : 1 mark.

31. If .52 oz. of gold is worth 43s. 4d., how many ounces can be bought for £35 18s.?

32. A boy laid by a certain sum of money each week. At the end of 1 yr. 3 mo. 2 wk. he had saved \$88.50. How much did he save each week? (Take 1 mo. = 4 wk. 2 days.)

33. A man changed \$350, half into English and half into German money. How much of each kind of money did he have?

34. How many feet are 4 ch. 30 links?

35. What is the ratio of 1760 yd. to 1 mi. 32 rods?

36. The mast of a ship was 78 ft. 4 in. high. During a storm .3 of it was broken off. How high was the remaining piece?

37. A four-sided field had sides of the following lengths: 63 ch. 2 rd.; 49 ch. 14 li.; 53 ch. 1 rd. 16 li. and 38 ch. 24 li. How far is it around the field?

38. A man walked $\frac{1}{8}$ of the length of a breakwater, which was 1 mi. 243 rd. 5 yd. long. How far did he walk?

39. A knot, or geographic mile, equals 6086 ft. What is the speed in common or statute miles per hour of a vessel that runs 21 knots per hour?

40. A wheel, 12 ft. in circumference, makes how many revolutions in $1\frac{1}{2}$ miles?

41. A telegraph wire is 14 mi. 140 rd. long, and is supported by 386 poles, which are placed at equal distances apart, a pole being at each end of the wire. How many feet apart are the poles?

42. What is the cost of 18 A. 120 sq. rd. of land at $52 per acre?

43. A man owned 14 A. of land, and sold 1428 sq. rd. How many acres did he have left?

44. If 1000 shingles are needed to cover 100 sq. ft. of roof, how many shingles are required to cover a roof 40 ft. long and 25 ft. wide, at the same rate?

45. A lawn tennis court 120 ft. long and 85 ft. wide is to be surrounded by a strip of sod 15 ft. wide at each end and 8 ft. wide at each side. What will the sodding cost at $.35 per square yard?

46. Find the cost of a half section of land at $45 per acre.

47. How many square rods are there in a rectangular field 24 ch. 45 li. long by 16 ch. 34 li. wide?

48. How many cubic inches are there in a tank containing 120 gal.? how many cubic feet?

49. How many cubic feet are there in a block of stone 5 ft. × 4 ft. × 6 inches?

50. How much must I pay for a board 16 ft. long and 8 in. wide, the board being 1 in. thick and lumber costing $35 per thousand? (A board foot means 144 sq. in. of surface, not over 1 in. thick.)

51. I bought 3 boards 12′ long, 6″ wide, 8 boards 16′ long, and 9″ wide, and 2 boards 10′ long, 12″ wide and all were 2″ thick; what did the whole cost at $30 per thousand?

52. What is the value of a straight pile of wood 16 ft. long, 8 ft. wide and 6 ft. high, at $7.50 per cord?

53. What is the weight of a pile of oak boards 14 ft. long 8 ft. 4 in. wide and 5 ft. high, at an average weight of 54 lb. per cubic foot?

54. At \$.75 a load of 1 cu. yd. what will be the cost of removing a pile of earth 60 ft. long 24 li. wide and 1 yd. high?

55. In one year the quarries of Minnesota yielded 4,000,000 cu. ft. of sandstone. What was this worth, at \$.76 a perch?

56. Find the cost of building a stone wall $150' \times 10' \times 2\frac{1}{2}'$, at \$3.50 a perch?

57. A bin $12' \times 8'6'' \times 5'$ is filled with wheat. What is the weight of the wheat at 60 lb. per bushel?

58. What is the value of a straight pile of pine slabs $32' \times 7' \times 4'$, at \$3.25 per cord?

59. A grocer paid \$4.50 for a barrel of vinegar ($31\frac{1}{2}$ gal.), and sold it at 5¢ per quart. What was his profit?

60. A jug contained 214 cu. in. of molasses. How much did it lack of containing $1\frac{1}{2}$ gallons?

61. What will 1 mi. of right of way for a railroad cost at \$60 per acre, the width of the right of way being 100 feet?

62. What will be the cost per mile for railroad ties at \$.45 apiece, the ties being laid one every 2 ft. (Omit the odd tie.)

63. Find the cost of the rails for 1 mi. of the road, the rails weighing 77 lb. per linear yard and costing \$35 per long ton.

64. Find the cost of fencing 1 mi. on both sides of the track, placing posts costing \$.25 apiece 16 ft. from center to center, and using wire weighing 2160 lb. per mile, and 30 lb. of staples @ 4¢. The fence is to be 4 wires high, and labor costs 21¢ per rod of fence. (Here count the odd post.)

65. Reduce .865 gal. to smaller units.

SOLUTION.—$.865 \times 4$ qt. $= 3.46$ qt.; $.46 \times 2$ pt. $= .92$ pt.; $.92 \times 4$ gi. $= 3.68$ gills. *Ans.* 3 qt. 3.68 gills.

66. Reduce .168 gal. to smaller units.

67. If oil is \$.11 per gallon, what will be the cost of three 42-gallon barrels of kerosene?

68. If a certain spring regularly yields 25 gal. daily, how many barrels of the capacity of $31\frac{1}{2}$ gal. would it fill in 20 days?

69. A merchant bought 16 gal. 3 qt. of syrup at 38¢ a qt., and sold 27 qt. for $12, 18 qt. for $9, and the remainder at 35¢ per quart. Did he gain or lose, and how much?

70. Reduce 2 pk. 7 qt. 1 pt. to the decimal of a bushel.

SOLUTION.—1 pt. = .5 qt. 7.5 qt. $= \frac{7.5}{8}$ pk. = .9375 pk. 2.9375 pk. = $\frac{2.9375}{4}$ bu. = .734375 bushels.

71. Reduce 3 pk. 5 qt. 1 pt. to bushels.

72. How many bushels are there in a bin 14 ft. long 7 ft. 6 in. wide and 5 ft. 8 in. high?

73. A farmer sold 6 loads of corn, each load averaging 36 bu. 2 pk. at 35 cents per bushel. What did he receive for the whole?

74. A boy had a bushel of hickory nuts, and sold 3 pk. 7 qt. 1 pt. What fraction of a bushel had he left?

75. If 9 bu. of potatoes cost $4.80, what is the average cost per peck?

76. A bin contained 5376.25 cu. in. of rye. How much did it lack of containing $3\frac{1}{2}$ bushels?

77. How many oz. of quinine will be required to prepare 12 gross of 3-grain capsules?

78. A coal dealer buys two car-loads of coal each weighing 67,200 lb., at $4.50 a long ton, and sells it at $5.75 a short ton. What is his gain?

THE METRIC SYSTEM

§149. Historical.

The metric system is a decimal system of weights and measures adopted by the French Government soon after the French Revolution of 1789. The aim of the system is to base all measures upon an invariable standard, and to secure the simplest possible relations between the different units of the system.

The unit of length, which is fundamental to the whole system, is called the *meter*. It was attempted to make the meter 1 ten-millionth of the length of the part of a meridian of the earth, which reaches from the equator to the pole, called a *quadrant* of the earth's meridian. The meridian of the earth was measured, and $\frac{1}{10000000}$ of the quadrant was obtained. A platinum bar equal to this length was very accurately cut and stored in the Government archives as the official standard of reference.

Later measurements of the earth's meridian showed the former length of the meridian to be incorrect. The length of the meter as obtained from the erroneous measures was, however, retained, and the length of this bar is the standard meter. From it all the other units of the system are derived.

The metric system is used for all purposes in France, and for nearly all scientific purposes in Germany, England, and the United States.

The length of the meter is 39.37079 in., or about 1.1 yards.

§150. Tables of Metric Measures.

Fig. 136 shows a scale graduated along the upper edge to 16ths of an inch, and along the lower edge to 1000ths of a meter, called *millimeters.*

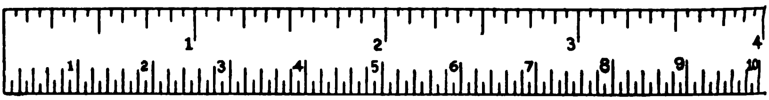


FIGURE 136

Decimal parts of the standard units are denoted by the Latin prefixes, abbreviated in each case to a single, small letter:

milli,	meaning	1000th	written m.
centi,	meaning	100th	written c.
deci,	meaning	10th	written d.

Multiples of the standard units are denoted by the Greek prefixes, abbreviated in each case to a single, capital letter:

deka,	meaning	10 times	written D
hekto,	meaning	100 times	written H.
kilo,	meaning	1000 times	written K.
myria,	meaning	10000 times	written M.

TABLE OF MEASURES OF LENGTH

10 *millimeters* (mm.)	= 1 *centimeter* (cm.)	= about	.4 in.
10 centimeters	= 1 decimeter (dm.)	= about	4.0 in.
10 decimeters	= 1 METER (m.)	= about	1.1 yd.
10 meters	= 1 dekameter (Dm.)	= about	32.8 ft.
10 dekameters	= 1 hektometer (Hm.)	= about	328 ft.
10 hektometers	= 1 *kilometer* (Km.)	= about	.62 mi
10 kilometers	**= 1 myriameter (Mm.)**	**= about**	**6.21 mi**

Table of Surface Measure

100 *sq. millimeters* (mm².)	= 1 *sq. centimeter* (cm².)
100 sq. centimeters	= 1 sq. decimeter (dm².)
100 sq. decimeters	= 1 SQ. METER (m².)
100 sq. meters	= 1 sq. dekameter (Dm².)
100 sq. dekameters	= 1 sq. hektometer (Hm².)
100 sq. hektometers	= 1 *sq. kilometer* (Km².)

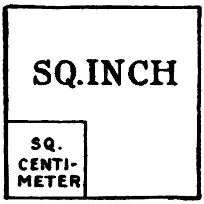


Figure 137

The cm².	=	.155 sq. in.
The m².	=	10.764 sq. ft.
The Km².	=	247.114 A.
1 m²	=	1 centare (ca.)

Table of Land Measure

100 centares	= 1 âre (pronounced ăir) (a.)
100 ares	= 1 *hektare* (Ha.)

Table of Measures of Volume

The standard unit of volume is the cubic meter = 35.314 cu. ft. = about 1.2 cubic yards.

1000 cubic millimeters (mm³.)	= 1 cu. centimeter (cm³.)
1000 cubic centimeters	= 1 cu. decimeter (dm³.)
1000 cubic decimeters	= 1 cu. meter (m³.)
	etc., etc.

Table of Measures of Capacity

The standard unit of capacity is the *liter* (leeter). It is equal to 1 cu. decimeter, and equivalent to .908 dry quarts.

10 milliliters (ml.)	= 1 *centiliter* (cl.)
10 centiliters	= 1 deciliter (dl.)
10 deciliters	= 1 *liter* (l.)
10 liters	= 1 dekaliter (Dl.)
10 dekaliters	= 1 *hektoliter* (Hl.)
10 hektoliters	= 1 kiloliter (Kl.)

Table of Measures of Weight

The standard unit of weight is the gram, which is the weight of 1 cu. centimeter of distilled water at its temperature of greatest density (39.1° F.).

10 milligrams (mg.)	= 1 *centigram* (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 *gram* (g.)
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hektogram (Hg.)
10 hektograms	= 1 *kilogram* (Kg.)
10 kilograms	= 1 myriagram (Mg).
10 myriagrams	= 1 quintal (Q.)
10 quintals	= 1 *metric ton* (T.)

1 centigram	= .15432 grain
1 gram	= 15.432 grains
1 kilogram	= 2.20462 lb.
1 metric ton	= 2204.621 lb.

§151. Metric and U. S. Equivalents.

The equivalents will be of assistance in changing the metric to the common system of measures.

1 m.	= 39.37 in.	1 mile	= 1.6093 Km.
1 Km.	= .6214 mi.	1 yard	= .9144 m.
1 m^2.	= 1.196 sq. yd.	1 square yard	= .8361 m^2.
1 Km^2.	= .3861 sq. mi.	1 square mile	= 2.59 Km^2.
1 are	= .0247 A.	1 acre	= 40.47 a.
1 m^3.	= 1.308 cu. yd.	1 cubic yard	= .7645 m^3.
1 stere	= .2759 cord	1 cord	= 3.624 st.
1 liter	= { 1.0567 liquid qt. / .9081 dry qt. }	61 022 cu. in.	= 1 l.
		1 liquid qt.	= .9436 l.
1 Hl.	= { 2.8376 bu. / 26.417 liq. gal. }	1 dry qt.	= 1.101 l.
		1 bushel	= .3524 Hl.
1 g.	= 15.432 grains	1 grain	= .0648 g.
1 Kg.	= 2.2046 lb. avoir.	1 lb. avoir.	= .4536 Kg.
1 metric ton	= 1.1023 T.	1 T.	= .9072 T.

PROBLEMS WITH THE METRIC UNITS

1. Take a smooth lath, or plane one, lay a straight strip of paper beside the lower edge of the metric rule, Fig. 136, page 230, and with a sharp pencil transfer the graduations from the rule to the strip of paper, and then from the strip of paper to the lath.

Continue the *centimeter* graduation marks along your lath until you get a meter stick (= 100 centimeters). How does this meter stick compare in length with a yard stick?

2. Explain which digit in 6.8752 m. stands for m.; which for dm.; which for cm., and which for millimeters.

3. Find by measurement the length and the width of your schoolroom in meters, and write the result of your measurements in meters and decimals of a meter.

4. How many centimeters long, wide, and thick is your arithmetic?

5. Express the length of your pencil in centimeters.

6. A sidewalk is 2112 m. long, how many kilometers long is it?

7. What is the cost of 158 cm. of ribbon at $.45 a meter?

8. How many steel rods, each 16 cm. long, can be cut from a rod 7.68 m. long?

9. From a piece of wire, 98.52 m. long, a piece .047 Km. was cut. How long was the remainder?

10. A bicycle track is .807 Km. long. How many meters would one go in riding around the track 5 times?

11. Express 1 Km^2. in square dekameters; in square meters.

12. Find the number of square meters in the surface of your schoolroom. Express the same in square dekameters.

13. Express the area of your desk in square decimeters; in square inches. How do the two compare?

14. Using the decimeter as a measure, find the number of square centimeters in the surface of your geography.

15. Express the same in square decimeters; in square inches.

16. Find the area of the door in square meters. What part of an *are* is this area?

17. A certain plot of ground contains 20 m^2. How many square feet in the plot?

18. With the aid of your meter stick, lay off a square meter upon the floor. Within this area, lay off a square yard. Note results.

19. With the aid of your decimeter measure, draw a square decimeter. In the corner, draw a square centimeter. How many square centimeters could be drawn within the square decimeter?

20. What is the area of a floor 8.4 m. long and 5 m. wide?

21. A lot is 7.64 m. wide and .033 Km. deep. What is the area?

22. What is the value of a piece of land 8 Dm. long and 6.5 Dm. wide, at $32 an *are*?

23. How many *ares* in a street 1.5 Dm. wide and 2.48 Km. long?

24. How many square meters in a floor 700 cm. long and 500 cm. wide?

25. A man had 3 pieces of land as follows: 8.4 Ha.; 3846 m^2., and 2.5 Km^2. How many hektares of land are there in all?

26. 20 liters equal how many centiliters? what part of a Dl.? of a Kl.? Find the difference between 12 l. and 12 quarts.

27. Using your decimeter measure as a basis, model a cubic decimeter or liter.

28. Using your square centimeter as a basis, model a cubic centimeter. What is the relation of a cubic centimeter to a gram? How many cubic centimeters in a liter?

29. How many cubic meters are there in the volume of your schoolroom?

30. What is the difference between an ordinary ton and a metric ton (tonneau)?

31. A gram is the weight of 1 cm^3. of distilled water. What is the weight in grams of 1 l. of distilled water?

32. A liter of water weighs nearly $2\frac{1}{5}$ lb. What is the weight of 6 l. of water in pounds? in grams?

33. A package of silver weighs 2.58 grams. What is its weight in grains?

34. How many pounds in 1 Q.? in 5 quintals?

35. How many 2 gr. capsules will 5 g. of quinine fill?

36. What would be the cost of the quinine at 10¢ per dozen capsules?

37. What is the cost of 5500 Kg. of coal at $6.50 per ton?

38. What is the cost of 2 Q. of sugar at $.08$\frac{1}{2}$ a kilogram?

39. If a book weighs 3.2 Dg., how many such books will weigh 1.792 kilograms?

40. A box contains 2500 packages of quinine, each package weighing 1.25 g. How many hektograms does the contents of the box weigh?

41. A vessel, 18 cm. long, 14 cm. wide, and 22 cm. high, is filled with lead, which weighs 11.35 times as much as water. What is the weight of the lead? (1 cu. ft. of water weighs 62.5 lb.)

PERCENTAGE AND INTEREST

§152. Percentage.

ORAL WORK

In a city block there are 100 houses. 50 of them are stores; 40 are dwelling houses; 5 are warehouses; 3 are hotels, and 2 are banks. 10 of them are built of brick; 30 are stone, and 60 are frame houses; 35 of them are 3 stories high, 25 are two stories, and the rest are one story high.

1. What part of the houses are stores? How many hundredths of the houses are stores?

NOTE.—"How many per cent?" "What per cent?" means "How many hundredths?"

2. How many per cent of the houses in the block are stores?

3. $\frac{1}{2}$ equals how many per cent?

4. What part of the houses are 2 stories high? What per cent of them are 2 stories high?

5. What per cent are warehouses? hotels? banks? dwellings?

6. What per cent are brick? stone? frame?

7. What per cent are 3-story buildings? 1-story buildings?

8. What part of the houses are warehouses? $\frac{1}{20}$ equals how many per cent?

9. What part of the houses are dwellings? $\frac{2}{5}$ equals how many per cent?

10. What part of the houses are frame houses? $\frac{3}{5}$ equals how many per cent?

A sidewalk 200 rd. long is made up of 10 rd. of stone walk, 25 rd. of cement walk, 40 rd. gravel walk, 50 rd. board walk, and 75 rd. cinder walk.

11. What per cent of the entire walk is stone? What per cent cement? gravel? board? cinder?

12. What per cent of the whole was the stone and the cement? the cement and the cinder? the gravel and the board? the stone and the board?

In a certain nursery there are 100 oak, 75 ash, 50 maple, 30 elm, 25 hickory, 15 poplar, and 5 walnut trees.

13. What per cent of the trees were oak? What part were oak?

14. What per cent of the trees were ash? What part were ash?

15. What per cent of the trees were maple? elm? hickory? poplar? walnut?

16. $2 equals what part of $8? 9 bu. equals what part of 12 bu.? 8 hr. equals what part of 24 hr. $6 of $9?

17. To how many hundredths of a number are the following fractional parts of that number equal:

$\frac{1}{2}$? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{1}{3}$? $\frac{2}{3}$? $\frac{3}{3}$? $\frac{1}{5}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{1}{10}$? $\frac{3}{10}$? $\frac{7}{10}$? $\frac{1}{20}$? $\frac{3}{20}$? $\frac{4}{25}$?

18. Express the following as hundredths:

$\frac{3}{50}$; $\frac{9}{25}$; $\frac{7}{20}$; $\frac{9}{10}$; $\frac{1}{6}$; $\frac{5}{6}$; $\frac{1}{12}$; $\frac{5}{12}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{3}{16}$.

DEFINITIONS.—The words *per cent* mean *hundredth*, or *hundredths*.

The sign, "%," is a short way of writing *per cent*, or *hundredths*; thus, 2%, 8%, $33\frac{1}{3}$%, mean $\frac{2}{100}$, $\frac{8}{100}$, $\frac{33\frac{1}{3}}{100}$, and are read, "2 per cent," "8 per cent," "$33\frac{1}{3}$ per cent."

The number (as 2, 8, $33\frac{1}{3}$), written before the sign, "%," is called the *rate per cent*.

It is well to recall that we have the following ways of writing such numbers as 6 per cent: (1) 6 per cent; (2) 6%; (3) 6 hundredths; (4) $\frac{6}{100}$; and (5) .06. The sign, "%," is merely an abbreviation for $\frac{1}{100}$, or .01.

19. Of 100 kinds of animals of a neighborhood, 27 die, 18 migrate and 10 hibernate as winter comes on. What per cent die? migrate? hibernate?

20. 10 kinds store food, 20 remain and feed abroad and 10 kinds appear only in winter. What per cent store food? feed abroad? appear only in winter.

WRITTEN WORK

Review §80, pp. 123-125, as oral work.

NOTE.—Whenever you can answer a problem orally, do so. Form the habit of using your pencil only when it is necessary.

1. In a schoolroom containing 40 pupils, $\frac{3}{5}$ of the pupils were girls. How many girls were there?

2. In the last problem, what per cent of the pupils were girls?

3. In the month of September, 12 da. were cloudy. What per cent of the days were cloudy?

4. Express the equivalents of the following fractions as per cents, or hundredths:

$$\tfrac{5}{6};\ \tfrac{5}{8};\ \tfrac{11}{12};\ \tfrac{1}{7};\ \tfrac{5}{7};\ \tfrac{4}{9};\ \tfrac{7}{11};\ \tfrac{6}{13};\ \tfrac{9}{19};\ \tfrac{18}{37}.$$

5. In a sample of soil weighing 37 oz., 18 oz. were sand. What per cent of the soil was sand?

6. A sample stick of timber, weighing 19 lb., contained 2 lb. of water. What per cent of the weight of the timber was due to the water it contained?

SOLUTION.—(1) Annex zeros after the numerator and divide thus:

$$19\overline{)2.00}\quad .11\tfrac{1}{19}. \qquad \text{But } .11\tfrac{1}{19} = 11\tfrac{1}{19}\%. \quad \text{Why?}$$

7. What per cent of a yard is 1 ft.? 9 in.? 15 in.? 28 in.? 35 in.? 1 inch?

8. The first number of these pairs is what per cent of the second:

(1) 8 of 150?	(4) $4\frac{5}{8}$ of 60?	(7) 32.91 of 263.28?
(2) 13 of 700?	(5) $27\frac{3}{5}$ of 600?	(8) $\frac{1}{4}$ of $\frac{1}{2}$?
(3) $7\frac{1}{2}$ of 12?	(6) 38.45 of 769?	(9) $\frac{3}{5}$ of $12\frac{1}{2}$?

9. Change the following per cents to their fractional equivalents (fractions in their lowest terms):

5%; 8%; $8\frac{1}{3}$%; $12\frac{1}{2}$%; 12%; 16%; $16\frac{2}{3}$; $22\frac{1}{2}$%; $67\frac{1}{2}$%; $87\frac{1}{2}$%.

10. Change to their decimal equivalents the following:

7%; $6\frac{1}{4}$%; $83\frac{1}{3}$%; $87\frac{1}{2}$%; $1\frac{1}{2}$%; $2\frac{3}{4}$%; $\frac{3}{8}$%; $\frac{1}{10}$ of 1%; $\frac{1}{100}$ of 1%.

11. At the beginning of a school year, the lung capacity of a boy was 161 cu. in. By the middle of the year it was 7% larger. How many cubic inches had his lung capacity increased?

12. The lung capacity of a boy at the beginning of the year was 160 cu. in., and at the middle of the year it was 166.4 cu. in. What was the per cent of increase?

13. The standing of the several clubs in the National League during one season was determined from this table:

CLUB	GAMES WON	GAMES LOST	PER CENT WON
Pittsburg	90	49	64.7%
Philadelphia....	83	57	
Brooklyn........	79	57	
St. Louis........	76	64	
Boston	69	69	
Chicago	53	86	
New York......	52	85	
Cincinnati......	52	87	

Find for each club what per cent of the total number of games played were won. Carry results to the first decimal place, as shown for Pittsburg.

14. What per cent of the farm, Fig. 4, p. 7, is the cornfield? the wheatfield? the meadow? the south oat field? the pasture? the lot occupied by the house and grounds?

15. 60% of the value of a mill is \$5400; what is the value of the mill?

16. 3% of a school of 1200 children were absent; how many children were absent?

17. I pay 12% of the value of the property I occupy as rent every year. My rent is \$240 a year; what is the value of the house?

18. A house was damaged by fire, and an insurance company paid the owner \$840 damages, which was 40% of the original cost of the house; what was the original cost?

§153. Algebra.

ORAL WORK

1. 50 equals what per cent of 100? of 200? of 150? of 500?

2. 2 equals what per cent of 4? of 8? of 6? of 20?

3. Any number equals what per cent of a number twice as large? 4 times as large? 3 times as large? 10 times as large?

4. x equals what per cent of $2x$? of $4x$? of $3x$? of $10x$?

5. $7a$ equals what per cent of $14a$? of $28a$? of $21a$? of $70a$?

6. 13 equals what per cent of 17? of 28? of 45? of 55?

SOLUTION.— $\frac{13}{17}=\frac{13\times 100}{17\times 100}=\frac{\frac{1300}{17}}{100}=\frac{76\frac{8}{17}}{100}=76\frac{8}{17}\%$.

7. $13x$ equals what per cent of $17x$? of $28x$? of $45x$? of $55x$?

8. x equals what per cent of y? of z? of m? of p?

NOTE.— $\frac{x}{y} = \frac{100\frac{x}{y}}{100} = 100\frac{x}{y}\%$.

WRITTEN WORK

1. What is 2% of \$200? of \$375? How is 2% of any number of dollars found?

2. How is 2% of \$$a$ found? What is 2% of \$$a$? of \$$b$? of \$$x$? of \$$6x$?

3. How is 5% of any number found? What is 5% of the number a? of b? of x? of z? of $8z$?

4. How is $12\frac{1}{2}\%$ of any number found? What is $12\frac{1}{2}\%$ of a? of x? of $10x$?

5. How is *any* per cent of a number found?

6. How can you express a as hundredths? x, as hundredths? $9x$? $12y$? $45z$?

7. Express these numbers as hundredths: 16; 29; a; x; m; $16x$.

8. What is $r\%$ of 160? of 350? of a? of R? of C? of $2m$?

DEFINITIONS.—The result of finding a given per cent of any amount, or number, is called the *percentage*. The amount, or number, on which the percentage is computed is often called the *base*.

9. Calling the percentage, p, the rate per cent, r, and the base, b, show by an equation how to find p, from b and r.

10. Replacing the symbols in your equation by the words for which they stand, translate the equation into the common language of percentage. This translation, properly made, is the fundamental principle of percentage.

PRINCIPLE.—*The percentage equals the product of the base and rate divided by 100, or more briefly,*

$$\text{(I)} \qquad p = \frac{br}{100}.$$

All of percentage is contained in this equation, called a *formula*, because *it formulates* (*expresses in brief form*) *a law.*

Multiplying both sides of the formula by 100, we have

$$(A) \qquad 100\,p = br.$$

Now divide both sides of this equation by r, and write the second number (see §69, p. 97) first, and obtain (note that $\frac{br}{r} = b$),

$$\text{(II)} \quad b = \frac{100p}{r}.$$

11. Translate (II) into words. How would you find the base (b) if the *percentage* (p) and rate (r) were given?

12. Divide both sides of (A) by b, write the second number first, and make a rule for finding r when p and b are given.

§154. Gain and Loss.

1. A merchant paid \$16 for a suit of clothes. At what price must he sell the suit to gain 25 per cent?

2. A stationer pays \$1.80 a dozen for blank books, and retails them at a profit of $66\frac{2}{3}\%$. At what price per book does he sell them?

3. A grocer buys eggs at wholesale at 10¢ a doz., and retails them at a profit of 30%. Supposing there is no loss due to breakage or spoiling, at what price per dozen does he sell them?

4. Allen paid \$1 for a sled, and sold it at a loss of 20%. How much did he receive for the sled?

5. From a box containing 200 oranges, 80% were sold in one day. How many oranges were sold?

6. What is the percentage on \$640 at 20%? at 28%? at 35%? at $6\frac{1}{4}\%$? at $12\frac{1}{2}\%$? at $87\frac{1}{2}\%$?

7. A farm, costing \$4400, was sold at a gain of 18%; for how much did the farm sell?

8. A farm sold for \$5192, which was 18% more than was paid for it. How much was paid for it?

SOLUTION.—The 18% here means 18% of what was paid for the farm (the cost price). \$5192 is then equal to what per cent of the cost? The statement may be written thus:

$$1.18 \text{ times the cost price} = \$5192,$$

or, more briefly, thus: $1.18x = \$5192$. Hence, $x = \frac{\$5192}{1.18}$. Divide and find the value of x.

9. A center fielder threw a ball 90 ft., which was 12% farther than the third baseman threw it. How far did the third baseman throw it?

SUGGESTION.—First answer the question, 12% of what?

10. A boy bought oranges at the rate of 5 for 3¢, and sold them at the rate of 3 for 5¢; what was his rate per cent of gain or loss?

11. A merchant in shipping 150 crates of eggs lost 25 crates by freezing. What per cent did he lose?

12. Hats, costing \$2.75, sold for \$3.50. What was the per cent of profit?

13. An automobile, costing \$750, sold for \$1250. What was the per cent of profit?

14. In a certain battle, 32,000 men were engaged, and 35% of all engaged lost their lives. How many lost their lives?

15. A horse, costing \$88, sold at a loss of 40%. For how much did the horse sell? (40% of what?)

16. I paid \$35 for a bicycle, and sold it the next season for 60% less than I paid for it. What was the selling price?

17. A cow gave 13 lb. of milk Nov. 1, and 15.6 lb. on Nov. 15. What was the rate % of gain in 14 days, in her daily yield of milk?

18. Make similar problems on the table, page 16.

19. The increase in population in one year in a certain town was 2000, which was $6\frac{1}{4}$% of the population at the end of the year? What was this population?

20. The death rate the same year was 1.8% of this population. How many deaths occurred?

21. A man lost 27% of the books of his library by fire. He lost 540 books. How many books did his library contain before the fire?

22. Anthracite coal cost \$8.50 a ton in Nov., and rose 15% in 1 mo. What was the price of anthracite coal after the rise in price?

23. I sold a lot for \$600, at a loss of 20%. What did the lot cost?

24. Find the cost of a piano which sold for \$187.50 at a loss of 25%.

25. I bought a tennis outfit for \$35, and sold it at a loss of 15%. Find the selling price.

26. Carpet, marked at \$1.25, sold at a reduction (discount) of $6\frac{2}{3}$%. Find the selling price.

27. A man lost 20% of his money and after losing 10% of the remainder had $3,600 left. How much had he at first?

28. A dealer sells a cask of 30 gal. of oil, and receives $30. In delivering it 4.5 gal. were spilled. What per cent was spilled, and how much should the dealer pay back?

29. Goods damaged by fire were sold at a loss of 40%. The amount received was $555. What was the original cost?

30. A suit of clothes was marked at $45, which was 50% more than the cost. It sold at a reduction of 20% from the marked price. What was the per cent of profit on the first cost of the suit?

§155. Meteorology.

The number of clear, cloudy, partly cloudy, and rainy or snowy days for the first 4 mo. of 1903, at Chicago, are here tabulated:

1903	Clear	Cloudy	Partly Cloudy	Rain or Snow
Jan.	9	15	7	9
Feb.	10	11	7	9
Mar.	12	14	5	11
Apr.	10	13	7	13

1. What per cent of the number of days of Jan. were clear? cloudy? partly cloudy? rainy or snowy?

2. Answer similar questions for Feb.; for March; for April.

3. What per cent of the total number of clear days of 1903 to May 1st fell in Jan.? in Feb.? in March? in April?

4. Answer similar questions for cloudy, partly cloudy and rainy, or snowy days.

5. The highest velocities (speeds) in miles per hour of the wind in Chicago for each of the 12 mo. (beginning with Jan.) of a certain year were: 50; 48; 57; 70; 54; 45; 56; 47; 52; 58; 52; 58. What per cent of the highest velocity for April was the highest velocity for Jan.? for Feb.? for each of the remaining months?

6. The total movement, in miles, of the wind in Chicago for each of the 12 mo. of a certain year was: in Jan. 12,736; 10,279; 13,999; 12,820; 12,356; 10,900; 11,231; 9,839; 11,834; 13,148; 13,583; 15,476. What per cent of the total wind movement in Dec. was the total movement for each of the other months?

The table below gives the height in feet (elevation), above sea level and the total wind movement in miles for an entire year, of a number of important weather signal stations. Block Island is on the Rhode Island coast, and Mount Tamalpais is a mountain station near San Francisco. Roseburg (Oregon) is the quietest place, as to wind movement, reported in the United States:

STATION	ELEVATION	WIND MOVEMENT
Mount Tamalpais	2,375	163,203
Block Island	26	152,838
Chicago	823	145,193
Cleveland	762	128,566
New York	314	127,267
Buffalo	767	125,042
Boston	125	98,755
Philadelphia	117	95,319
St. Louis	567	84,482
New Orleans	51	74,299
Louisville	525	70,396
Washington	112	63,629
Roseburg (Oregon)	518	30,471

7. What per cent is the wind movement of Chicago of that of each of the other places in the list?

8. What per cent is the elevation of Chicago of that of each of the other places?

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9. 100 green oak leaves weighed 100 grams. When thoroughly dried they weighed 39.5 grams. The dried leaves were then burned, and the ash weighed 2.4 grams. What per cent of the leaves was dry solid? mineral matter (ash)?

10. 100 green elm leaves weighed 60 grams. $38\frac{1}{3}\%$, by weight, was dry solid, and $2\frac{2}{3}\%$ was mineral matter. What was the weight of the dry solid? of the mineral matter?

11. A school garden, $15' \times 40'$, was seeded as follows: 25%, potatoes; $16\frac{2}{3}\%$ of the remainder, peas; 20% of the remainder, beans; 25% of the remainder, lettuce; $33\frac{1}{3}\%$ of the remainder, radishes; 20% of the remainder, parsley, and the rest in beets. How many square feet were seeded to beets?

12. The precipitation (rainfall) at Chicago, in inches, for Jan., Feb., Mar., and Apr. for 1903 was 1.09, 3.03, 1.67, and 3.77, and the averages for these same months for 33 yr. are 2.05, 2.11, 2.52, and 2.73. Find the rate per cent of excess, or deficiency, for each of the four months of 1903, on the 33-yr. average.

13. Problems may be made on the following table of data relating to Chicago weather:

1902	Temperature		Precipitation		Weather		
	Monthly Mean	Mean for 31 Years	In Inches	Average for 32 Yr.	Clear Days	Fair Days	Cloudy Days
Jan. ..	25.2	23.8	.66	2.08	10	14	7
Feb. ..	20.8	25.9	1.53	2.30	12	9	7
Mar. ..	38.6	34.3	4.16	2.56	6	13	12
Apr. ..	46.4	46.4	2.26	2.70	10	12	8
May ..	59.0	56.5	5.08	3.59	10	19	2
June ..	64.2	66.6	6.45	3.79	7	16	7
July ..	72.4	72.3	5.78	3.61	12	13	6
Aug...	68.4	71.1	1.44	2.83	15	10	6
Sept...	60.8	64.4	4.83	2.91	9	8	13
Oct....	55.2	53.1	1.45	2.63	14	10	7
Nov. ..	47.0	38.5	2.03	2.66	8	13	9
Dec. ..	26.5	29.0	1.90	2.71	7	8	16

§156. The Almanac.

NOTE.—In this section "length of the day" means the length of daylight, or the time interval from sunrise to sunset.

1. On Jan. 1, at Chicago, the sun rose at 7 hr. 29 min., and set at 4 hr. 38 min., and on July 1, it rose at 4 hr. 28 min., and set at 7 hr. 39 min. The length of the day on Jan. 1 equals what per cent of the length on July 1?

2. On Jan. 1, the length of the day in St. Louis was 103.8%, and in St. Paul 96.4%, of the length of the same day in Chicago. Find the day's length in each of these cities.

3. On July1, in the same places, the day's lengths were, respectively, 97.9% and 102.2% of its length in Chicago. Find the day's length in each of these cities on this date.

4. Make and solve similar problems for the calendar pages of a common almanac.

5. On the first of Feb., of March, of April, of May, of June, of July, of Aug., of Sept., of Oct., of Nov., and of Dec., the day's lengths in Chicago were 108%, 122.3%, 137.8%, 152.6%, 163.2%, 163.5%, 157.9%, 144.6%, 129.2%, 132.5%, and 102.6% of the day's length on Jan. 1. Find the day's length on the dates named.

6. On vertical lines, one for each month, plot these rates to scale and draw through the points a smooth freehand curve.

7. Similar problems may be obtained from the calendar pages of a common almanac.

8. From the almanac find what per cent the period from "moon rises" to "moon sets" on the first of some month is of the same period on the 15th or 20th of the same month.

9. Find what per cent the time from full moon to new moon is of the time from new moon to first quarter; from new moon to second quarter, or full moon.

§157. Geography.

Find these per cents to one place of decimals.

1. Refer to the table of p. 30, and find what per cent of the population (1900) of your state was in elementary and secondary (high) schools.

2. Compare the result of problem 1 with the corresponding results for any other states.

Data for the following problems will be found on pp. 30 and 39.

3. What per cent of the area of the United States (without outlying territory) is the area of the North Atlantic division? of the Western division?

4. What per cent of the total population of the United States (continental) is the population of the North Atlantic division? of the Western division?

5. Find what per cent the area and population of your state is of the area and population of the division to which your state belongs.

6. Find the per cent of increase of population of your state from 1890 to 1900.

7. Find the per cent of increase of population of the (continental) United States during the same time.

8. Find the per cent of increase in population from 1890 to 1900 of the division to which your state belongs.

9. The following table shows the growth in territory of the United States and the dates when the additions were made. Find the per cent of increase of each addition on the date of acquiring to 1899 inclusive.

Acquisition	Date	Area in Sq. Mi.
Original territory	1783	827,844
Louisiana purchase	1803	1,182,752
Florida	1819	59,268
Texas	1845	371,063
Mexican purchase	1848	522,568
Texas purchase	1850	96,707
Gadsden purchase	1853	45,535
Alaska	1867	590,884
Hawaii	1898	6,449
Porto Rico	1899	3,600
Philippine Islands	1899	114,000
Guam	1899	200
Isle of Pines	1899	882

10. The area in millions of square miles of the surface of the earth is 148.18; of this surface 108.77 is water, and 39.41 land. Find what per cent of the surface of the earth is water; land. The land surface equals what per cent of the water surface?

The following table contains the actual lengths of the coast lines of the continent and also the lengths that would be needed to enclose them if they were solid, with smooth outlines, also the ratio of low land to high land:

Continent	Actual Lengths	Length, if Solid	Ratio Low to High Land
North America	24,040	10,380	$6\frac{1}{2}:9$
South America	13,600	9,030	$9\frac{1}{4}:2\frac{3}{4}$
Asia	30,800	13,780	$10\frac{1}{8}:13$
Africa	14,080	11,760	$6\frac{1}{8}:14\frac{1}{2}$
Europe	17,200	6,630	$4\frac{3}{8}:1\frac{1}{3}$
Australia	7,600	5,860	$3\frac{1}{2}:1$

11. Find what per cent of each number in column 2 equals the corresponding number in column 3. What does each per cent mean? Which continent has the longest coast to defend in comparison with its area?

12. Express the given ratios of column 4 of low land to high land in per cent, and tell what the per cents mean?

The table below contains the heights, in feet, of mountains and peaks of the world:

Mt. Everest	29,002	Popocatapetl (volc.)	17,784
Aconcagua	23,910	Mt. Wrangell	17,500
Chimborazo (volcano)	20,500	Mt. Blanc	15,744
Kilimanjaro Mts.	20,000	Mt. Shasta	14,350
Mt. Logan	19,500	Longs Peak	14,271
Karakoram Mts.	18,500	Pikes Peak	14,147
Orizaba (volcano)	18,312	Mt. Etna (volcano)	10,875
Mt. St. Elias	18,100	Rocky Mts	10,000

13. The height of Pikes Peak equals what per cent of that of Mt. Everest? of Mt. Logan? of Chimborazo? of Mt. Shasta?

14. Answer other similar questions on the table.

15. What per cent of North America (16,130,269 sq. mi.) is drained through the Mississippi basin (1,250,000 sq. mi.)? through the St. Lawrence basin (360,000 sq. mi.)? the Columbia basin (290,000 sq. mi.)? the Colorado basin (230,000 sq. mi.)?

16. What per cent of the length of the Mississippi River (4,200 mi.) is the length of the St. Lawrence (2,000 mi.)? of the Columbia River (1,400 mi.)? of the Colorado River (2,000 mi.)? Yukon River (2,000 mi.)?

The following table gives the lengths, in miles, and the areas of basins, in square miles, of 24 of the longest rivers of the world:

River	Length	Area of Basin	River	Length	Area of Basin
Mississippi	4,200	1,250,000	Mackenzie	2,400	680,000
Nile	3,900	1,300,000	Volga	2,400	590,000
Amazon	3,600	2,500,000	St. Lawrence	2,000	360,000
Yangtzekiang	3,300	650,000	Yukon	2,000	440,000
Obi	3,000	1,000,000	Brahmaputra	2,000	426,000
Yenisei	3,000	1,400,000	Colorado	2,000	230,000
Congo	3,000	1,500,000	Indus	2,000	325,000
Niger	2,900	1,000,000	Euphrates	2,000	490,000
Hoangho	2,800	390,000	Danube	1,900	320,000
Amur	2,700	780,000	Rio Grande	1,800	225,000
Lena	2,700	900,000	Ganges	1,800	450,000
La Plata	2,500	1,250,000	Orinoco	1,500	400,000

17. What per cent of the length of the Mississippi equals that of the Nile? What per cent of the area of the Mississippi basin equals the Nile basin?

18. Solve other similar problems on the table.

The following table contains the areas, in sq. mi., the altitudes, in ft. (above sea level), and the greatest depths, in ft., of the great lakes of the world:

LAKE	AREA	ALTITUDE	DEPTH
Superior..............	31,200	602	1,008
Huron	23,800	581	700
Michigan.............	22,450	581	875
Erie..................	9,950	573	212
Ontario	7,242	248	738
Victoria..............	22,167	4,000	620
Winnipeg.............	9,400	710	72

19. The area of Lake Michigan equals what per cent of that of Lake Superior?

20. Similarly compare the altitudes and greatest depths of these lakes.

21. Solve similar problems on the table.

§158. Commission.

DEFINITION.—Commission is a sum of money paid by a person or firm called the *principal,* to an agent for doing business for the principal. It is usually reckoned as some per cent of the amount of money received or expended for the principal.

1. An agent sells 250 bu. @ 80¢ on a 2% commission. How much money was due the agent?

2. An agent bought for his firm 1000 bbl. of apples @ $1.25, and received a commission of 5%. How much money did he receive?

3. What is the commission on 100 baskets of peaches @ 50¢, the rate of commission being 3%?

4. What is the commission on the following sales:

(1) 80 A. land @ $75, rate of commission being 5%?

(2) City property sold @ $8500, rate of commission 4½%?

(3) 40 bu. tomatoes @ 75¢, rate of commission 10%?

(4) 1000 bu. wheat @ 95¢, rate of commission 1½%?

(5) 100 doz. eggs @ 25¢, rate of commission 12%?

(6) 1000 bu. green vegetables @ 90¢, rate of commission 12½%?

(7) $500 collected debts, rate of commission 15%?

(8) $1200 bad debts collected, rate of commission 20%?

5. Find the commission on $1850 at 2½%; at 3½%; at 12½%.

6. A principal sent his agent $2652 to be used for the firm after deducting (subtracting) the agent's commission of 2%. How much money was used for the firm?

7. A clothing house sent its agent $1224, which included a sum of money to be invested and 2% commission on this sum for the agent. What sum was to be invested?

8. 5% commission on a certain amount of money was $684.20. What was the amount? (Statement: $.05x = \$684.20$. Find x.)

DEFINITION.—A shipment of goods sent to an agent to be sold is called a *consignment*.

9. A consignment of 4560 bu. of wheat was sold by an agent at 78⅝¢ per bushel. What was the agent's commission at 1½ per cent?

10. A commission agent sold the following consignment of goods: 20 doz. eggs at 14¾¢; 40 lb. creamery butter at 21¢; 36 lb. cheese at 13¼¢; 80 lb. chickens at 12½¢; 8 doz. live chickens at $3.75; 4 bbl. apples at $3.25; 16 boxes oranges at $2.25; 4 boxes oranges at $2.50; 12 boxes grape fruit at $2.75; 25 bunches bananas at $1.25; 12 boxes lemons at $2.75; 16 bbl. potatoes at $4.25. Find the agent's commission at 6¼ per cent.

11. An agent's commission at 2½% on a certain collection amounted to $97.16. What was the amount of the collection?

§159. Trade Discount.

DEFINITION.—A *discount* is a certain rate per cent of reduction (sum thrown off) from the prices at which articles are listed or quoted. The discount is usually allowed for cash payments or for payment within a specified time.

1. A retail merchant buys silk at $1.20 a yard. He is allowed a discount of 10% for cash. He pays cash. How much does the silk cost him?

SOLUTION.—$1.20—10% of $1.20 = $1.08.

2. A publisher sells 50 books at \$1.50 and allows 20% discount. How much does he receive for the books?

Merchants and manufacturers often publish expensive catalogues containing their price lists of articles, whose prices change rapidly. When prices fall, instead of publishing new lists, they mark off an additional discount. For example, the price of a certain article may be catalogued thus: \$60, discount, 20%, 10%, 5%. This means a 20% reduction, then a 10% reduction on the reduced price, and then a 5% reduction on the second reduced price. It would be computed thus:

\$60	\$48	\$43.20
.20	.10	.05
\$12.00	\$4.80	\$2.1600

\$60 − \$12 = \$48. \$48 − \$4.80 = \$43.20. \$43.20 − \$2.16 = \$41.04.

NOTE.—Notice this is not the same as a discount of 20% + 10% + 5% (= 35%). Wherein is it different?

3. A New York merchant sells to a customer goods marked thus: Price, \$3500; disc't, 10% 60 da. and 5% for cash. What must the customer pay to settle the bill by cash payment?

NOTE.—The customer gets the benefit of both discounts.

What would the customer have paid if he had been given a single discount of 15 per cent?

4. A bill of plumber's supplies was marked thus:

Price: \$7.50, disc't. 20% and 7% and 5%.

How much did the supplies really cost?

5. Compute the amount of money needed to settle the following bills if paid in cash or within the shortest time mentioned:

(1) \$45; discount 25% 60 da. and 10% 5 days.
(2) \$180; discount $16\frac{2}{3}$% 30 da., and 10% 10 da., and 5% cash.
(3) \$1800; discount 30% and 10% and 7% cash.
(4) \$54; discount $12\frac{1}{2}$%, and 8%, and 5% 30 days.
(5) 46 T. coal at \$5.75; discount 10% and 2% cash.
(6) 50 men's suits at \$18; discount 20% and 5% 10 days.
(7) 4 gross tablets at 50¢ per doz.; discount 20% and 7% cash.

6. To what single rate of discount is a discount of 20% and 5% equivalent?

SUGGESTION.—Take a base of \$100.

§160. Marking Goods.

In marking his goods a merchant uses the *key* word "harmonizes." He writes the *selling price* above a horizontal line, and the *cost price* below it, using the letters *h-a-r-m-o-n-i-z-e-s* in order for the digits 1-2-3-4-5-6-7-8-9-0. For example a book sells at \$2.50 and costs \$1.75. The mark would be $\frac{a\,o\,s}{h\,i\,o}$.

1. Using the key "harmonizes," interpret the following cost marks and find the per cent of profit for each cost mark:

$$(1)\ \frac{n\,s}{m\,a};\ (2)\ \frac{i\,o}{o\,r};\ (3)\ \frac{a\,a\,o}{h\,i\,o};\ (4)\ \frac{m\,r\,z}{r\,s\,s};\ (5)\ \frac{m\,a\,o}{r\,r\,n}.$$

2. Complete these marks so that the selling price may be 40% greater than the cost price:

$$(1)\ \frac{\cdots}{a\,o};\ (2)\ \frac{\cdot\,\cdot}{m\,s};\ (3)\ \frac{\cdots}{i\,o};\ (4)\ \frac{\cdots}{z\,s};$$

$$(5)\ \frac{\cdots}{h\,a\,o};\ (6)\ \frac{\cdots}{r\,a\,o};\ (7)\ \frac{\cdots}{m\,o\,s}.$$

3. Articles bearing the following selling marks are marked to sell at a profit of $33\frac{1}{3}\%$; fill in the cost mark, using the same key as above:

$$(1)\ \frac{m\,s}{\cdots};\ (2)\ \frac{z\,s}{\cdots};\ (3)\ \frac{m\,a\,s}{\cdots};\ (4)\ \frac{h\,a\,s}{\cdots};\ (5)\ \frac{r\,n\,s}{\cdots};\ (6)\ \frac{z\,n\,s}{\cdots}.$$

4. Using the same key, mark articles costing the following prices to sell at a profit of $37\frac{1}{2}\%$:

(1) 40¢; (2) 72¢; (3) \$1.68; (4) \$2.88; (5) \$6.40; (6) \$8.24.

5. Supply complete cost marks for articles sold at 50% profit the selling prices of which are as follows:

(1) 90¢; (2) \$1.50; (3) \$2.50; (4) \$3.66; (5) \$7.50; (6) \$8.10.

6. Solve problems 4 and 5, using the key "black horse."

7. Choose a key word and with it solve problems 4 and 5.

8. A merchant wishes to mark his goods so that he may drop 10% below the marked price and still make 20% of the cost price. Using the key "harmonizes," how must he mark articles costing the following prices:

(1) 50¢? (2) 60¢? (3) \$1.20? (4) \$2.50? (5) \$6.40?

§161. Interest. ORAL WORK

Review §81, pp. 125-26. The method of §81 is known as the *six per cent method.*

In reckoning interest the year is regarded as containing 12 mo. of 30 da. each.

1. A man is charged \$6 for the use of \$100 for 1 yr. What per cent of the sum borrowed (\$100) equals the sum (\$6) he is charged for its use?

2. A man is charged \$21 for the use of \$350 for 1 yr. The sum charged equals what per cent of the sum borrowed?

DEFINITIONS.—*Interest* is money charged for the use of money. It is reckoned at a certain rate per cent of the sum borrowed for each year it is borrowed.

When money earns 3, 6, 7, or 10 cents on the dollar *annually* (each year) the *rate* is said to be 3%, 6%, 7%, or 10% *per annum* (by the year) and the *rate per cent* is said to be 3, 6, 7, or 10.

3. At 6% per annum, how much interest does \$360 earn in 1 yr.? in 3 yr.? in $4\frac{1}{2}$ yr.? in $2\frac{3}{4}$ yr.? in t years?

4. Make a rule for computing the interest on any sum of money at 6% when the time is in years.

5. At 6% per annum, how much interest does \$1 earn in 1 yr.? in 2 mo.? in 1 mo.? in 6 da.? in 12 da.? in 18 days?

6. When the time is in months, how may the interest on any sum of money at 6% per annum be computed?

7. How may the interest at 6% per annum on any sum of money be computed when the time is given in months and days? in years, months, and days?

DEFINITIONS.—The sum of money on which the interest is computed is called the *principal.* The principal plus the interest is called the *amount.* Since the borrower must not only pay the interest on the borrowed principal, but also return the principal, the debt he must discharge (pay) is the *amount.*

8. How long will it require any principal (say \$1) to amount to twice its value (\$2) or to double itself at 6% per annum?

9. Give the reasons for these statements:

Any principal at 6%—(1) doubles itself in 200 months; (2) earns $\frac{1}{100}$ of itself in 2 mo. or 60 days.

10. Let I stand for the interest on \$$p$ at $r\%$ for t yr. and let i denote the interest on \$$p$ at 6% for t yr. Explain the meaning of the equations:

$$i = \frac{6pt}{100};\quad I = \left(\frac{r}{6}\right) \times i$$

WRITTEN WORK

To find the Interest.

1. What is .07 of \$450? What is the interest on \$450 at 7% for 1 yr.? for 2 yr.? for 5 yr.? for $3\frac{1}{2}$ yr.? for $4\frac{3}{4}$ yr.? for t years?

2. What is .08 of \$1250? What is the interest at 8% on \$1240 for 1 yr.? for 3 yr.? for $5\frac{10}{12}$ yr.? for x years?

3. \$640 was on interest at 5% from July 1, 1896, to July 1, 1900. Find the interest.

4. \$1800 was on interest at 7% for 3 yr. 7 mo. 21 da. Find the interest.

Convenient Forms

I. Interest computed first at the given rate.

	\$1800	principal
	.07	rate
	\$126.00	int. for 1 yr.
	3	whole years
	\$378.00	int. for 3 yr.
$\frac{1}{2}$ of \$126.00	63.00	int. for 6 mo.
$\frac{1}{6}$ of 63.00	10.50	int. for 1 mo.
$\frac{1}{2}$ of 10.50	5.25	int. for 15 da.
$\frac{1}{5}$ of 10.50	2.10	int. for 6 da.
	\$458.85	int. for 3 yr. 7 mo. 21 da.

II. Interest computed first at 6%.

.18	= int. on \$1 for 3 yr. at 6%
.035	= int. on \$1 for 7 mo. at 6%
.0035	= int. on \$1 for 21 da. at 6%
.2185	= int. on \$1 for 3 yr. 7 mo. 21 da. at 6%
1800	
1748000	
2185	
393.3000	= int. on \$1800 for given time at 6%
65.55	= int. on \$1800 for given time at 1%
\$458.85	= int. on \$1800 for given time at 7%

Suggestions for II.—
(1) If the interest on \$1 for 1 yr. is 6¢, what is the interest for 3 years?
(2) If the interest on \$1 for 2 mo. is 1¢, what is the interest for 7 mo.?
(3) If the interest on \$1 for 6 da. is 1 mill, what is the interest for 21 da.?

5. How much interest must I pay for the use of $600 for 1 yr. 5 mo. 24 da. at 7 per cent?

6. Find the amount of $300 for 2 yr. 4 mo. 25 da. at $4\frac{1}{2}$ per cent.

7. Find the interest and the amount under the following conditions:

(1) $700 for 1 yr. 7 mo. 15 da. at 3 per cent.
(2) $400 for 2 yr. 9 mo. 27 da. at 3 per cent.
(3) $210 for 2 yr. 5 mo. 28 da. at 6 per cent.
(4) $150 for 1 yr. 11 mo. 13 da. at 7 per cent.
(5) $280 for 1 yr. 6 mo. 19 da. at $4\frac{1}{2}$ per cent.
(6) $360 for 1 yr. 4 mo. 5 da. at 4 per cent.
(7) $260 for 2 yr. 3 mo. 11 da. at $3\frac{1}{2}$ per cent.
(8) $500 for 1 yr. 11 mo. 14 da. at 7 per cent.
(9) $300 for 2 yr. 7 mo. 12 da. at 5 per cent.
(10) $625 for 3 yr. 9 mo. 18 da. at 6 per cent.

To find the Principal.

8. What principal at 8% will furnish $16 interest in 2 years?

SUGGESTION.—What interest will $1 produce at 8% in 2 yr.? How many dollars will yield $16 at 8% in 2 years?

9. What principal at 8% will produce $30 interest in $2\frac{1}{2}$ years?

10. What principal at 6% will amount to $112 in 1 yr. 6 months?

SUGGESTION.—What is the amount of $1 at 6% interest for 1 yr. 6 mo.?

11. Find the principal which will yield $61.25 interest in 3 yr. 6 mo. at 7 per cent.

12. Find the principal which will amount to $972.40 in 3 yr. 2 mo. 12 da. at $4\frac{1}{2}$ per cent.

13. Make a rule for finding the principal when the rate, time, and interest are given.

14. Make a rule for finding the principal when the rate, time, and amount are given.

15. Supply the correct value for the letter in each of the following cases:

	Rate	Time	Interest	Principal	Amount
(1)	6 %	1 yr. 7 mo. 15 da.	$19.50	*P*	*A*
(2)	8 %	3 yr. 3 mo. 18 da.	$169.02	*P*	*A*
(3)	$8\frac{1}{3}$%	4 yr. 7 mo. 21 da.	*I*	*P*	$998.50
(4)	7 %	6 yr. 8 mo. 16 da.	*I*	*P*	$198.42
(5)	4 %	2 yr. 6 mo. 18 da.	$122.50	*P*	*A*
(6)	5 %	1 yr. 10 da.	$176.00	*P*	*A*
(7)	$4\frac{1}{2}$%	90 da.	$32.50	*P*	*A*

To find the Rate.

16. At what rate per cent will $320 yield $34 interest in 2 yr. 9 months?

SUGGESTION.—How much interest will $320 yield in 2 yr. 9 mo. at 1%? At what rate per cent then will the same sum yield $34 interest in 2 yr. 9 months?

17. At what rate will $780 yield $486.60 in 5 yr. 8 months?

18. A man invested $2000 for 2 yr. 7 mo. 27 da. and received $2638 at the end of this time. What rate per cent of interest did his investment earn for him?

19. A man bought 120 A. of land at $85 and sold it 2 yr. 8 mo. later for $100 per A., after having received $900 in rents from it and having twice paid taxes on it at 75 cents per acre. What was his annual rate per cent of profit?

20. Make a rule for finding the rate when the principal, time, and interest are known.

21. Find the correct value to the first decimal place for the letter in each of the following problems:

(1)	$580.00	1 yr. 5 mo.	$46.50	*r*%
(2)	$1280.00	3 yr. 10 mo.	$500.00	*r*%
(3)	$798.45	2 yr. 8 mo. 15 da.	$258.65	*r*%
(4)	$3698.50	1 yr. 5 mo. 19 da.	$568.75	*r*%

To find the Time.

22. How long will it take $80 to earn $14 interest at 4% annually?

SUGGESTION.—How much interest will $80 earn in one year at 4%? In how many years then will $80 earn $14 at the same rate?

23. How long will it take \$125 to earn \$57.50 interest at 8% per annum?

24. At 7%, how long will it take \$648 to yield \$69.84?

NOTE.—When the time results in decimals of a year the decimal may be reduced to months and days by the method of problem 65, p. 228.

25. How long will it take \$750 to yield \$750 interest at 8%?

26. How long will it take \$10 to double itself at 6%? at 7%?

27. How long will it take \$975 to amount to \$1225 at 5%?

SUGGESTION.—What is the total interest? What is the interest on \$975 at 5% for 1 year?

28. Make a rule for finding the time, T, required for a given principal, P, to amount to a given sum, A, at a given rate per cent, r?

29. Make a rule for finding how long it will take a given principal to earn a given interest at a given rate per cent.

30. Find the time, T, in years, months and days under the conditions stated in each of the following problems:

	PRINCIPAL	RATE	INTEREST	TIME
(1)	\$66.00	7 %	\$28.60	T
(2)	\$460.00	6 %	\$31.05	T
(3)	\$750.00	5½%	\$147.475	T
(4)	\$1260.00	5 %	\$213.15	T
(5)	\$2460.00	4 %	\$321.44	T

31. Supply the value for which each letter stands in the problems of the following table:

	PRINCIPAL	RATE	TIME	INTEREST	AMOUNT
(1)	\$60.00	6%	3 yr. 3 mo. 8 da.	I	A
(2)	\$175.00	5%	4 yr. 9 mo. 15 da.	I	A
(3)	\$800.00	7%	T	I	\$926.00
(4)	\$475.00	8%	T	\$142.50	A
(5)	\$1266.00	4%	T	I	\$1349.85
(6)	P	6%	5 yr. 7 mo. 27 da.	\$509.25	A
(7)	\$1575 30	r%	1 yr. 4 mo. 18 da.	I	\$1662.46⅜
(8)	\$728.25	8%	T	\$209.736	A
(9)	\$364.75	r%	2 yr. 1 mo. 15 da.	\$69.29	A

32. Complete, to mills, the following interest table:

INTEREST TABLE: PRINCIPAL $100.

TIME	RATE				
	3%	4%	5%	6%	7%
1 da	$.008	$.011	$.014	$.017	$.019
2 da					
3 da					
4 da					
5 da					
6 da					
1 mo..........					
2 mo..........					
3 mo..........					
6 mo..........					
1 yr...........					

33. Compute by the table the interest on $758 at 7% for 4 mo. 12 da.; for 7 mo. 8 da.; for 1 yr. 3 mo. 10 days.

§162. Algebra.

1. Compute the interest on $750 at 5% for each of the following times: (1) 1 yr.; (2) 2 yr.; (3) $6\frac{1}{2}$ yr.; (4) $3\frac{2}{3}$ yr.; (5) $25\frac{5}{6}$ yr.; (6) x yr.; (7) t years.

2. Find the interest and the amount on $1250 for each of the following:

(1) 5%, 1 yr.	(4) 6%, $3\frac{1}{3}$ yr.	(7) r%, t yr.
(2) 6%, $2\frac{3}{4}$ yr.	(5) 4%, $7\frac{3}{8}$ yr.	(8) r%, x yr.
(3) 7%, 6 yr.	(6) 3%, t yr.	(9) r%, n yr.

3. Denote the interest on a certain principal, P, by I, the rate by r, and the time (in years) by t, write an equation showing how to find I from P, r, and t, and translate into words the meaning of the equations.

SOLUTION.—

$$
\text{(I)} \begin{cases} (1)\ I = P \times \dfrac{r}{100} \times t, \text{ or} \\ (2)\ I = \dfrac{P \times r \times t}{100}, \text{ or} \\ (3)\ I = \dfrac{Prt}{100}. \end{cases}
$$

Translated into words: (1) means, "Interest equals the product of the principal, the rate divided by 100, and the time (in years)."

(2) and (3) mean, "Interest equals the product of principal, rate, and time, divided by 100."

4. Calling A the amount, I the interest, and P the principal, write an equation showing how to find A from I and P.

5. Write an equation to show how to find P from I, r, and t, and state in words the meaning of the equation.

If we multiply both sides of the equation, $I = \frac{Prt}{100}$, by 100 we have the equation, $100 I = Prt$. Now to show how to find P from I, r, and t, divide both sides by rt, and write the second member on the left. We then have:

$$\text{(II)} \qquad P = \frac{100\,I}{rt}.$$

6. State in words the meaning of formula (II).

7. Show, by proper multiplications and divisions of equation (I) (3), that the rate r may be found from P, I, and t, by the equation,

$$\text{(III)} \qquad r = \frac{100\,I}{Pt}.$$

8. State in words the meaning of formula (III).

9. Show from (I) (3) that the time t may be found from I, P, and r by the formula,

$$\text{(IV)} \qquad t = \frac{100\,I}{Pr}.$$

10. State in words the meaning of formula (IV).

11. Solve by formulas I-IV the following problems:

(1) $P = \$64$, $r = 8$, and $t = 2\frac{1}{4}$; find I.
(2) $I = \$24$, $r = 6$, and $t = 1\frac{1}{2}$; find P.
(3) $I = \$75$, $P = \$850$, and $t = 2$; find r.
(4) $I = \$120$, $P = \$600$, and $r = 10$; find t.
(5) $I = \$230.13\frac{3}{4}$, $P = \$722$, and $t = 3\frac{3}{4}$; find r.

§163. Promissory Notes.

DEFINITIONS.—A *promissory note* is a written promise, made by one person or party, called the *maker*, to pay another person or party, called the *payee*, a specified sum of money at a stated time.

The sum of money for which the note is drawn is called *the face value*, or the *face*, of the note.

The date on which the note falls due is called *the date of maturity*, and *the time to run* is the time yet to elapse before the note falls due.

\$250.00 KANKAKEE, ILL., *June 3, 1903.*

Ninety days after date I promise to pay to the order of *A. M. Baker, Two Hundred Fifty and* $\frac{00}{100}$ Dollars, for value received, with interest at *six* per cent per annum from date.

Due Aug. 30, 1903. *H. R. Newman.*

1. Who is the maker of the above note? the payee? What is the face of the note? the date? the rate of interest? the date of maturity? the time to run?

2. A promissory note, unless otherwise specified in the note, draws interest on its face value at the rate mentioned in the note from the date of the note until it is paid. Compute the interest and the amount on the foregoing note if it was paid Sept. 1, 1903.

3. Find the interest on the following note, paid Aug. 30, 1903:

\$875. Urbana, Ill., May 18, 1897.

One year after date I promise to pay to James Black, or order, Eight Hundred Seventy-five and $\frac{00}{100}$ Dollars, at Busey's Bank, for value received, with interest at the rate of seven per cent per annum from date.

Due May 18, 1898. HENRY OSBORN.

NOTE.—To find the time for which interest is to be computed, proceed thus:

CONVENIENT FORM

Date of payment,	1903	2	10
Date of note,	1897	5	18
Time,	5	8	22
	yr.	mo.	da.

EXPLANATION.—

2 mo. 10 da. = 1 mo. 40 da.
1 mo. 40 da. — 18 da. = 1 mo. 22 da.
1903 1 mo. = 1902 13 mo.
1902 13 mo. — 5 mo. = 1902 8 mo.
1902 — 1897 = 5 yr.

4. Find the amount of each of the following notes:

	Face	Rate	Date of Note	Date of Payment	Interest	Am'nt
(1)	$250	6 %	Mar. 12, 1899	Jan. 1, 1902	......	......
(2)	$635	7 %	Nov. 20, 1896	July 15, 1900	......	......
(3)	$2400	$5\frac{1}{2}$%	Dec. 12, 1899	Jan. 8, 1903	......	......
(4)	$3865	$4\frac{1}{2}$%	Oct. 13, 1901	Sept. 7, 1903	......	......
(5)	$3640	$8\frac{1}{3}$%	Aug. 17, 1900	Mar. 3, 1903	......	......

§164. Discounting Notes.

DEFINITION.—*Discount* is a deduction from the amount due on a note at the date of maturity.

In some cases promissory notes do not draw interest. The following is an example:

John C. Cannon purchased a self-binding harvester from A. R. Crow for $120 and gave him the following note in payment:

> $120. Peoria, Ill., June 20, 1899.
>
> Eighteen months after date, for value received, I promise to pay to A. R. Crow, or order, One Hundred Twenty and $\frac{00}{100}$ Dollars, without interest until due.
>
> Due Dec. 20, 1900. JOHN C. CANNON.

1. On Sept. 20, 1899, Crow sold the note to Adams at such a price that Adams received his purchase money and 8% interest on it until the date of maturity (Dec. 20, 1900). How much did Adams pay for the note?

SUGGESTIONS.—Any principal at 8% will amount to 110% of itself in 1 yr. 3 mo. Why? Hence, $120 = 110% of what number? Or, better, $1.10x = \$120$. Find the value of x.

DEFINITIONS.—The sum of money which, at the specified rate and in the time the note is to run before falling due, will amount to the value of the note, when due, is called the *present worth* of the note. The difference between the value of the note, when due, and the present worth is called the *true discount*.

The *bank discount* of a note is the interest upon the value of the note when due, from the date of discount until the date of maturity.

2. If Crow had sold the above note to a banker at a discount of 8%, the banker would have computed the interest at 8% on $120 from the date of sale (Sept. 20, 1899) until the date of maturity. How much would he have received for the note? Find the difference between the true and the bank discount of the note.

3. C. A. Thomas bought a road wagon of J. K. Duncan, giving the following note in payment:

> \$65. Pekin, Ill., Sept. 10, 1900.
>
> Two years after date I promise to pay J. K. Duncan, or order, Sixty-five Dollars, value received, with interest at 7% per annum. C. A. THOMAS.
>
> Due Sept. 10, 1902.

On March 10, 1901, Duncan sold this note to a bank at $7\frac{1}{2}\%$ discount. How much did Duncan receive for the note?

NOTE.—Remember, bank discount is computed on the *amount* of the note *when due*, for the time to run from date of sale.

4. Counting money worth 7%, how much did Duncan receive for the wagon?

5. A man bought a horse, giving in payment his note for 1 yr. for \$85, dated Feb. 26, 1903, and drawing interest at 7% from date. Two months later the holder of the note discounted it at a bank at 6%. What was the discount? What did the bank pay for the note?

6. Find the bank discount and proceeds on the following notes:

	FACE	RATE	DATE OF NOTE	DATE OF MATURITY	DATE OF SALE	RATE OF DISC.	BANK DIS-COUNT
(1)	\$60	6 %	Apr. 6, 1897	Oct. 6, 1898	June 15, 1897	7 %	.....
(2)	\$275	$6\frac{1}{2}\%$	Aug. 7, 1899	Nov. 7, 1901	Mar. 13, 1900	6 %	.....
(3)	\$350	$7\frac{1}{2}\%$	Sept. 10, 1900	Dec. 10, 1902	Jan. 15, 1901	6 %	.....
(4)	\$700	8 %	Feb. 15, 1901	Nov. 15, 1903	June 20, 1901	6 %	.....
(5)	\$858	6 %	Jan. 1, 1900	July 15, 1903	May 19, 1900	7 %	.....
(6)	\$1260	5 %	Feb. 28, 1896	Aug. 31, 1900	Aug. 8, 1896	6 %	.....
(7)	\$1800	$5\frac{1}{2}\%$	June 19, 1897	Aug. 19, 1901	Dec. 29, 1898	6 %	.....
(8)	\$2450	7 %	Nov. 18, 1899	Feb. 28, 1902	Dec. 1, 1900	$5\frac{1}{2}\%$	.....
(9)	\$3865	6 %	Dec. 20, 1901	Nov. 20, 1902	Feb. 12, 1902	5 %	.....
(10)	\$8600	$4\frac{1}{2}\%$	Oct. 18, 1900	Jan. 18, 1904	Apr. 2, 1901	4 %	.....

§165. Partial Payments.

DEFINITION.—When a note or bond is paid in part the fact is acknowledged by the holder by his writing the date of payment, the sum paid, and his signature on the back of the note or bond. This is called an *indorsement*.

Partial payments are made only (1) on notes which read, "On or before, etc.," (2) by private agreement between the maker and the holder of the note.

For calculating the balance due on a note or bond on which partial payments have been made, nearly all the states have adopted the following rule, known as "The United States Rule of Partial Payments," which has been made the legal rule by a decision of the Supreme Court of the United States:

RULE.—*Find the amount of the principal to the time when the payment or the sum of the payments equals or exceeds the interest due; subtract from this amount the payment or the sum of the payments. Treat the remainder as a new principal and proceed as before.*

ILLUSTRATIVE EXAMPLES

1. Find the balance due on the following note at maturity:

$1250. **Chicago, Ill., May 21, 1900.**

On or before two years after date I promise to pay to the Order of P. A. Hopper Twelve Hundred Fifty and $\frac{00}{100}$ Dollars at the Corn Exchange National Bank, for value received, with interest at 6 per cent per annum.

Due May 21, 1902. **JOHN P. MILLER.**

The indorsements on the back of this note were as follows:

Nov. 21, 1900 .... $80.00

P. A. Hopper

Feb. 21, 1901 .... $10.00

P. A. Hopper

May 21, 1901 .... $150.00

P. A. Hopper

Feb. 21, 1902 .... $500.00

P. A. Hopper

SOLUTION BY RULE

Principal on May 21, 1900	$1250.00
Interest for 6 mo. on $1	.03
Interest due Nov. 21, 1900	$ 37.50
Amount Nov. 21, 1900 (date of first payment of $80)	$1287.50
First payment	80.00
New principal Nov. 21, 1900	$1207.50
Interest for 3 mo. on $1	.015
Interest to Feb. 21, 1901 (date of second payment of $10)	$ 18.11
Payment being less than interest no settlement is made.	
	$1207.50
Interest for 6 mo. on $1	.03
Interest to May 21, 1901 (date of third payment of $150)	$ 36.23
Amount May 21, 1901	$1243.73
Sum of second and third payments ($10 + $150)	160.00
New principal May 21, 1901	$1083.73
Interest for 9 mo. on $1	.045
Interest to Feb. 21, 1902 (date of fourth payment of $500)	$ 48.77
Amount Feb. 21, 1902	$1132.50
Fourth payment	500.00
New principal Feb. 21, 1902	$ 632.50
Interest for 3 mo. on $1	.015
Interest to date of maturity, May 21, 1902	$ 9.49
Balance due at maturity, May 21, 1902	$ 641.99

2. The following payments were made on a $650 note, bearing 7% interest and dated April 20, 1901:

July 30, 1901	$75.00
Jan. 15, 1902	$15.00
Aug. 12, 1902	$175.00
Jan. 1, 1903	$50.00

Find the amount due April 20, 1903.

3. A note of $2800, dated Feb. 23, 1900, and bearing 7% interest, carried the following indorsements:

Feb. 23, 1901	$100.00
July 16, 1901	$50.00
Jan. 1, 1902	$800.00
July 15, 1902	$85.00
Nov. 28, 1902	$380.00

Find the amount due Feb. 23, 1903.

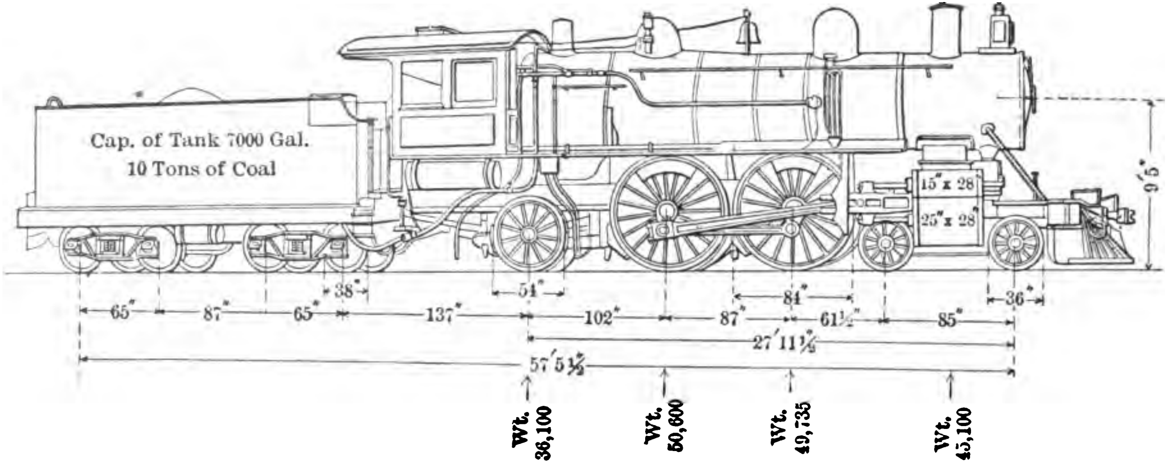


FIGURE 138

§166. Locomotive Engine.

1. The small wheels under the front of the engine are called *engine truck wheels*, or *leaders*. How many leaders are there under the engine (on both sides)?

The large wheels are called *drivers*. The smaller wheels just behind the drivers are called *trailers*.

Answer the following questions by referring to Fig. 138:

2. How high is the center line of the boiler above the top surface of the track?

3. Give the following distances in feet:

(1) Between the centers of the leaders;

(2) Between the centers of the rear leader and of the front driver;

(3) Between the centers of the drivers;

(4) Between the centers of the rear driver and of the trailer;

(5) Between the centers of the trailer and of the front wheel of the tender (tank);

(6) Between the centers of the front two wheels of the tender;

(7) Between the centers of the front leader and of the trailer;

(8) Between the centers of the front leader and of the trailer;

(9) Between the centers of the front leader and of the rear tender truck wheel.

4. In the drawing the distance between the centers of the engine truck wheels is $\frac{1}{16}''$. The actual distance between the centers of the engine truck wheels is 85″. Find the *scale* of the drawing of Fig. 138.

5. Using the scale found in problem 4, find by measuring with a ruler graduated to 16ths of an inch the actual distance from the center of the front engine truck wheel to the front tip of the pilot (cow-catcher).

6. The actual distance from the center of the rear tender truck to the rear end of the tender is 5′ $3\frac{1}{2}''$. What is the actual distance from the front tip of the pilot to the rear end of the tender?

7. In the drawing it is $1\frac{3}{8}''$ from the top surface of the rails to the top of the smokestack. How high is the top of the smokestack above the rails?

8. In the drawing, the distance from the top surface of the rails to the top of the boiler is 1″. How high is the top of the boiler above the top of the rails?

9. How high is the top of the engine cab above the top of the rails? The top line of the tender? The top of the lantern containing the headlight?

10. Find the size of the windows of the engine cab.

11. Give the distance in inches (1) between the nearest points

on the rims of the drivers; (2) between the nearest points on the rims of the leaders; (3) between the nearest points on the rims of the front tender wheels.

12. How long are the radii of the leaders shown in the cut? How long are the circumferences of these wheels?

13. Compute the radii and the circumferences of the tender wheels.

14. Find the circumference of the drivers; of the trailers.

15. When the drivers turn round 240 times a minute, how fast does the engine go?

16. How many times do the leaders turn while the drivers turn round once?

17. The weights written beneath indicate the number of pounds of the weight of the engine which is borne by the different pairs of wheels. Find the total weight of the engine.

18. The weight of the empty tender is 43,000 lb. When the tender is loaded it carries 10 T. coal and 7000 gal. of water. A cubic foot of water weighs $62\frac{1}{2}$ lb., and contains $7\frac{1}{2}$ gal. Find the total weight of the loaded tender.

19. The engine shown in the cut drew a train of 3 sleepers, averaging 93,000 lb.; 5 passenger coaches, averaging 78,000 lb.; an express car, weighing 34,000 lb.; and a mail car, weighing 75,000 lb. What was the total weight of the train, including both engine and tender?

20. The force (pull) exerted by an engine to draw a train is different for different speeds. For a speed of 10 mi. an hour it has been found that on straight, level track an engine must exert a force of $4\frac{3}{5}$ lb. for each ton of weight of the train, including the weight of both engine and tender. Find the force required to draw the train of problem 19 under these conditions.

21. For a speed of 15 mi. an hour a force of $5\frac{1}{2}$ lb. per ton of train weight is needed. What force will draw the train of problem 19 at this speed?

22. Following are the forces for different speeds from 20 to 75 mi. an hour. Find the pulling (tractive) force to be exerted by

the engine to draw the train of problem 19 at each indicated speed:

SPEED	FORCE IN LB. PER T.	TRACTIVE FORCE	SPEED	FORCE IN LB. PER T.	TRACTIVE FORCE
20	$6\frac{1}{3}$		50	$11\frac{1}{3}$	
25	$7\frac{1}{6}$		55	$12\frac{1}{6}$	
30	8		60	13	
35	$8\frac{5}{6}$		65	$13\frac{5}{6}$	
40	$9\frac{2}{3}$		70	$14\frac{2}{3}$	
45	$10\frac{1}{2}$		75	$15\frac{1}{2}$	

23. Find the difference between each number and the number next above it in the table. What do you find?

24. In the equation $F = \frac{V}{6} + 3$, let F stand for the force in pounds per ton and let V stand for the speed in miles per hour. Let $V = 20$ in the equation. Find F by dividing 20 by 6 and adding 3 to the quotient. Compare your result with the number of column 2, and in line with 20. What do you find?

25. Let $V = 25$. Find F and compare with the number of the table in line with 25.

26. Let V equal other numbers of columns 1 or 4 and find F for each speed.

DEFINITIONS.—Replacing V in this way by numbers like 20, 25, and so on is called *substituting* for V the numbers 20, 25, and so on.

Performing the operations indicated in the equation and obtaining the number for F is called *finding the value of F*.

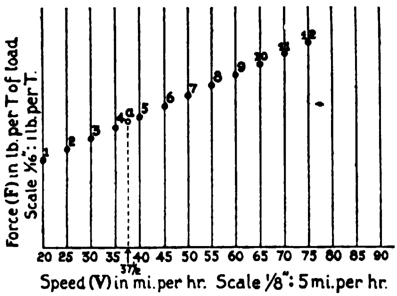


FIGURE 139

27. How does the equation say that the force in lb. per ton (F) needed to draw the train can be obtained when the speed (V) of the train is known?

28. The speeds of the table (problem 15) are plotted to scale on the horizontal line of Fig. 139, and the forces, in lb. per ton of load, are plotted to a different scale on the vertical parallels. The points 1 to 12 are

the upper ends of the vertical lines, whose lengths represent the successive numbers of columns 2 and 5 of the table. Place the edge of a ruler along these points, or stretch a thread taut just over them. How do the points seem to lie?

29. Notice the horizontal and the vertical scales and make a drawing like that of Fig. 139 to a scale 4 times as large.

30. Assuming that the same law, $F = \frac{V}{6} + 3$, relating force and speed, holds also for a speed of 80 mi. per hour, substitute $V = 80$ in the equation and compute F, the force in pounds per tonneeded to draw the train 80 mi. per hour. Make a similar computation for $V = 85$ and for $V = 90$.

31. Plot to scale on the 80, 85, and 90 lines of your enlarged drawing the computed values of F, these lines becoming 13, 14 and 15. Be careful to get each computed, F, on the proper vertical line.

32. Stretch a string, or place a ruler, along the points 1 to 15 of your drawing. Do the points added from your computed values seem to lie on the straight line through the points 1 to 12? With a ruler draw a single straight line through all the points of your drawing.

33. Mark a point on the horizontal line midway between the 35 and 40 points. What speed does this point represent? Draw a vertical from this point up to the line through the points 1 to 12. Mark the upper end of this vertical *a*. What force in pounds per ton does the line from *a* to $37\frac{1}{2}$ represent?

34. How could you find from the drawing the number of pounds per ton needed to draw the train $42\frac{1}{2}$ mi. per hour? $67\frac{1}{2}$ mi. per hour? $22\frac{1}{2}$ mi. per hour? 15 mi. per hour? $17\frac{1}{2}$ mi. per hour? 10 mi. per hour? 5 mi. per hour?

35. How could you find from the equation, $F = \frac{V}{6} + 3$, the numbers of pounds per ton needed to draw the train at the speeds 18, 36, 42, 57, and 69 mi. per hour? Compute these forces and compare them with the numbers found from measurements on the drawing of Fig. 139.

REMARK.—The straight line through the points 1 to 12 is said to represent the equation, $F = \frac{V}{6} + 3$.

§167. Laws for the Drawing of Loads.

1. The force (F), say 200 lb., needed to draw a load (L), say 800 lb., on a common road wagon over loose sand is given by the law $F = \frac{1}{4}L$, or $F = .25L$. Find F for these loads.

(1) $L = 864$ lb.; (2) $L = 1280$ lb.; (3) $L = 2648$ lb.; (4) $L = 3268$ lb.; (5) $L = 4893$ lb.

2. Over fresh earth, the law is $F = .125 L$. What forces (F, in pounds) are needed to draw the following loads, in pounds, over fresh earth:

(1) $L = 680$? (2) $L = 1624$? (3) $L = 2160$? (4) $L = 3840$? (5) $L = 4580$?

3. With common road vehicles on dry level highways the law of pulling (tractive) force (in pounds) is $F = .025\ L$. L denotes the combined weight of the wagon and load in pounds. What forces will be needed to draw the following loads:

(1) 45 bu. wheat on a 1200-lb. wagon? (2) 2 T. coal on a 2200-lb. wagon? (3) $1\frac{1}{2}$ T. hay on a 1680-lb. wagon? (4) A traction engine weighing $8\frac{1}{2}$ T.? (5) A thresher, weighing $4\frac{1}{4}$ T.

4. On well packed gravel roads the law is $F = .052\ L$. To draw a certain load on gravel road a tractive force of 44.72 lb. was exerted. What was the load L?

SOLUTION.—Dividing both sides of the equation by .052, we have $\frac{F}{.052} = L$, or, what is the same thing, $L = \frac{F}{.052}$. We have then, by substituting, $L = \frac{44.72}{.052} = 860$. *Ans.* $L = 860$ lb.

5. Under the conditions expressed in problem 4, find the loads the following forces will draw:

(1) $F = 54.08$ lb.; (2) $F = 66.56$ lb.; (3) $F = 137.8$ lb.; (4) 201.76 lb.; (5) 234 pounds.

6. The law of tractive force, on good, straight, level railroad track is $F = .0035\ L$; on fair track, straight and level, it is $F = .0059\ L$. Weights of cars are given in problems 15 and 17, p. 129, and in problem 19, p. 266. Make and solve problems based on these facts.

7. The tractive force (F) that can be exerted by a locomotive on dry track, straight and level, is about .3 of the part of the weight of the locomotive which rests on the *drivers*. The loads that can just be moved along by a locomotive are given by the equations of the last problem. Make and solve problems based on the following actual weights upon the drivers of certain locomotives:

(1) 80,890 lb.; (2) 85,850 lb.; (3) 86,030 lb.; (4) 106,875 lb.; (5) 112,190 lb.; (6) 131,225 lb.; (7) 141,320 lb.; (8) 202,232 lb. (See also Fig. 139).

8. Problems may also be made on the following laws for road wagons:

(1) Broken stone (fair) . . . $F = .028\ L$;
(2) Broken stone (good) . . $F = .015\ L$;
(3) Worn macadam . . . . $F = .033\ L$;
(4) Nicholson pavement . . $F = .019\ L$;
(5) Asphalt pavement . . . $F = .012\ L$;
(6) Stone pavement . . . $F = .019\ L$;
(7) Granite pavement . . . $F = .008\ L$;
(8) Plank road . . . . . . $F = .010\ L$.

9. Find the value of x in these equations:

(1) $.35x = 7$; (2) $.58x = 11.6$; (3) $1.28x = 3.84$; (4) $.092x = 3.68$; (5) $.019x = 133$; (6) $7.51x = 302.8$; (7) $.175x = 10.5$; (8) $1.093x = 13.116$.

SOLUTION of (1): $x = \frac{7}{.35} = 20$

10. Writing law (1) of problem 8 in the form $.028L = F$, find the loads (L) to 2 decimals, the following forces (F) will draw under the conditions of problem 8 (1):

(1) 50 lb.;	(3) 56.28 lb.;	(5) 68.58 lb.;
(2) 135 lb.;	(4) $165\frac{1}{2}$ lb.;	(6) 110.35 lb.

11. To find the loads (L) that given forces (F) will draw, the laws of problem 8 are more conveniently written thus: $L = \frac{1}{.028} F = 35.71\ F$. Change other laws of problem 8 to this form.

12. Find the load, L, in $L = 35.71\ F$, for each of the following values of the force F:

(1) $F = 280$ lb.; (3) $F = 78.6$ lb.; (5) $F = 140$ lb.;
(2) $F = 145$ lb.; (4) $F = 175.8$ lb.; (6) $F = 700$ lb.

13. Find the load, L, in $L = \frac{1}{.033} F$, or $L = \frac{F}{.033}$, for these values of F:

(1) $F = 66$ lb.; (3) $F = 132$ lb.; (5) $F = 26.4$ lb.;
(2) $F = 363$ lb.; (4) $F = 528$ lb.; (6) $F = 52.8$ lb.

14. Find L in $F = .012\ L$ for the following values of F:

(1) $F = 120$ lb.; (3) $F = 720$ lb.; (5) $F = 840$ lb.;
(2) $F = 600$ lb.; (4) $F = 480$ lb.; (6) $R = 84.48$ lb.

15. Find x in $y = .325\ x$ for each of the following values of y:

(1) $y = 65$; (4) $y = 29.25$; (7) $y = 526.5$;
(2) $y = 97.5$; (5) $y = 58.5$; (8) $y = 10.53$;
(3) $y = 48.65$; (6) $y = 17.55$; (9) $y = 357.5$.

16. On fair track, straight and level, the force which an engine can exert to pull a load is about $\frac{4}{15}$ of that part of its weight which is carried by the drivers. Show that this may be written as an equation thus:

$$F = \frac{4}{15} W.$$

Tell what the letters stand for.

17. By the aid of the law $F = \frac{4}{15} W$, find the pulling force of engines having the following weights on the driving wheels:

(1) $W = 105000$ lb.; (4) $W = 99000$ lb.; (7) $W = 87300$ lb.;
(2) $W = 90000$ lb.; (5) $W = 107200$ lb.; (8) $W = 171300$ lb.;
(3) $W = 101500$ lb.; (6) $W = 121500$ lb.; (9) $W = 136350$ lb.

18. Show how to change $F = \frac{4}{15} W$ first into $15\ F = 4\ W$ and then into $W = 3.75\ F$. The letters mean the same here as in prob. 12. State the law $W = 3.75\ F$ in *words*.

16. Find the weights on the driving wheels of engines that can exert the following pulling forces:

(1) $F = 28000$ lb.; (3) $F = 26400$ lb.; (5) 37600 lb.;
(2) $F = 30400$ lb.; (4) $F = 32640$ lb.; (6) 39640 lb.

20. Find the values y in the equations if $x = 138.5$:

(1) $y = 32\ x$; (3) $y = .036\ x$; (5) $y = .0036\ x$;
(2) $y = 16.8\ x$; (4) $y = .065\ x$; (6) $y = .0159\ x$.

CONSTRUCTIVE GEOMETRY

§168. Problems.

PROBLEM I.—Draw a perpendicular to a given line from a point upon the given line.

EXPLANATION.—Let *AB* denote the given line, and let *P* be a point upon *AB*.

We wish to construct a perpendicular to *AB* at *P*. Place the pin foot on *P* and close the compass feet until their distance apart is less than either *PA* or *PB*. *P1* is such a distance. Then with *P1* as radius draw a short arc across *AB* at both *1* and *2*.

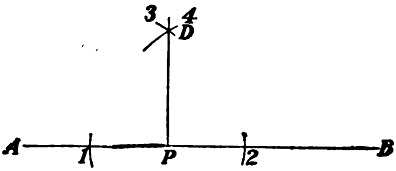


FIGURE 140

Now spread the feet a little wider than *P1*, place the pin foot on *1* and draw arc *3*. Arc *3* may be drawn either above or below *P*. In Fig. 140 it is above *P*. Without change of radius put the pin foot on *2* and draw arc *4* across arc *3*. Let *D* be the intersection of arcs *3* and *4*.

With the straight edge of your ruler draw the line *PD*. *PD* is the desired perpendicular to *AB*.

EXERCISES

1. Draw straight lines on your paper, mark a point upon each line and draw perpendiculars through the marked points until you understand fully how it is done.

2. Draw the perpendicular to any line you may draw on your paper through a marked point on the line, drawing arcs *3* and *4* below the line.

3. Draw a straight line on the blackboard, mark a point upon it, and with crayon, string, and ruler draw a perpendicular to the line through the marked point.

4. Draw a straight line on the ground and along the edge of a board, mark a point on it, and with cord, stake, and a straight board, draw a perpendicular through the point.

PROBLEM II.—Draw a perpendicular to a given line through a point outside of (not on) the given line.

Explanation.—Let AB denote the given line and P the marked point outside of AB. We wish to draw a perpendicular to AB through P.

Put the pin foot on P and spread the feet far enough apart so that the pencil foot will reach below the line AB. Then draw a short arc at *1* and at *2*.

Put the pin foot on *1* and draw arc *3*. Then with pin foot on *2*, draw arc *4* crossing *3* at D.

Connect P and D with ruler and pencil.

PD is the desired perpendicular.

Note.—The arcs *3* and *4* might be drawn with radii of any length greater than half the distance from *1* to *2*; but both *3* and *4* must be drawn with the same radius.

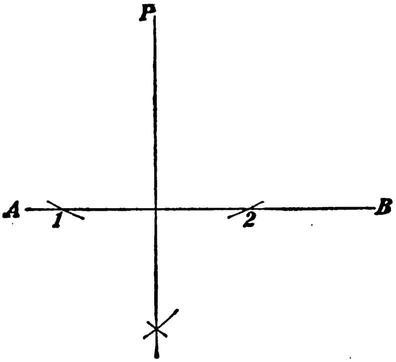


FIGURE 141

EXERCISES

1. Draw straight lines on paper or on the blackboard, mark points outside of them, and draw perpendiculars to the lines, until the way of doing it is fully understood.

2. With cord, stake, and edge of a board draw a perpendicular on the ground through a point not on a line. (Fig. 37, p. 101).

Problem III.—To draw a square inside of a circle of radius $\frac{11}{16}''$.

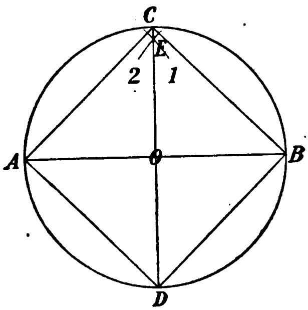


FIGURE 142

Explanation.—Let O, Fig. 142, be the center and let $OB = \frac{11}{16}''$.

Draw a diameter AB.

Spread the compass feet apart wider than $\frac{11}{16}''$ and putting the pin foot first on A, then on B, draw the arcs 1 and 2. Through E and O draw a line and extend it both upward and downward until it cuts the circle at C and D.

How do AB and CD compare in length?

With ruler and pencil connect B and C; C and A; A and D; D and B.

The figure $BCAD$ is the desired square.

EXERCISES

1. Draw a square inside of a circle of 1″ radius; of $1\frac{1}{2}''$ diameter; of $1\frac{3}{4}''$ diameter.

§169. To Model a 3-Inch Cube.

The cube is much used by architects in buildings and parts of buildings. See the lower part of the tower of Old Independence Hall, Fig. 143. Some crystals found in nature also have the cubical form. Can you suggest any uses or occurrences of the cube?

FIGURE 143
Old Independence Hall, Philadelphia

Draw the development, or pattern, of a 3″ cube.

Draw a line as *ac* nine inches long and mark it off into 3-inch spaces. Mark the points of division. At *p* draw a perpendicular line to *ac*, prolong the perpendicular and lay off 3-inch spaces on it as in Fig. 144. Through *b* draw a line parallel to the perpendicular, *d p*, as in Problem II or III, p. 178, or Problem VI, p. 181. On this line set off 3-inch spaces and complete the development, as shown in the figure. Leave flaps as indicated. Crease the paper along the lines over the edge of a ruler and fold up a 3-inch cube. Paste all edges excepting those at the top.

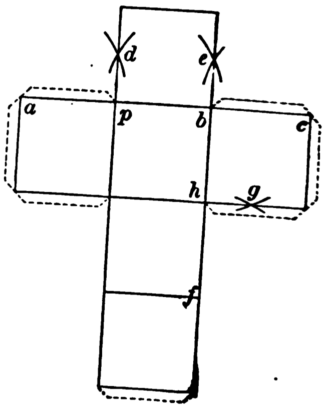


FIGURE 144
Development of 3″ cube

1. What is the area of each surface? of all the surfaces?

2. How many edges has the cube? How many faces meet in each edge?

The surfaces of the cube meet each other in the edges forming *lines*. Call the corners *vertices* (ver′-tĭ-sēs). A single corner is a *vertex*.

3. How many vertices has the cube? How many edges meet at each vertex?

The edges meet each other in the corners of the cube forming *points*.

4. In how many faces does each vertex lie?

5. Do cubes have length? breadth? thickness? Do surfaces? Do lines? Do points?

6. What are the limits of cubes? of surfaces? of lines?

Patterns of the surfaces of figures like those of Fig. 144 will hereafter be called *developments*.

7. Place an inch cube in the corner of the 3-inch cube. How many inch cubes will fill the row along one edge of the larger cube?

8. How many such rows will form a layer on the bottom?

9. How many such layers will fill the cube?

10. How many inch cubes will fill a 3-inch cube?

11. How many cubic inches are there in a 4-inch cube? in a 10-inch cube? in an x-inch cube?

§170. To Model a Square Prism.

Draw a straight line *ad* (Fig. 145) 12 inches long, and make *ab* and *dc* each 2 inches long. Draw *en* and *fo* perpendicular to *ad* at *b* and *c*. Lay off distances *eb*, *bg*, *gl*, *ln*, *fc*, *ch*, *hm*, *mo*, each equal to 2 inches. Draw *ef*, *jk*, etc., and complete the development. Provide flaps; cut and paste the model.

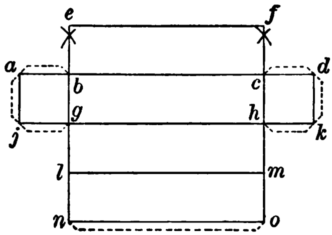


FIGURE 145
Development of the Square Prism

1. What is the area of each of the end surfaces?

2. Give the area of each side surface; of all the side surfaces.

3. How many inch cubes will the model hold?

4. By measuring the edges how could you find the number of cubic inches the model will hold?

5. How many cubic inches would it hold if it were 1 inch longer? twice as long?

6. If every line in the pattern were twice as long as in the development, how would you answer questions 1 to 4?

Pupils should make their own models, different pupils using different lengths of lines and even devising different forms of pattern. For example, let one pupil use a 3-inch fundamental distance instead of an inch, another a 4-inch distance, and so on, each comparing his completed model with other models.

§171. To Model a Flat Prism.

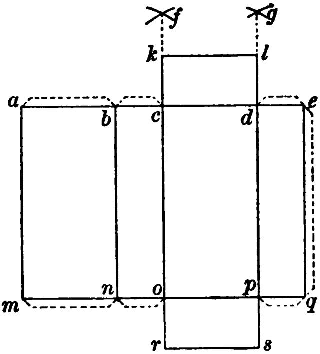


FIGURE 146
Development of Flat Prism

Draw a line *ae* (Fig. 146) 18 in. long and mark off $ed = 3$ in., $dc = 6$ in., $cb = 3$ in., and $ba = 6$ inches.

At *c* and *d* draw perpendiculars to *ae* and extend them 11 in. below *c* and *d* to *r* and *s*, and 3″ above *c* and *d* to *k* and *l*.

Make *co* and *dp* 8 in. long and draw *mq* through *o* and *p*.

Complete the development by drawing the necessary parallels.

Provide it with the necessary flaps and paste up the model, leaving the top *ckdl* open.

1. With the aid of the model of an inch cube, find how many cubic inches would fill the flat prism.

2. To what is the product of the 3 different edges equal?

3. How could you obtain the capacity in cubic inches of such a prism by measuring the lengths of its edges?

4. How many square inches in the whole surface of the flat prism of Fig. 146?

§172. Comparison of Prisms.

(a) A right prism.

Using the lengths of lines as shown in Fig. 147, draw a pattern on heavy paper and construct the model of a right prism as was done in Fig. 145. Provide the edges with flaps and paste up the model, leaving one end open, so that it may be filled with sand. How many cubic inches of sand will just fill the model?

How can you find the capacity of the model by using the lengths of the edges?

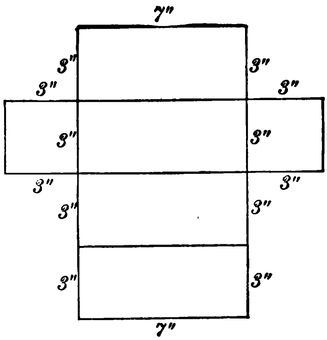


FIGURE 147
Development of Right Prism

(b) An oblique prism with parallelograms for bases.

Similarly draw a pattern, cut, fold, and paste up a model of the oblique prism, as shown in Fig. 148, leaving an end unpasted for sand.

When the model of the right prism is just full of sand, the sides not being bulged out with the sand, pour the sand into the model of the oblique prism. Does it fill the second model?

What is the ratio of the capacities of the two models?

What is the ratio of the areas of the bases (ends)?

(c) A triangular prism.

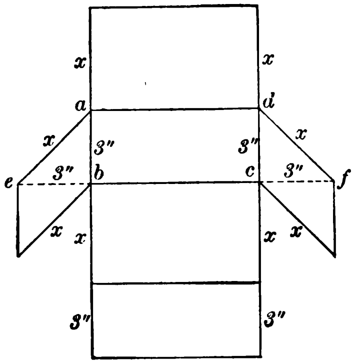


FIGURE 148
Development of Parallelogram Prism

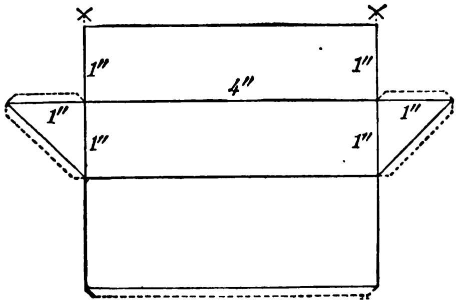


FIGURE 149
Development of Triangular Prism

Model a triangular prism like the one shown in Fig. 149.

Compare the capacity of this model with that of the model of a square prism 1″×1″×4″. What is their ratio?

Model a triangular prism such as that of Fig. 149, but use for lengths 3″ instead of 1″ and 7″ instead of 4″. How does its capacity compare with that of the prism of Fig. 147? of Fig. 148? Give the ratio in each case.

Compare the ratios of the bases.

§173. Volume of an Oblique Prism.

The volume of a figure is the number of cubical units in the space enclosed by its bounding surfaces.

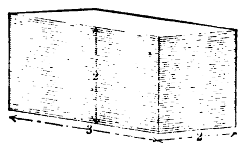


FIGURE 150
Square Pile. or Right Prism

1. How many cubic inches are there in a straight pile of visiting cards 2″ high, if each card is 2″ × 3″ ?

2. How many cubic inches would there be in the pile if it were 5″ high? 9″ high? a in. high?

3. Push the straight pile of problem 1 over as in Fig. 151. How many cubic inches of paper are there in this oblique pile?

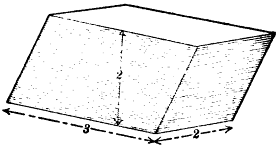


FIGURE 151
Oblique Pile, or Oblique Prism

4. Has the height of the pile been changed in Fig. 151? has the area of the base? has the volume?

5. How can you find the number of cubic units in a right prism from the area of its base and its height? in an oblique prism?

6. Find the volumes of square prisms having edges of the following lengths:

(1) $3'' \times 5'' \times 6''$;	(4) $4\frac{2}{3}' \times 6' \times 12'$;	(7) a in. $\times$ b in. $\times$ c in.;
(2) $6'' \times 5'' \times 9''$;	(5) $16' \times 10\frac{3}{8}' \times 20'$;	(8) x ft. $\times$ v ft. $\times$ z ft.;
(3) $15'' \times 9'' \times 8''$;	(6) $45'' \times 16\frac{2}{9}'' \times 21''$;	(9) m yd. $\times$ n yd. $\times$ p yd.

§174. Paper-Folding.

For the following exercises in paper-folding any moderately thick, glazed paper will do. Tinted or colored paper, without lines, will however show the creases more clearly. It is convenient to have the paper cut into pieces, about 4″ square. Such paper is inexpensive and may be had of any stationery dealer.

PROBLEM I.—At a chosen point on a line, make a perpendicular to the line, by folding paper.

EXPLANATION.—Fold one part of a piece of paper over upon the other and crease the paper along the fold, as at AB, by drawing the finger along the fold. Taking D to denote the chosen point, fold the paper over the point D, and bring the two parts, DA and DB, of the crease AB exactly together. Hold the paper firmly in this position and crease the paper along the line DC.

Compare the portion of the paper between the creases DB and DC with the portion between the creases DA and DC. How do the angles BDC and CDA compare in size?

DEFINITIONS.—When two lines meet in this way making the angles at their point of meeting (intersection) equal, the lines are said to be *perpendicular* to each other, and each is called a *perpendicular* to the other.

The angles thus formed are called *right angles*.

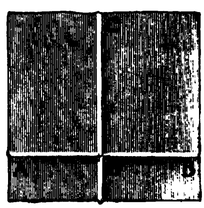


FIGURE 152

PROBLEM II.—Bisect an angle, by folding paper.

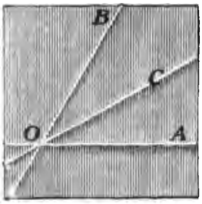


FIGURE 153

EXPLANATION.—Crease two lines, as *OA* and *OB*, lying across each other. They make the angle *AOB*.

Now fold the paper over, and bring the crease *OA* down on *OB*. Holding the creases firmly together crease the bisector *OC*.

How do the angles *AOC* and *BOC* compare in size? Do they fit?

What is the ratio of angle *AOC* to angle *BOC*? of *AOB* to *AOC*?

How could the angle *AOB* be divided by creases into 4 equal parts?

PROBLEM III.—Crease three non-parallel lines.

EXPLANATION.—Crease *AB* in any position. Crease *CD* in any position, not parallel to *AB*. Finally, crease *EF* in any position not parallel to either *AB* or *CD*, Fig. 154.

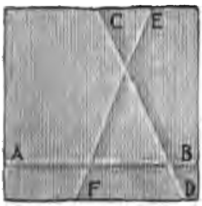


FIGURE 154

In general, in how many points do three non-parallel lines cross each other?

Can you crease three lines, no two of which are parallel, in such a way as to obtain just two crossing points (intersections)? Try it.

Crease three non-parallel lines in such positions as to give but one intersection (see Fig. 155). Lines which go through the same point are called *concurrent lines.*

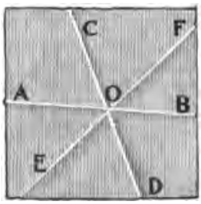


FIGURE 155

PROBLEM IV.—Crease the bisectors of the 3 angles of a triangle.

EXPLANATION.—Crease out a triangle such as *ABC*.

Then crease the bisector of each angle, as in Problem II. Work carefully.

1. How do the bisectors cross each other?

2. Since the three bisectors of the three angles of a triangle all go through *O*, what name would be applied to them

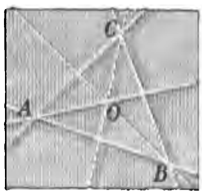


FIGURE 156

PROBLEM V.—Bisect a given line, by paper-folding.

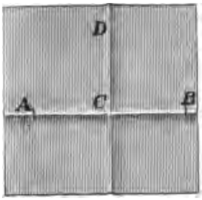


FIGURE 157

EXPLANATION.—Crease a line and stick the point of a pin through the crease at *A* and at *B*.

AB is the line to be bisected.

Fold the paper over and bring the pin hole at *B* down on the pin hole at *A*. Press the paper down and crease a line across *AB*, as at *C*. *CD* is the perpendicular bisector of *AB*.

Crease a line from *D* to *B* and another from *D* to *A*. When the paper is folded over the line *CD*, how do these two creases seem to lie? Compare the lengths of *DB* and of *DA*.

1. Crease the perpendicular bisectors of the 3 sides of a triangle and find how they cross each other.

2. What kind of lines are the perpendicular bisectors of the sides of a triangle?

PROBLEM VI.—Crease a square and its diagonals.

EXPLANATION.—Crease 2 lines, as *AB* and *AX*, perpendicular to each other. (See Problem I.)

Fold the paper over the point *B* and when the two parts of the crease *AB* fit, crease the line *BY*.

Now fold the paper over so that crease *BY* comes down along crease *BA*, and crease the line *BC*.

Fold the paper over a line through *C*, bringing *CX* down along *CA*, and crease *CD*.

ABCD is the required square.

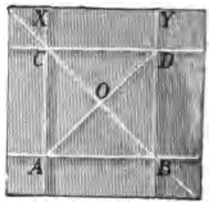


FIGURE 158

1. When the square is folded over the diagonal *BC*, where does *D* fall? Along what crease does *BD* lie? *CD*?

2. How does the diagonal divide the area of the square?

3. Fold over and crease the diagonal *AD*? How does it divide the square?

4. Compare *OB* with *OC*; *OD* with *OA*.

5. How do the diagonals of a square divide each other?

6. How do the diagonals divide the area of the square?

7. How does *AD* divide the angle *BAC*? the angle *BDC*?

8. Fold a second square and crease its diagonals (Fig. 159). Fold over *O*, bringing *D* down on *HC* and crease *HG*. Similarly crease *EF*.

9. When the paper is folded over OF along what line does OD fall? OH?

10. How does OB compare with OD in length? How then do AD and BC compare?

11. How do EF and HG divide the square?

12. Crease the following lines: GF, FH, HE, and EG. How does the area of the square $GFHE$ compare with that of $ABDC$?

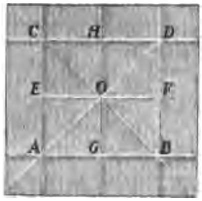


FIGURE 159

PROBLEM VII.—Crease the three perpendiculars from the vertices of a triangle to the opposite sides.

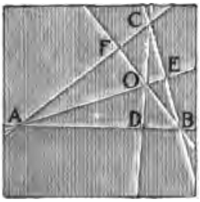


FIGURE 160

EXPLANATION.—Crease the triangle ABC, Fig. 160. Fold the paper over the vertex C, and bring the crease DB down along DA, and crease the perpendicular CD.

Similarly crease AE and BF. Work carefully. How do the creases CD, AE and BF cross each other?

What kind of lines are the three perpendiculars from the vertices to the opposite sides of a triangle?

PROBLEM VIII.—Crease the three perpendicular bisectors of the sides of a triangle, Fig. 161.

How do they cross? What kind of lines are they?

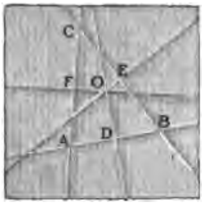


FIGURE 161

PROBLEM IX.—Crease a rectangle and its diagonals.

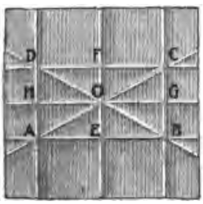


FIGURE 162

EXPLANATION.—Crease AB, Fig. 162, making the distance from A to B two inches. By Problem I, crease the perpendiculars AD and BC. Make AD and BC each 1″ long and crease CD.

Crease the diagonals, one through A and C, and the other through B and D. Call their crossing point O.

Fold the paper over the perpendicular FE, bringing B down on A. Where does C fall? What other line equals CB? OB? OC?

Fold the paper over the point O so that B falls on C, and crease the perpendicular HG. What other line equals AB? OG? OD?

How does the intersection, O, of the diagonals divide the diagonals?

§175. **Perimeters.**—The perimeter of any figure is the sum of the lines bounding the figure. Thus, in form (6), Fig. 163, if p denote the perimeter,

(I) $$p = x + y + z.$$

1. What does $x + 2x + 4x$ mean? How may it be more briefly written?
2. What is the coefficient of x in the answer to problem 1?
3. Write the perimeter, p, of (1), Fig. 163, in two ways.
4. Write the perimeter, p, of (13) in three ways.
5. Write the perimeter, p, of (12) in three ways.
6. Write $\frac{1}{2}$ of the perimeter, p, of (13) in two ways.
7. Write an expression showing that the two answers to 6 are equal.

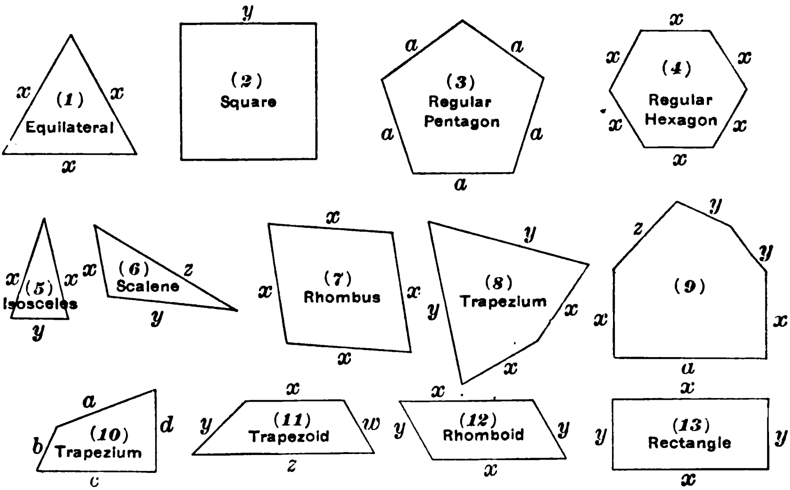


FIGURE 163

8. In (12), if $x = 80$ rods and $y = 40$ rods, how many feet in the perimeter?
9. In (1) and (2), if x and y each = 20 rods and one side of the triangle rests on the square, making a new figure, omitting the common line, what is the perimeter of this new form, in feet? How many sides has the new figure thus formed?
10. Forms (1) and (13) are combined into a single figure. Write p for the new figure in two ways, supposing x the same in both.

§176. Quadrilaterals.

1. Forms (2), (7), (8), (10), (11), (12), and (13), Fig. 163, are different kinds of quadrilaterals. What is a *quadrilateral?*

2. What quadrilaterals have their opposite sides parallel? These figures are parallelograms. Define a *parallelogram.*

3. What parallelograms have *all* their sides *equal?* What is a *rhombus?*

4. What rhombus has *all* its angles *equal?* What is a *square?*

5. What quadrilaterals have their opposite sides equal but consecutive angles not equal? Define a *rhomboid.*

6. What parallelograms have their angles all equal? Define a *rectangle.*

7. What quadrilateral has only *one* pair of sides parallel? Define a *trapezoid.*

8. Is a trapezoid a parallelogram? Is it a quadrilateral?

9. Is a rectangle necessarily a quadrilateral? Is it a parallelogram? a square? *May* a rectangle be a square?

10. Is a square necessarily a quadrilateral? Is it a parallelogram? a rectangle? a rhombus?

§177. Perimeters of Miscellaneous Figures.

1. Denote the perimeter of each of the forms in Fig. 163 by p and write an equation like (I) in §175, showing the value of p for each figure.

2. Omitting the lines on which the forms join, write an equation showing the value of p when (2) and (6), Fig. 163, are joined on y, which has the same value in both.

3. In the same way join (3) and (10) on a, which has the same value in each, and write an equation showing the value of p. What is the name of the figure thus formed?

4. Join (2) and (7) in which x and y are equal. If the perimeter of the figure thus formed is 240 rods, find the value of x in feet.

5. What is the length of the perimeter of a figure like (6), Fig. 163, whose sides are 2 in., a in., and b in. long? whose sides are x in., 8 in., and z in. long? like (10), whose sides are c ft., $2c$ ft., d ft., and x ft.? like (12), whose sides are $2x$ rd., $4y$ rd., $2x$ rd., and $4y$ rd. long?

Write the perimeter (p) of each of the figures below.

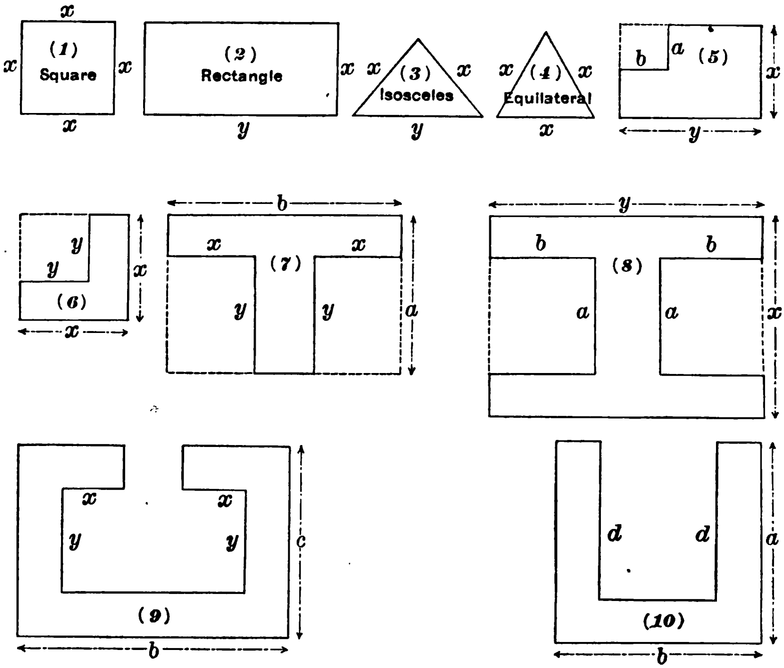


FIGURE 164

In such forms as those from (5) to (10), Fig. 164, some line, or lines, must be found by subtracting others. In (6) for example, note that the ends are each $x-y$. The perimeter is then $x+x+(x-y)+y+y+(x-y) = 4x-y-y+y+y = 4x-2y+2y = 4x$.

All sides not lettered must be expressed without using other letters than those given on the figure. The perimeter means the sum of all the lines that bound the strip, or surface, of the figure.

NOTE.—One-half the difference of a line m and a line n is written $\frac{m-n}{2}$ or $\frac{1}{2}(m-n)$.

1. In (8), $a = 15'$, $b = 12'$, $x = 25'$ and $y = 30'$. Find the length of the perimeter of the figure.
2. Find the area of (8) enclosed by the solid lines.
3. Make and solve other similar problems.

§178. Measuring Angles and Arcs.

ORAL WORK

We may measure the amount of turning of each clock hand in either of two ways:

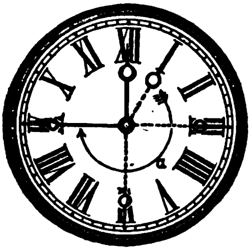


FIGURE 165

(1) By the length of the circular arc passed over by the tip of the rotating hand;

(2) By the wedge-shaped space having its point at the hand-post, A, over which the stem, A-III, A-VI, etc., of the rotating hand moves.

While the clock hand turns from XII to III its *tip* moves over 1 quadrant of arc and its *stem* moves over a right angle.

1. While the hand turns from XII to VI over how many quadrants does its tip move? Over how many right angles does its stem turn?

2. Answer same questions for a turn of the hand from XII to IX; from XII to XII again; from XII around through XII to III.

3. Over what part of a right angle does the stem of the hand move while the hand is passing from XII to I? from XII to II? from III to IV? from VI to VIII?

4. Over what part of a quadrant does the tip of the hand pass in each case of problem 3?

5. Over what part of a right angle does the stem of the hand move while the tip moves 1 min. along the arc? In the same case over what part of a quadrant does the tip move?

DEFINITIONS.—Any wedge-shaped part of the face moved over by the stem of a clock hand as it turns around the hand-post is called an *angle*. The curve passed over by the tip of the hand, while the stem of the hand moves over the angle, is called the *arc* of the angle.

ILLUSTRATIONS.—The wedge-shaped spaces, having their points at A, and included between any two positions of the hand, as A-XII and A-I, A-I and A-III, A-I and A-VI, are all angles; the space swept over by the hand, A-I, as it moves around through II, III, IV, V, VI, etc., to IX is also an angle.

6. As the hand moves from A-XII to A-VI, that is, so that the two positions of the hand are in the same straight line, the hand moves over a *straight angle*. A straight angle equals how many right angles?

7. The arc of a straight angle equals how many quadrants? How many quadrants make a complete circle?

8. How many 5-min. spaces make the circumference of a complete circle? How many 1-min. spaces make the circumference of a complete circle?

9. What part of a right angle is passed over by the stem of the hand as its tip moves from one end to the other of a 1-min. space?

10. If lines were drawn from the hand-post to the ends of all the 1-min. spaces, the whole face of the clock would be divided up into how many small angles?

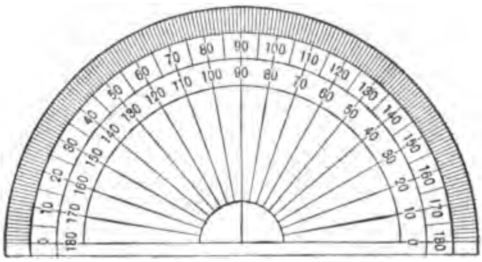


FIGURE 166

The instrument in common use for measuring angles and arcs is the *protractor*. (See Fig. 166.)

11. Instead of dividing the right angle up by radiating lines into 15 equal parts, the protractor divides the right angle into 90 equal parts, one of which is called the *angular degree*. These same lines would divide the quadrant up into how many equal parts? Each of these parts is a *degree of arc*.

12. How many angular degrees are there in a straight angle? How many degrees of arc in the arc of a straight angle?

13. How many angular degrees are swept over by a clock hand while moving entirely around once? How many degrees of arc in the circumference of a circle?

14. How many degrees of angle are passed over by the stem of the minute hand in two hours? In the same time how many degrees of arc are passed over by the tip of the hand?

15. A sextant is $\frac{1}{6}$ of a circumference; how many degrees of arc are there in a sextant?

16. An octant is $\frac{1}{8}$ of a circumference; how many degrees of arc are there in an octant?

17. Study the protractor, Figs. 166 and 167; notice how its marks are numbered. How many degrees of arc are there between

the closest lines on the outer edge? How many degrees of angle are there between the lines which converge toward the center?

18. For smaller angles a shorter unit is $\frac{1}{60}$ of a degree of angle and the smaller unit is called the *minute* of angle. $\frac{1}{60}$ of the

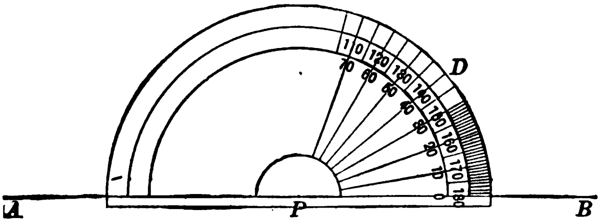


FIGURE 167

minute, called the *second*, is a still smaller unit. How many minutes of angle in a right angle? in a straight angle? in a complete revolution, or *perigon*? How many seconds, in each case?

19. The arcs between the sides of the minute and the second angles are the minute and the second of arc. How many minutes of arc in a quadrant, in a sextant, in an octant, in a circumference? How many seconds, in each case?

A paper protractor can be purchased at any bookstore, and each pupil should supply himself with one.

To measure an angle the center of the protractor is placed on the vertex A, and the 0 line through A is placed along one side AD (see Fig. 168). The reading of the mark below which the other

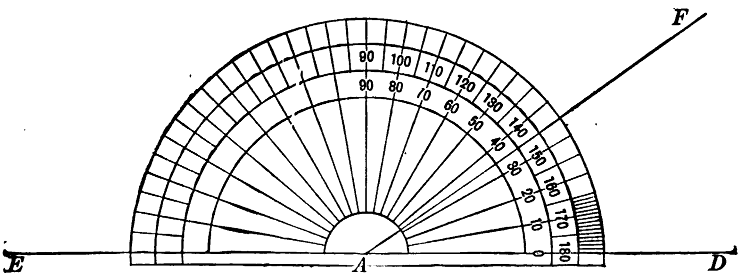


FIGURE 168

side, AF, falls is the number of degrees in the angle, or in the arc of the protractor included between AD and AF.

WRITTEN WORK

20. Draw a triangle, and, with a protractor, measure its angles. To what is the sum of all three equal?

21. With ruler draw a half a dozen triangles of different shapes, carefully measure each angle, and find the sum of the three for each triangle.

22. Measure with the protractor the number of degrees in the angle at one corner of the page of this book.

23. Draw two straight lines crossing each other as in Fig. 169. With the protractor measure and compare *a* and *c*; *b* and *d*.

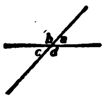


FIGURE 169

The angles *a* and *c*, or *b* and *d*, lying opposite each other, are called *opposite angles*, or *vertical angles*.

24. Draw two crossing lines in different positions, and in each case compare the measures of a pair of opposite angles. What do you find to be true?

25. Draw a pair of parallel lines, as *a* and *b*, Fig. 170, and draw a third straight line *c* cutting across the parallels. Measure with a protractor and compare the four angles, in which 1 is written in Fig. 170. Measure and compare those in which 2 is written. What do you find to be true?

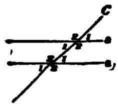


FIGURE 170

26. Draw a line perpendicular to another, see §168, and measure all the angles formed. What do you find?

DEFINITIONS.—The lines which include the angle are *sides* of the angle. The point where the sides meet is the *vertex* of the angle.

27. Draw a quadrilateral of any irregular shape and measure each of its 4 angles. To what is the sum equal? Try another quadrilateral and see whether you obtain the same sum.

28. Draw a given angle at a point on a given line.

EXPLANATION.—Let the given line be *ED*, the given angle 35°, and the given point *A*, Fig. 168. Place the protractor with its center at *A*, and with the diameter of the protractor along the line *ED*. Make a dot opposite the 35° mark on the protractor. With the ruler draw a straight line through *A* and this dot to *F*. The angle *DAF* is the required angle.

29. Draw a straight line, mark a point on the line and draw the angle 75°; 95°; 100°; 135°; 55°30′.

Table of Units of Angle and Arc Measurement

60 seconds (″)	=	1 minute (′)
60 minutes	=	1 degree (°)
360 degrees	=	1 circumference (or perigon)
90 degrees	=	1 quadrant (or right angle)

Table of Equivalents

$$\left.\begin{array}{r} 1{,}296{,}000'' \\ 21{,}600' \\ 360^\circ \\ 4 \text{ quadrants} \end{array}\right\} = 1 \text{ circle}$$

§179. The Sum and Difference of Angles.

Exercise 1.—With ruler and compasses, draw AB and CD perpendicular to each other.

1. Suppose a protractor supplied with a pointer as shown in Fig. 171, the center of the protractor being placed at the point, O, over how many degrees of arc would the tip of the pointer turn while the stem of the hand turns from line OB to line OC? from OB through OC to OA?

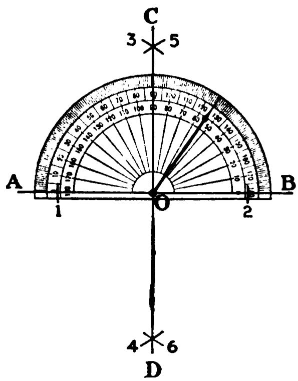
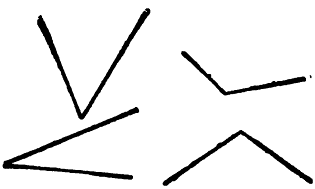


Figure 171

2. How many degrees are there in the angle BOC? in angle BOA? in COD? in AOD? in the angle from OA around through OD to OB? from OA around to OC? from OA entirely around to OA again?

3. How many degrees of angle fill the space around a point, as O, on one side of a straight line, as AB? on both sides?



Acute Angles Obtuse Angles

Figure 172

Definitions.—An angle that is smaller than a right angle is called an *acute angle* (see Fig. 172). An angle that is larger than a right angle is called an *obtuse angle.*

EXERCISE 2.—Find the sum of two angles

1. Draw two acute angles like those at the top of Fig. 173, carefully cut them out, and place them as in the lower part of the figure. Push their vertices and nearer sides entirely together. Draw two lines along the other sides of the angle. Remove the angles and have an angle like that on the left, which is the *sum* of the two angles.

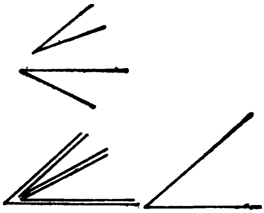


FIGURE 173

2. Calling the larger angle x, and the smaller y, what denotes the last angle?

3. Draw an angle equal to the sum of an acute and an obtuse angle.

EXERCISE 3.—Find the difference of two angles.

1. Draw two angles like those at the top of Fig. 174, cut them out, and place them as in the lower, left-hand part of the figure. Mark along the lower side of the upper angle and cut along the mark. The lower part of the larger angle (shown on the right) equals the *difference* of the two angles.

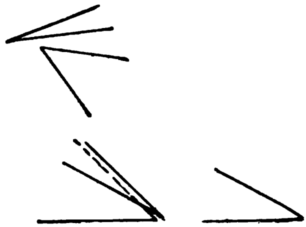


FIGURE 174

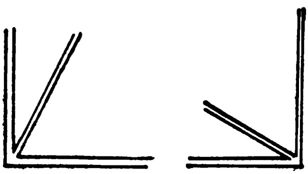


FIGURE 175

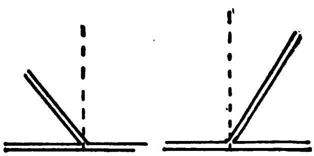


FIGURE 176

2. If the larger angle equals x degrees and the smaller equals y degrees, how many degrees are there in the difference?

DEFINITIONS.—Two angles whose sum equals a right angle, or 90°, are called *complemental angles* (Fig. 175). Two angles whose sum equals 2 right angles, or 180°, are called *supplemental angles* (Fig. 176).

The sum of all the angles that just cover the plane on one side of a straight line is equal to how many right angles (Fig. 177)? To how many degrees is this sum equal?

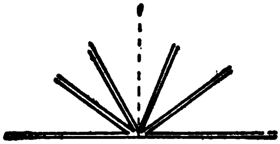


FIGURE 177

EXERCISE 4.—Draw an equilateral triangle (Problem VII, p. 102). Tear off a corner and fit it over each of the other corners in turn. How do the three angles compare in size. Tear off the other corners and place them as in Fig. 178. To what is the sum of all three angles equal?

If one of the angles is x degrees, how many degrees are there in the sum of all three angles? As the sum of the three angles

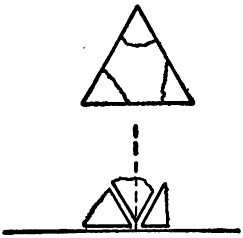


FIGURE 178

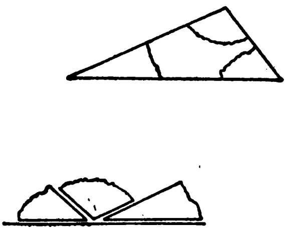


FIGURE 179

equals both $3x$ degrees and also 180°, we may write the equation

$$3x = 180.$$

If $3x = 180$, to what is x equal? How many degrees are there in one of the angles of an equilateral triangle?

EXERCISE 5.—Draw any scalene triangle. Tear off the corners and place them as in Fig. 179. To how many right angles is the sum of all the angles of the triangle equal? To how many degrees?

1. Letting x, y, and z denote the numbers of degrees in the respective angles, in what other way may we write the sum of all three? What equation may we then write? Tell what the equation $x + y + z = 180°$ means.

2. Draw other scalene triangles, tear off the corners, place them as in the figure, and find whether $x + y + z = 180°$ in all cases.

3. Crease a right triangle and find whether the equation, $x+y+z=180°$, is true for it.

4. Cut along the creases and tear off the two acute angles of a carefully creased right triangle and fit them over a carefully creased right angle? What seems to be true? If this is true what equation may you write for the sum of the two acute angles (x and y) of a right triangle?

EXERCISE 6.—Draw a quadrilateral and cut it along a straight line from one corner to the opposite corner as in Fig. 180. Such a line as is indicated by the cut is called a diagonal.

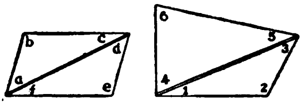


FIGURE 180

1. The cut divides the quadrilateral into figures of what shape?

2. To what is the sum of the three angles of each part equal?

3. To what is the sum of all six angles of the two triangles equal?

4. To what is the sum of all 4 angles of the quadrilateral equal?

5. Do your answers hold true for the parallelogram of Fig. 180?

6. Do they hold good for any shape of quadrilateral you can draw?

7. To what then is the sum of the four angles, x, y, z, and w, of any quadrilateral equal?

EXERCISE 7.—Draw a hexagon and cut it along diagonals as shown in Fig. 181.

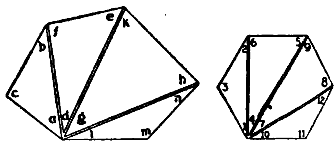


FIGURE 181

1. Into figures of what shape is the hexagon divided?

2. Into how many such figures do the diagonal cuts divide the hexagon?

3. To how many degrees is the sum of all the angles of all the triangles equal?

4. To how many right angles is the sum of all the angles of the hexagon equal?

5. Do all your answers hold true for the regular hexagon (sides equal and angles equal) of Fig. 181 also?

6. In each of these hexagons how many less triangles than sides of the uncut hexagons are there?

7. Draw an 8-sided figure and find how many triangles the diagonal cuts from any vertex would give.

8. How many less triangles than sides were given by the quadrilaterals of Fig. 180?

9. To how many right angles is the sum of all the angles of any 15-sided figure equal? of an n-sided figure?

10. The angles of a regular hexagon are all equal. How many degrees does each contain?

EXERCISE 8.—Draw a rectangle and cut it out. Cut it along a diagonal, and denote the angles by letters, as shown in Fig. 182.

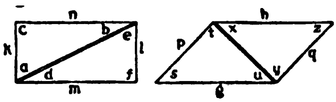


FIGURE 182

1. Turn the lower right-hand triangle around and place it on the upper piece so that e may fall at a, f at c, and d at b. Can you make the parts fit?

2. How does a diagonal divide a rectangle?

3. Show the relative length of sides m and n by an equation; of k and l; of the angles a and e; c and f; b and d; $b+e$ and $a+d$.

4. From these equations point out those which show that the opposite sides of a rectangle are equal.

5. Show from your equations how the opposite angles compare in size.

6. With parallel rulers draw a parallelogram and cut it as suggested by Fig. 182. Answer questions 1-5 for the parallelogram.

7. Write the perimeters of the parallelogram and of the rectangle of Figs. 180 and 182.

§180. Products of Sums and Differences of Lines.

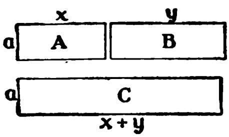


FIGURE 183

1. What is the area of A? of B? of C?

NOTE.—The product of a and $x+y$ is written $a(x+y)$ and is read "a times the sum $x+y$."

2. Read the equation $ax+ay=a(x+y)$ and explain its meaning from Fig. 183.

3. Draw a figure and show that

$$ax+ay+az=a(x+y+z).$$

4. What is the area of each of the parts of Fig. 184?

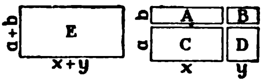


FIGURE 184

NOTE. $a+b$ times $x+y$ is written $(a+b)(x+y)$.

5. From Fig. 184 show the meaning of each product in the equation $(a+b)(x+y) = ax + ay + bx + by$. Also show from Fig. 184 why the equation is true.

6. What is the area of each part of Fig. 185?

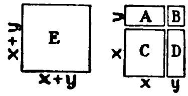


FIGURE 185

7. Write an equation showing how to multiply $x+y$ by itself, or how to square $x+y$.

8. Point out from Fig. 185 the meaning of

$$(x+y)^2 = x^2 + 2xy + y^2.$$

9. Point out from Fig. 186 the meaning of

$$b(a-c) = ab - bc.$$

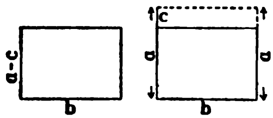


FIGURE 186

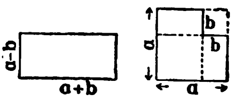


FIGURE 187

10. Draw a figure and show that

$$b(a+d-c) = ab + bd - bc.$$

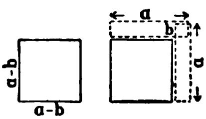


FIGURE 188

11. Show from Fig. 187 that

$$(a+b)(a-b) = a^2 - b^2.$$

12. Show from Fig. 188 that

$$(a-b)(a-b) = a^2 - 2ab + b^2.$$

13. What is the area of the rectangle $ABCD$ (Fig. 189)? of S?

14. What is the length of the dotted part of BC?

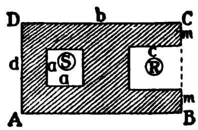


FIGURE 189

15. What is the area of R?

16. What is the area of the cross-ruled part?

17. Write the areas of the cross-lined figures below (Fig. 190).

Call the difference of the two areas d in each case, and answer with an equation.

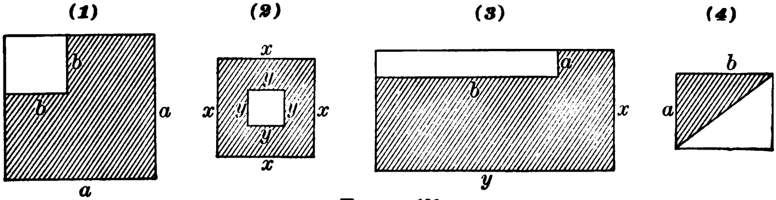


FIGURE 190

18. A lot, $50' \times 175'$, is occupied by a house which, with its porch, covers a $40'$ square. A cement walk $3\frac{1}{2}'$ wide runs from the front line of the lot to the porch, and from the back door of the house to the rear line of the lot. The rest of the lot is covered with grass. How many square feet of grass are there?

19. A man cuts an $18''$ strip of grass entirely around a rectangular lawn, $75' \times 175'$; how many square feet of grass does he cut?

20. He then cuts another $18''$ strip around just inside of the strip mentioned in problem 19; how many square feet of grass does he cut the second time round?

21. A farmer reaps a $12'$ swath entirely round a rectangular wheat-field that is 40 rd. $\times$ 80 rd. How many square rods does he reap the first time round. ($12' = \frac{8}{11}$ rd.)

22. He again reaps a $12'$ swath entirely round the field. How many square rods does he reap the second round? the third round?

23. A farmer plows an $18''$ strip entirely round a rectangular piece of ground 20 rd. $\times$ 80 rd. How many sq. ft. does he plow?

24. He continues plowing round the rectangle (prob. 23) until he has a strip plowed 1 rd. wide entirely round it. How many acres has he then plowed?

25. How many acres has he plowed when the plowed strip is 2 rd. wide? 4 rd. wide?

26. How many acres has he plowed when the unplowed strip of prob. 23 is 4 rd. wide, the plowed strip being of the same width all round the rectangle?

27. How many acres have been plowed when the unplowed strip remaining is $3\frac{1}{2}$ rd. wide? $2\frac{1}{2}$ rd. wide?

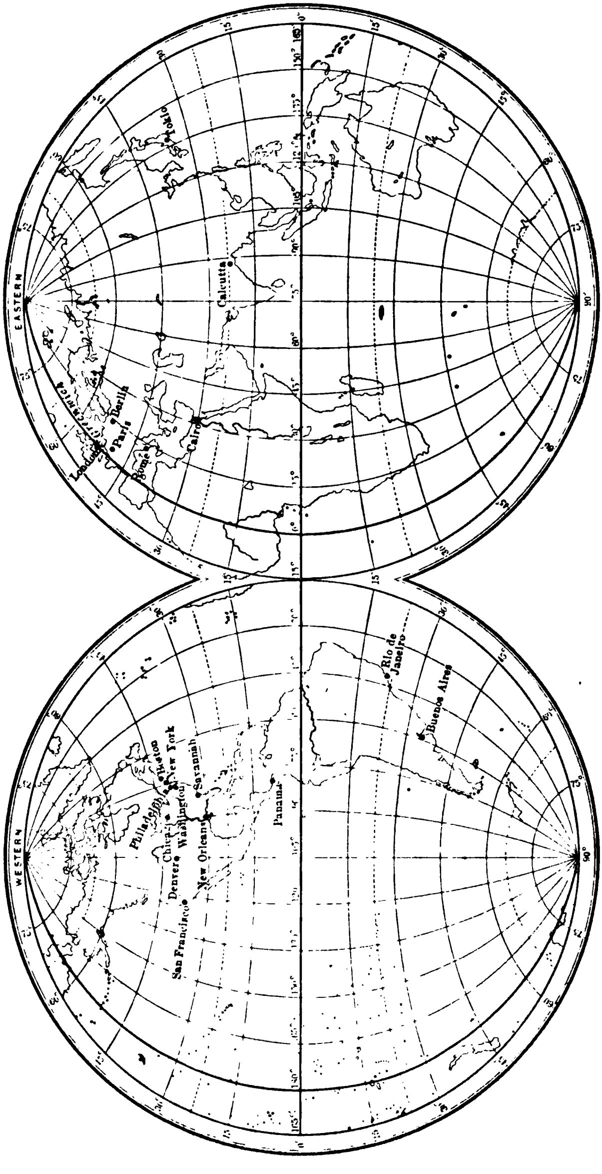


FIGURE 191. MAP OF THE WORLD

§181. Locating and Describing Places on the Earth.

Fig. 191 is a map of the eastern and the western hemispheres of the earth, showing the lines that are used for locating and describing places on the earth. An imaginary line, called the *equator*, runs entirely round the earth, dividing its surface into the northern and the southern hemispheres. This equator is shown on the map by the heavy horizontal line.

The heavy curved line going through the north and south poles of the eastern half of the map and crossing the equator at 0° is called the *prime meridian.*

ORAL WORK

1. What do the numbers 15°, 30°, 45°, etc., mean, that stand along the equator to the right and left of 0°?

2. What part of the map is north? south? east? west?

3. How many degrees are there between the curves that run up and down on the map through the poles?

4. How many degrees are there between the curves that run across the map from right to left?

5. Show the place on the map that is 15° (meaning 15 degrees) west of the zero (0°) point; 30° (30 degrees) west of 0°; 60° W.; 105° W.; 150° W.

6. Show the place that is 15° east of the 0° point; 45° E.; 60° E.; 90° E.; 120° E.; 150° E.; 165° E.

7. Point to the place that is 15° north of 0°; 30° N.; 30° south; 45° S.; 60° N.; 75° S.; 90° N.; 90° S.

8. Show the place that is 30° W. and 15° N.; 30° W. and 30° N.; 30° W. and 15° S.; 60° W. and 30° S.; 60° E. and 15° N.; 60° E. and 30° S.

9. Point out the places that have the following positions:

(1) 30° W. and 75° N.;	(8) 105° W. and 30° N.;
(2) 30° W. and 30° S.;	(9) 60° W. and 60° S.;
(3) 45° E. and 15° S.;	(10) 105° W. and 45° S.;
(4) 60° E. and 45° N.;	(11) 90° E. and 30° N.;
(5) 45° W. and 45° N.;	(12) 90° E. and 90° N.;
(6) 45° W. and 45° S.;	(13) 180° W. and 75° S.;
(7) 45° E. and 45° S.;	(14) 90° W. and 90° N.

Degrees measured toward the west and toward the north are written with a plus (+) sign before them. Degrees measured toward the east and toward the south are marked with a minus (–) sign before them.

10. Point out the following places:

(1) +15°, +30°;	(7) +100°, –45°;
(2) +15°, –30°;	(8) +100°, +75°;
(3) +60°, –75°;	(9) +160°, +80°;
(4) –60°, –75°;	(10) – 40°, +65°;
(5) –75°, +60°;	(11) –140°, –35°;
(6) –75°, –60°;	(12) –120°, –65°.

WRITTEN WORK

1. Write down about the correct distances in degrees westward or eastward, and northward or southward, of the mouth of the Amazon River; of the Mississippi River; of the Nile River; of the Ohio River; of the Niger River (Africa).

2. Write about the positions of these places, using + or – signs: Washington, D. C.; New York City; Boston; Chicago; Denver; New Orleans; London; Paris; Berlin; the south point of Florida; the south point of South America; the east point of South America.

3. About how many degrees long is South America? Africa? The Nile River?

4. About how many degrees wide is South America? Africa? The United States?

The curved lines running from right to left are called *parallels;* those running from north to south are *meridians.*

5. About how many degrees wide is the United States along the 30° parallel? along the 45° parallel?

6. About how many degrees wide is South America along the equator? along the –30° parallel? the –15° parallel?

7. Answer similar questions for Africa.

8. How many degrees are there between the prime meridian and the meridian of the mouth of the Nile River? between the prime meridian and the meridian of the southeast point of Arabia? of the southwest point of Arabia?

9. How many degrees are there between the 45th meridian and the meridian of the mouth of the Mississippi River?

10. Chicago is about how many degrees west of New York City, that is, about how many degrees are there between a meridian through New York City and a meridian through Chicago?

11. About how many degrees is Chicago north of New Orleans? of Panama? of Rio de Janeiro? of the mouth of the Amazon River?

12. About how many degrees west from New York City is Denver? San Francisco? The mouth of the Ohio River?

Distances measured westward or eastward in degrees along a *parallel* from the prime meridian are called *longitudes*. Distances measured in degrees along a meridian northward or southward are called *latitudes*.

13. About what is the difference between the longitude of New York and of Chicago? Boston and Denver? Philadelphia and New Orleans? New Orleans and San Francisco? Washington and Savannah?

14. Give the differences of latitude of the same pairs of places.

The latitude and longitude in degrees of some important seaports are given here:

PLACE	LATITUDE	LONGITUDE	PLACE	LATITUDE	LONGITUDE
Cape Town ...	−33.93	− 18.47	Queenstown ...	+51.85	+ 8.28
Hamburg	+53.55	− 9.97	San Francisco.	+37.78	+122.43
Honolulu.....	+21.30	+157.85	Savannah.....	+32.02	+ 81.12
Manila	+14.67	−121.00	Stockholm	+59.33	+ 18.05
Panama	+ 9.00	+ 79.52	Valparaiso.....	−33.00	+ 71.63
Quebec	+46.80	+ 71.32	Yokohama	+35.50	−139.58

15. What is the difference of latitude of San Francisco and Honolulu? The difference of longitude?

16. What are the differences of latitude and of longitude of San Francisco and Yokohama? of San Francisco and Valparaiso? of Hamburg and Savannah? of Queenstown and Quebec? of Cape Town and Stockholm? of Panama and Manila? of Panama and Valparaiso? of Manila and Yokohama? of Quebec and Panama?

17. Other problems may be made from the table.

18. Read the differences of latitude and of longitude of any places given on maps in your geographies.

§182. Longitude and Time.

The earth may be regarded as an immense sphere turning, like a top, round one of its diameters as an axis from west to east once every 24 hr. This carries the surface of the earth and all objects fixed upon it (as a schoolhouse) round through 360° in 24 hours.

Measuring one complete turn (rotation) of the earth in degrees, it may be said to be equivalent to 360°; measuring it in time, it may be said to be equivalent to 24 hours.

This gives us the following tables of equivalent measures:

360° correspond to 24 hr.;
1° corresponds to $\frac{1}{360}$ of 24 hr. = $\frac{1}{15}$ hr. = 4 min. of time;
1′ corresponds to $\frac{1}{60}$ of 4 min. = $\frac{1}{15}$ min. = 4 sec. of time;
1″ corresponds to $\frac{1}{60}$ of 4 sec. = $\frac{1}{15}$ sec. of time.

24 hr. correspond to 360°;
1 hr. corresponds to 15°;
1 min. of time corresponds to $\frac{1}{60}$ of 15° = $\frac{1}{4}$° = 15′;
1 sec. of time corresponds to $\frac{1}{60}$ of 15′ = $\frac{1}{4}$′ = 15″.

All objects seen on the sky, as the sun, may be *regarded* as stationary, while the turning of the earth carries us past them from the west toward the east. This makes the sun *appear* to rise in the east, move over, and set in the west.

1. Will the sun pass over eastern or western places earlier? Which places have later local* times, those over which the sun passes earlier, or later? Which places have earlier local times, eastern places, or western places?

2. What time is it at a place 45° west of Washington when it is 10 o'clock at Washington? At the same instant, what time is it at a place 30° east of Washington?

DEFINITION.—*Longitude* is the distance in degrees, minutes and seconds (of arc) due eastward or westward from a chosen meridian, called the *prime meridian*. Astronomers and navigators have agreed that the prime meridian shall be the meridian of the Royal Observatory at Greenwich, England.

3. The difference between the local times of two places is 4 hr. 3 min.; what is the difference in longitude between them?

---

*Local sun time is obtained by setting timepieces at XII as the sun crosses the meridian.

4. The difference of longitude between two places is 105° 45′; what is the difference of their local times?

5. It is 4:50 (4 hr. 50 min.) p.m. at a certain place and 1:48 p.m. at another; which place is east of the other and what is the difference of their longitudes?

6. Two men met in Chicago with their watches keeping the correct local times of the places whence they came. On comparing their times one watch showed 10:47 p.m., and the other 3:18 p.m., when it was 9:26 p.m. in Chicago. From which direction did each man come?

7. If the watches (problem 6) were keeping the correct local times of their places, what were the differences of longitude between the places and Chicago?

8. Explain how it happened that the announcement of Queen Victoria's death was read in the Chicago dailies at an earlier hour than that borne by the announcement itself.

9. How may it happen that a cable message sent Wednesday forenoon may be received at a remote place Tuesday?

10. The local times of two ships at sea differ by 4 hr. 18 min. 15.6 sec.; what is the difference of their longitudes?

11. One ship is in longitude 186° 40′ 12″ west and another is in longitude 20° 16′ 48″ west; what is the difference of their longitudes? of their times?

12. The longitude of one ship is 3° 28′ 10″ west and that of another is 18′ 38″ east. What is the difference of their longitudes? of their times?

13. The longitude of the Observatory of Madrid, Spain, is 3° 41′ 17″ west and that of the Berlin Observatory is 13° 23′ 43″ east. What is the difference of their longitudes? of their times?

14. The longitudes of the Cambridge (Eng.) and of the Paris Observatories are 5′ 41.25″ east and 2° 21′ 14.55″ east respectively. What are the differences of their longitudes? of their times?

15. When the sun is on your meridian, that is, when it is noon at your place, at what places on the earth is it forenoon? afternoon? night? midnight?

The longitudes from Greenwich, Eng., of places are also often given (as below) in hours, minutes and seconds *of time.* The plus (+) sign means that the place is west of the meridian of Greenwich, and the minus (–), that the place is east of this meridian.

PLACE	LONGITUDE FROM GREENWICH			PLACE	LONGITUDE FROM GREENWICH		
	H.	M.	S.		H.	M.	S.
Albany	+4	54	59.99	Denver	+6	59	47.63
Algiers	–0	12	08.55	Edinburgh	+0	12	44.2
Allegheny, Penn.	+5	20	02.93	Glasgow	+0	17	10.55
Ann Arbor, Mich.	+5	34	55.19	Madison, Wis.	+5	57	37.93
Berkeley, Calif.	+8	09	02.72	Madrid	+0	14	45.12
Berlin	–0	53	34.85	Mexico	+6	36	26.73
Bombay	–4	51	15.74	New York	+4	55	53.64
Cambridge (Eng.)	–0	00	22.75	Paris	–0	09	20.97
Cambridge (Mass.)	+4	44	31.05	Philadelphia	+5	00	38.51
Cape of Good Hope	–1	13	54.76	St. Petersburg	–2	01	13.46
Chicago	+5	50	26.84	Washington	+5	08	15.78

16. Give from the table the difference of local times of Greenwich, England, and of each of the following places and state whether the time is earlier or later than Greenwich time: Albany; Algiers; Ann Arbor; Berlin; Bombay; Chicago; New York; Paris; St. Petersburg.

17. What is the longitude in degrees (°), minutes (′) and seconds (″) of arc of Albany? of Algiers? of Cambridge (Mass.)? of Washington?

18. Which of each of these pairs of places has the earlier local time and how much earlier is this time:

Albany and Allegheny?	Chicago and Paris?
Ann Arbor and Berkeley?	Edinburgh and Madison?
Ann Arbor and Bombay?	Cape of Good Hope and Bombay?
Chicago and Cambridge, Mass.?	Paris and Bombay?

19. Give the differences of longitude in each case of problem 18.

20. Solve other similar problems on the table.

§183. Standard Time.

For convenience of railway traffic a uniform system of time-keeping, known as Standard Time, was agreed upon in 1883 by the principal railroad companies of North America. It was decided that places within a belt of 15° extending (roughly) $7\frac{1}{2}$° on each side of the 75th meridian west of Greenwich should use the time of the 75th meridian. All places in similar belts extending about $7\frac{1}{2}$° on each side of the 90th, of the 105th, and of the 120th meridian should take the times of those meridians respectively, and hence should have times just 1 hr., 2 hr., and 3 hr. earlier (less) than the time of the 75th meridian *time belt*. This system has now been generally adopted by most civilized countries. In practice, however, the dividing lines of the time belts are irregular lines running through railroad terminals. The following map shows the time belts and the names used to distinguish the times of the several belts which cover continental United States. In this system the times at all places in the United States differ only by whole hours.

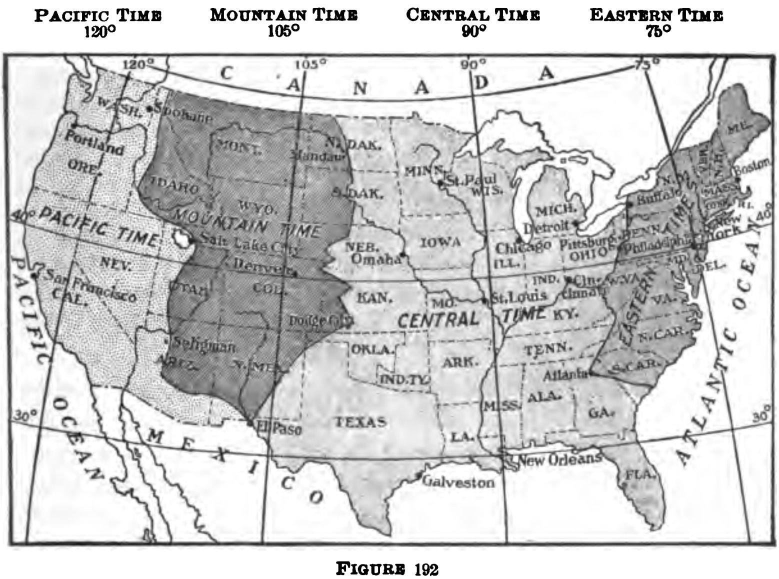


FIGURE 192

1. When it is 8 o'clock a.m. (Standard Time) at Chicago, what is the time at each of the following places: New York? Pittsburg?

St. Louis? Omaha? Denver? Spokane? San Francisco? (See the map, Fig. 192.)

2. When it is 9 hr. 10 min. 45 sec. p.m. at Omaha, what is the time at New Orleans? Philadelphia? Buffalo? Washington? Austin (Tex.)? Boise City? Los Angeles? El Paso? Salt Lake City? (Refer to your Geography.)

3. Answer the questions of problem 2 for 2:15 p.m. at St. Paul; for 3:25 a.m. at Dodge City, Kan.

4. Answer for other places marked on the map such questions as are asked in problems 2 and 3 for Omaha, for St. Paul, and for Dodge City.

The circle *SqNr* (Fig. 193) represents the earth and the curved lines represent the hourly meridians running from the equator, *qr*, and converging toward the poles *S* and *N*. If *O* represents some place in the northern hemisphere, *SqNr* is the meridian of the place and *BQPR* may be imagined to represent the meridian of the sky (*hour circle* of the sun), on which the sun, T, is situated. The hour circle of the sun in the apparatus is held in place at *A*. If now the crank *F* is turned so as to carry *q* forward and downward through *E* to *r*, the sun's meridian standing stationary, the motions which cause the differences of time and the changes of day and night may be understood, by recalling that only the half of the globe which is turned to the sun is light (in day). *SGN* denotes the *prime meridian.*

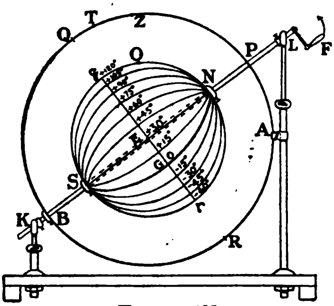


FIGURE 193

5. As the globe is standing in Fig. 193, what time is it at *O*? at places on the 105th meridian? on the 60th meridian? on the 15th meridian east* of Greenwich? on the 45th east?

6. If the meridian *SGN* is continued around on the other side of the globe, what will its number be?

7. Remembering that any place on the earth, as *O*, turns completely round through 360° in 24 hr., how long does it require

* Remember that east is the direction toward which the globe is turning, that is eastward means from *E* toward *G* through *r* and around to *E* again.

the space between any two adjacent meridians shown in Fig. 193 to pass under the sun's hour circle?

8. If a man should start from some place on the prime meridian (say London) on Friday noon and move westward just as fast as the globe turns eastward, what time (by the sun) would it be to him during his journey all the way round the globe? What hour and day would a Londoner call it when the traveler returned 24 hr. later?

The problem raises the question, "Where should the traveler have changed his date so that his date might agree with that of his starting place when he returns?" The answer is, "It has been agreed that the date should change at the 180th meridian." When vessels cross this meridian from the east toward the west they add a day to their reckoning. If they cross at noon on Friday, Friday noon instantly becomes Saturday noon. Crossing from the west toward the east, they repeat a day. In the case mentioned, Friday would "be done over again." The 180th meridian is for this reason called the *Date Line*. Trace it round the earth in your Geography.

9. The date begins on the 180th meridian of longitude and travels *westward* around the earth. Supposing the day begins at midnight, give the date and hour on the 180th meridian, when it is Wednesday, Oct. 15, 10 P. M., standard time, in Chicago; in Washington; in Denver; in San Francisco.

10. The U. S. Supreme Court has decided that *legal time* at any place is the *local time* of the place. A man in Akron, O., insured his house at 11:30 A. M. "standard" (90th meridian) time and the house took fire at that moment and was destroyed. The policy stated that the insurance would be in force at and after 12 o'clock (noon) of the day of insuring. The difference between Akron local time and 90th meridian time is 33 minutes. Would the law require the insurance company to pay the policy? Give reason for your answer.

11. If every place on the earth has the time of that meridian of the map, p. 296, nearest to it, what will be the time of the following places when it is 3:25 P. M. in England? (1) Washington? (2) Boston? (3) New Orleans? (4) Denver? (5) Rio Janeiro? (6) Cuba? (7) Japan? (8) Philippine Islands?

MENSURATION

§184. Areas, Roofing and Brickwork.

1. Give the rule for computing the area of a square from its measured sides; a rectangle; a parallelogram; a triangle. (See pp. 102-104 and 179.)

Fig. 194 represents a plot of the streets and blocks of a part of a certain city. The streets are all 75 ft. wide between sidewalks. The numbers written on the lines indicate their lengths in feet. The dotted lines show how to divide up the areas into parts for computation.

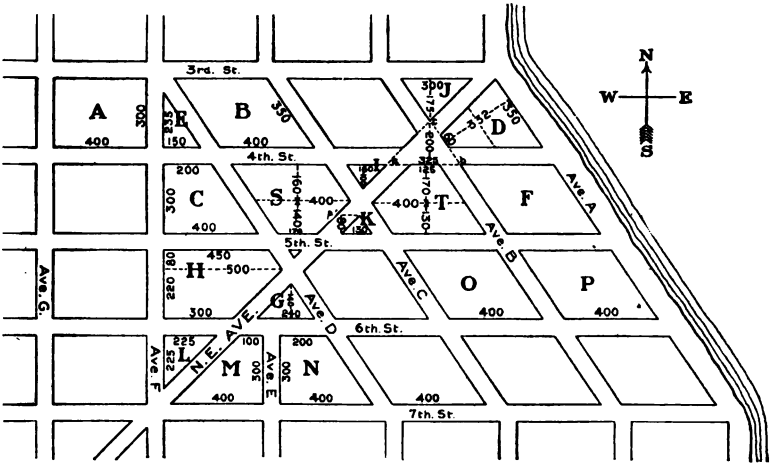


FIGURE 194

2. If $240 per foot of frontage on both 3rd and 4th streets was paid for block B (Fig. 194), how much did the block cost per square foot? per square yard?

3. Find the area in square yards of other parallelograms, such as O, P, etc., of Fig. 194.

4. Find the area in square feet of the following triangles of Fig. 194: E; G; I; J; K; L; *abc*.

Other areas of Fig. 194 require a knowledge of trapezoids.

FIGURE 195

DEFINITIONS.—A four-sided figure (a quadrilateral) like Fig. 195, having one pair of parallel sides, is called a *trapezoid.*

The altitude of a *trapezoid* is the distance square across between the two parallel sides (called the *bases.*)

5. Study the trapezoids of Fig. 196, and find how to get the length of EF, the line connecting the middle points of the two non-parallel sides, from the lengths of the two bases. After computing EF, find for each trapezoid a rectangle whose area equals the area of the trapezoid. Find the areas of the trapezoids of Fig. 196.

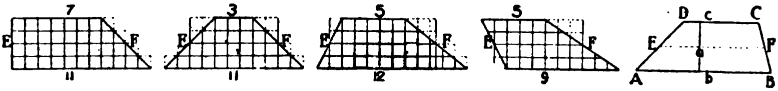


FIGURE 196

DEFINITION.—The lengths a, b, and c of the last trapezoid are called its *dimensions*.

NOTE.—$\frac{1}{2}$ the sum of x and y is written $\frac{1}{2}(x+y)$. n times the half sum is written $\frac{1}{2}n(x+y)$.

6. How do the altitude and the sum of the bases of a trapezoid compare with the altitude and base of a parallelogram having an area equal to the area of the trapezoid (Fig. 197)?

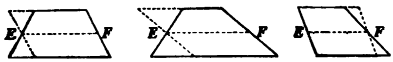


FIGURE 197

7. Supposing a, b, and c are the dimensions, in feet, of the trapezoids of Fig. 197, what are the areas of these trapezoids?

8. Find the areas in square feet of C; of D; of M; of N; of H (Fig. 194).

9. Calling Z the area of any trapezoid, whose bases are b and c and whose altitude is a, write an equation showing how to find Z from a, b, and c.

10. Find the area in square yards of A, B, N, S, T, Fig. 194.

DEFINITION.—A *square* of roofing means a 10′ square of roof surface, or 100 sq. ft. A shingle is said to be laid 4″, $4\frac{1}{2}$″, or 5″ *to the weather* when the lower end of each course of shingles on the roof extends 4″, $4\frac{1}{2}$″, or 5″ below the course next above it.

11. Draw two perpendicular center lines, and with the aid of the dimensions given in the left part of Fig. 198, complete an enlarged drawing to a convenient scale, of the development of the roof (shown on the right).

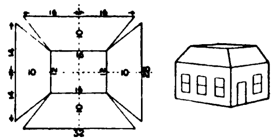


FIGURE 198

12. If 1000 shingles laid 4″ to the weather cover a square of roofing, how many shingles will be needed to cover a square, if laid 5″ to the weather? $4\frac{1}{2}$″ to the weather? 3″ to the weather? $3\frac{1}{2}$″?

13. The dimensions on the development (Fig. 198) being in feet, find the cost of the shingles, at $1.10 a bunch of 250, needed to cover the four sides of the deck roof (Fig. 198). Find the cost of enough tin to cover the 12′ × 16′ flat deck at 15¢ a 20″×28″ sheet, the long sides of the sheets being laid parallel to the long side of the deck, and allowing 10% loss for joints and overlap at edges.

14. When shingles are laid 4″ to the weather, 1000 shingles are estimated to cover a square (of roofing). Find the number of shingles, laid 4″ to the weather, needed to cover a roof which, with gables, is made up of the following parts:

SHAPES	ALTITUDES	BASES	SHAPES	ALTITUDES	BASES
2 rectangles	10′	40′	4 triangles	$8\frac{1}{2}$′	17′
2 parallelograms	12′	$26\frac{1}{2}$′	2 trapezoids	12	12′ and $20\frac{1}{2}$′
2 parallelograms	12′	14′	2 trapezoids	12′	5′ and $13\frac{1}{2}$′
2 triangles	7′	14′	8 trapezoids	12′	6′ and $14\frac{3}{4}$′

15. Find the cost of the shingles of Prob. 14 @ 90c a bunch of 250.

16. Ten years after building this house a new roof was put on it. It cost $1.25 per M. to remove the old shingles, $3 per M. to dip the new ones, and $2.75 per M. for the labor of putting on the shingles. How much did it cost to remove the old shingles and to re-roof with dipped shingles?

17. If it takes 14 bricks per square foot of outside surface to lay a 2-brick wall, how many bricks will be needed to lay the side wall of the building shown in Fig. 199, deducting for 5 windows each $3\frac{1}{2}' \times 6\frac{1}{2}'$ and for 5 windows each $3\frac{1}{2}' \times 9\frac{1}{4}'$.

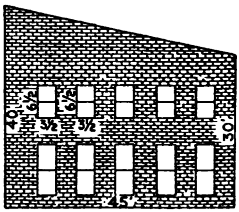


FIGURE 199

18. Make other similar problems from your own measurements or from dimensions obtained from an architect or builder.

19. The walls of a square brick house extend 6′ into the ground and rise 24′ above the ground to the eaves. From the bottom to 12′ above the ground the walls are 3 bricks thick and from this line to the eaves they are 2 bricks thick. Find the cost of laying the bricks of these walls @ \$4.25 per M.

NOTE.—No allowance for openings of any kind is made in figuring the *cost of laying* bricks in such walls, and outside dimensions are used, thus counting corners twice.

20. From the number of bricks found in problem 21 deduct for the following openings:

DOWNSTAIRS	UPSTAIRS
1 door 3′ 6″ × 6′ 9″;	2 doors 3′ 6″ × 6′ 6″:
1 door 3′ × 6′ 6″;	1 window 5′ × 6′ 6″;
2 windows 5′ × 6′ 6″;	12 windows 3′ × 6′;
9 windows 3′ × 6′;	

and 1400 bricks for double counting of corners, and find the cost of the bricks @ \$12½ per M.

21. My friend's house has two octagonal (8-sided) towers, the roofs being made up of equal triangles (Fig. 200). Find the number of slates for both roofs, 240 to the square (100 sq. ft.), and cost @ \$2.75 a hundred.

22. Draw the development of the roof and tower to a scale ¼″ to 1′, as shown in the figure.

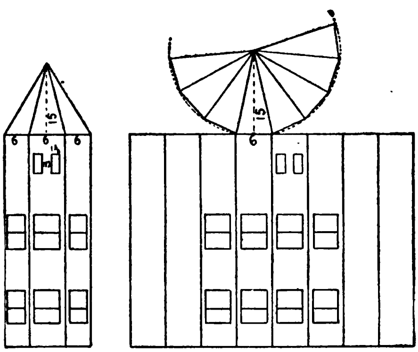


FIGURE 200

23. From the eaves to the ground the towers are octagonal prisms, built of brick, all faces being rectangles 6′ × 40′, and each prism has four faces exposed to the weather. How many square feet of brick surface are in the 8 exposed faces, not allowing for windows? Find the cost of laying the brick @ \$6.25 per M.

24. Deduct for 16 windows, each 3′ 6″ × 5′ 8″, and for 4 windows 1′ 6″ × 3′, and find the number of bricks in the two layers, and their cost @ \$14 per M.

§185. Land Measure.

Review §78, pp. 119 and 120.

The law requires land to be marked out or surveyed in divisions of the form of squares and rectangles. In the western states the land has been surveyed in accordance with this law.

To mark out the largest squares, north and south lines, called *meridians*, are first run 24 mi. apart and marked with corner-stones, or by trees, or other permanent objects. East and west lines, called *base* lines, are then run at right angles to these meridians at distances 24 mi. apart. This would divide the land up into 24-mi. squares were it not for the convergence of the meridians toward the poles of the earth. Notice this on a map in your Geography and on the map of Fig. 192.

Each 24-mi. tract is then divided into 16 nearly equal squares, called townships, by running north and south, and east and west lines through the quarter points of the sides of the large tract.

How long is a township? how wide? how many square miles does it contain?

Certain meridians, called *principal meridians*, are run with great care, and these principal meridians govern the surveys of lands lying along them for considerable distances both toward the east and toward the west. The tiers, or rows, of townships running north and south along the principal meridians are called *ranges*. The first tier on the east is called range No. 1 east, and is written R1E; the second range is No. 2 east, written R2E, and so on.

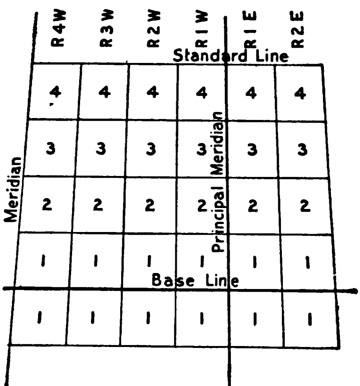


FIGURE 201

Point out on the drawing, Fig. 201, R1W; R2W; R3W; R2E; R4W.

Certain base lines are run with great care and are called *standard base* lines. The rows of townships running east and west are numbered with reference to these standard base lines. A township in the first row north of a base line is township No. 1 north, and is written T1N; one in the second row south is called township No. 2 south, written T2S, and so on.

Interpret the following symbols and point out on the drawing, Fig. 201, the townships indicated: T3N; T4N; T1S; T2N.

A township is identified by giving its number and range from some standard base and principal meridian.

Point out these townships on the drawing, Fig. 201: T1N, R2W; T3N, R1W; T4N, R2E; T1S, R3W.

The law also requires townships to be subdivided into smaller squares, called *sections*. Sections are numbered, beginning at the northeast corner and running toward the west to 6, then 7 is just south of 6, and so on, as in Fig. 202.

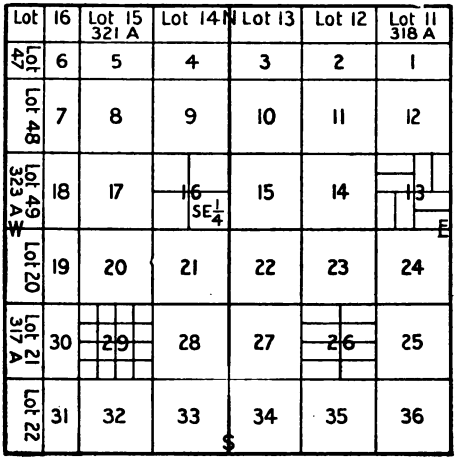


FIGURE 202

Sections are then subdivided into quarters (see section 16), half-quarters (see sections 13 and 26), and quarter-quarters (see section 29).

1. Referring to Fig. 202, read and write the descriptions of the divisions of section 16; of section 26; of 13; of 29.

Whatever deviations there may be from exactly 640 A., in the sections of any township, due to convergence of meridians or other causes, are required by law to be added to, or subtracted from, the north and west rows of half-sections. These tracts are then not called fractional sections, but are called *lots*, and are numbered in regular order as shown in Fig. 202.

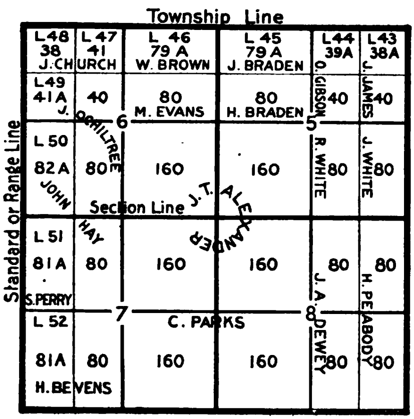


FIGURE 203

A section then always means exactly 640 A. Any fractional part of a section means the corresponding fractional part of 640 A. The lot, on the contrary, must always be measured before its area is known.

2. Following is an assessment list of farm property. Fill the blanks from Fig. 203:

Owner	Description	No. Acres
H. Peabody............	E½ SE¼ Sec. 8	....
H. Peabody............	E½ NE¼ Sec. 8	....
J. White..............	E½ SE¼ Sec. 5	....
J. James..............	SE¼ NE¼ Sec. 5	....
J. James..............	Lot 43	....
O. Gibson.............	Lot 44	....
O. Gibson.............	SW¼ NE¼ Sec. 5	....

3. Make out an assessment list for all owners in Sec. 6.

4. Make and solve a similar problem for owners in Sec. 7.

5. Correct the mistakes in this erroneous assessment list:

Owner	Description	No. Acres
S. Perry..............	W½ NW¼ Sec. 7	81
J. Hay................	W½ SW¼ Sec. 6	82
J. Hay................	W½ SW¼ Sec. 7	80
H. Ochiltree...........	SW¼ NW¼ Sec. 6	41
H. Ochiltree...........	SE¼ NE¼ Sec. 6	40
H. Ochiltree...........	E½ SE¼ Sec. 6	80
J. Church.............	NW¼ NW¼ Sec. 6	38
J. Church.............	NE¼ NW¼ Sec. 6	41

§186. Volumes.

DEFINITION.—The volume of any figure is the number of cubical units within its bounding surfaces.

1. Give the rule for finding the volume of a square prism (called also a *rectangular parallelepiped*). (See pp. 112, 113.)

2. Give a rule for finding the volume of an oblique parallelepiped (Fig. 148, p. 277) having the same base and the same altitude as a given rectangular parallelepiped. (See Figs. 150 and 151, pp. 277 and 278.)

3. How would you find the volume of a hollow beam 12 ft. long, having a cross section like (1) Fig. 65, p. 132? (2)? (3)? (5)? (6)? (4)? (9)?

4. Give the volumes of the beams of problem 3, if the length of the beam in each case is l feet.

5. A model of a square prism (like Fig. 145) having a base 2 in. square and an altitude of 8 in. will contain how many cubic inches of sand?

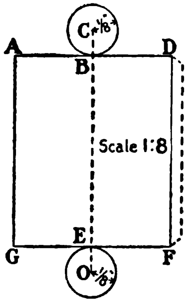


FIGURE 204

6. The model of a right circular cylinder, made as shown in the scale drawing of Fig. 204 and pasted along the flap, DF, and around one end with a strip of paper, was filled with sand and poured into the empty model of the square prism of problem 5. It filled the square prism a little more than $\frac{3}{4}$ full. About how many cubic inches are there in the model of the cylinder?

7. It is shown in geometry that the volume of any right circular cylinder is .7854 ($= \frac{\pi}{4}$) times the volume of a square prism of the same altitude and having for one side of its base the diameter of the cylinder. Find the volumes of these circular cylinders:

(1) Diameter	4″,	altitude 6″	(4) Diameter	$18\frac{1}{2}$″,	altitude 8″
(2) "	7″	" 7″	(5) "	6′	" 10.5′
(3) "	10″	" $8\frac{1}{2}$″	(6) "	$2r$	" a

8. How long is AB? How many square inches in the rectangle $ADFG$ (Fig. 204)? How many square inches are there in the entire outside surface of the cylinder?

9. The inside diameter of the cylindrical water tank of a street sprinkler is 3.6 ft., and its length is 10 ft.; how many liquid gallons does the tank hold?

10. The tank of the sprinkler is filled through a hose of $2\frac{1}{2}$″ inside diameter from a hydrant from which the water flows at the rate of 300 linear feet per minute. How long will it take to fill the tank?

SUGGESTION.—In 1 min. a cylinder of water as large as the inside of the hose and 300′ long flows into the tank.

11. Find the weight of an iron rod $\frac{1}{2}$″ in diameter and 24′ long, if iron weighs 450 lb. per cubic foot?

12. How many cubic inches of air are there in the hollow tire of a bicycle wheel 28″ in diameter (from center line to center line of tubes), the hollow having a diameter of $1\frac{3}{4}''$?

13. How many cubic inches of rubber are there in the hollow tire of a 32″ automobile wheel if the inside diameter of the cylindrical tube is 3″ and the outside diameter is 4 inches?

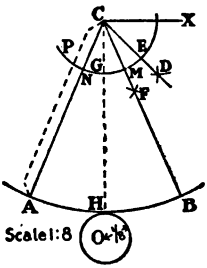


FIGURE 205

14. Fig. 205 is a scale drawing of a pattern for the paper model of a cone whose base is to be a circle 2″ in diameter and whose sloping side from the apex to the base is to be 8″. The arc *AHB* is just as long as the circumference of the base. Compute the length of the circumference of a circle whose radius is 2″, and of another whose radius is 8″, and find the ratio of the first to the second.

15. What part of the whole circumference (with center *C*) is the arc *AHB*? What part of 360° is the angle *ACB*?

16. Make a right angle, as *HCX*, and bisect it (See Problem II, p. 176). *CD* is the bisector. Bisect angle *HCD* and obtain *CB*.

17. With *C* as center and with a convenient radius, as *CE*, draw the indefinite arc *PNME*. Put the pin-foot on *G* and mark an arc across arc *GP* at *N*. Draw *CN* and prolong it. This makes angle *GCN* equal to angle *GCM*. Now with *C* as a center and with 8″ as a radius, draw the arc *AHB*. Prolong *CH*, make $HO = 1''$, and draw the lower circle. Provide the flap and paste up the model of the cone.

If such a model is carefully made and the height is measured and if the model of a cylinder has the same height and the same base, the model of the cone will be found to hold $\frac{1}{3}$ as much sand as does the model of the cylinder. It is proved in geometry that the volume of any cone equals $\frac{1}{3}$ of the volume of a cylinder having an equal base and an equal altitude.

18. Find the volumes of these circular cones:

(1)	Diameter of base	3″,	altitude,	7″
(2)	“ “	6″	“	20″
(3)	“ “	6.8″	“	2.25′
(4)	“ “	3.95″	“	10.25″

19. How many cubic inches of water are there in a funnel-shaped vessel, if the water is 4″ deep and the diameter of the surface of the water is 4.5″? (See Fig. 206).

20. How many cubic inches of water will it take to fill the same vessel to twice the depth, or 8″? (See Fig. 206.)

21. How many cubic inches had to be poured into the vessel to fill it to 8″ depth if the depth of the water was 4″ at the beginning?

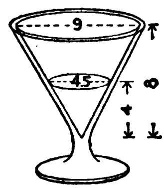


FIGURE 206

22. A mountain peak has the shape of a circular cone whose altitude is 1.25 mi. and the diameter of whose base is 2.35 miles. Find its volume in cubic miles.

23. How much water will it take to fill a conical vase 10″ deep and 3.225″ across at the top.

24. The conical tower of a building is 24.75′ across at the base and 36.8′ high. How many cubic feet of space does it occupy?

25. If a cord or waxed tape (bicycle repair tape) be wrapped around the curved outside surface of a *half* croquet ball, as a top is wound, until the surface is covered, and then if the same cord, or tape, be wrapped around on the flat circular base beginning at the center until the circle is covered, the length of the former cord will be found to be just twice the latter. The area of the surface of the *whole* ball is then how many times the area of the circular section of the ball? What radius has this circular section?

It is proved in geometry that the area of the surface of a sphere equals 4 times the area of a circular section going through the center of the sphere.

26. The radius of a ball is $1\frac{1}{2}''$; what is the area of the greatest circular section of the ball? What is the area of the surface of the ball?

27. Measure the circumference of a baseball and find how many square inches of leather there are in the cover of the ball.

28. How many square inches of paint will it take to cover the surface of a globe of 10″ radius? of 10″ diameter?

29. The average diameter of the earth is 7918 mi.; how many square miles are there in its surface?

30. Calling s the surface and r the radius of a sphere, write an equation showing the relation between s and r.

It is seen on p. 277 that the volume of a triangular right prism equals $\frac{1}{2}$ the volume of a square prism whose base and altitude are equal to the base and the altitude of the square prism.

31. Find the volume of a right triangular prism whose altitude is 18″ and whose base is a triangle having a base of 8″ and an altitude of 5″ inches.

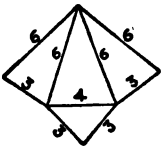


FIGURE 207

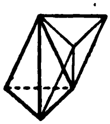


FIGURE 208

If a triangular pyramid be carefully modeled (Fig. 207) and filled with sand it will be found that just 3 times the volume of the pyramid is equal to the volume of the model of a triangular prism (see Fig. 208) of equal base and equal altitude. It is proved in geometry that the volume of any pyramid equals $\frac{1}{3}$ of the volume of a prism having an equal base and an equal altitude. Notice in Fig. 208 how a triangular prism may be completed on a triangular pyramid having the same base and the same altitude as the prism.

32. Calling V the volume, B the area of the base, and a the altitude of a triangular pyramid, write an equation, showing the way V would be computed from B and a.

33. Find the volume of a pyramid whose base contains 16 sq. in. and whose altitude is 12 inches.

34. The Great Pyramid of Egypt is 481 ft. high and its base is a 756′ square. If it were solid and had smooth faces, how many cubic feet of masonry would it contain?

35. At the close of the nineteenth century the United States had 195,887 mi. of railroad. How many times would these railroads, if placed end to end, encircle the earth? (Use $\pi = 3\frac{1}{7}$, and the radius of the earth = 3959 miles).

36. The ties used for these roads would contain wood enough to make a pyramid of the same shape 1395′ high with a 2192′ square for its base. How many cubic feet of wood were used for the railroad ties?

37. It has been computed that the materials used for the road beds for these railroads would make a solid pyramid 2470 ft. high

and having a 3870 ft. square for its base. How many cubic feet would this make?

If the entire surface of a globe, or sphere, were divided up into small triangles like the one shown in Fig. 209, and the sphere were cut up by planes cutting along the curved sides of the triangles and passing through the center, O, the volume of the sphere would be divided up *approximately* into small triangular pyramids, having their vertices at the center. The volume of each pyramid would be the product of its triangular base by $\frac{1}{3}$ of the the radius of the sphere. The sum of the areas of all triangular pyramids would equal the surface of the whole sphere and the sum of all the volumes of the pyramids would equal the surface of the whole sphere multiplied by $\frac{1}{3}$ of the radius of the sphere.

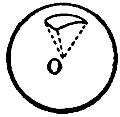


FIGURE 209

38. Calling V the volume and r the radius of a sphere, give the meaning of the formula:

$$V = \frac{r}{3} \times 4\pi r^2 = \frac{4}{3}\pi r^3.$$

39. Find the number of cubic inches in a sphere of 2″ radius.

40. How many cubic inches in a croquet ball 4″ in diameter? in a tennis ball 1.75″ in diameter? in a baseball 2.1″ in diameter? in a globe 10.15″ in diameter?

41. Calling the sun, the moon and the planets all spheres with diameters in miles as in the following table, compute their circumferences in miles, their surfaces in square miles and their volumes in cubic miles:

Moon 2,160;	Mars 4,230;	Uranus 31,900;
Mercury 3,030;	Jupiter 86,500;	Neptune 34,800;
Venus. 7,700;	Saturn 73,000;	Sun . . . . 866,400.
Earth. 7,918;		

§187. Constructive Geometry.

PROBLEM I.—To find the center of a given arc.

EXPLANATION.—Let AB, Fig. 210, be the given arc whose center is to be found.

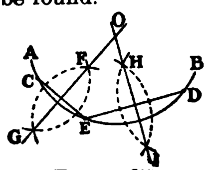


FIGURE 210

Mark any three points, as C, E, and D, on the arc. Draw the straight lines CE and ED. Bisect each of these lines as in Problem VI, pp. 100 and 101, and prolong these bisectors until they intersect as at O. O is the required center.

DEFINITION.—The lines CE and ED, each of which connects two points of the arc, are called *chords* of the arc.

To solve this problem is it necessary actually to draw the chords?

How could you find the center of a circle that would go through any three points not in a straight line? Mark 3 points not all in the same straight line and draw a circle through them.

PROBLEM II.—To bisect a given arc.

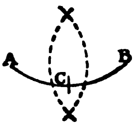


FIGURE 211

EXPLANATION.—Let AB, Fig. 211, be the given arc. With A as a center and then with B as a center and with a radius greater than the distance from A to the middle of the arc AB, draw the dotted arcs as indicated. Lay a ruler on the intersections of the two arcs and draw a short line across the arc, as at C. C is the mid-point of the arc, and arc $AC =$ arc CB.

PROBLEM III.—To circumscribe a circle around an equilateral triangle.

EXPLANATION.—Draw an equilateral triangle as in Problem VII, p. 102, and bisect two of its angles as shown in Fig. 212. With the intersection of the bisectors as a center and with a radius equal to the distance from this intersection to any vertex, draw a circle. This circle is said to be circumscribed around the triangle.

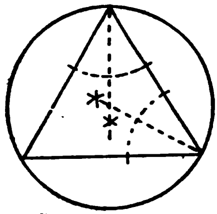


FIGURE 212

PROBLEM IV.—To draw a trefoil.

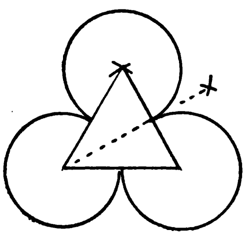


FIGURE 213

EXPLANATION.—Draw an equilateral triangle and bisect one of its sides, as shown in Fig. 213. With the upper vertex as center and with a radius equal to the distance from this vertex to the middle of the bisected side draw an arc of a circle around until it touches the sides of the triangle both ways. Draw arcs around the other vertices in the same way.

If desired, other circles, with slightly longer radii, may be drawn just outside of these until the arcs come together but do not cross.

PROBLEM V.—To construct a square and to draw a quatrefoil (a four-foil) upon it.

EXPLANATION.—Prolong BA through A far enough to draw a perpendicular, as AC, to AB at A. Draw this perpendicular and make $AC = AB$ With AB as a radius and (1) with C as a center, then (2) with B as a center, draw two intersecting arcs at D. Draw CD and BD and complete the drawing as shown in Fig. 214.

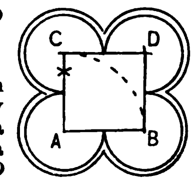


FIGURE 214

PROBLEM VI.—To draw the designs of Fig. 215.

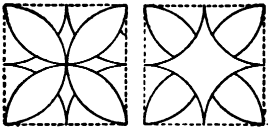


FIGURE 215

EXPLANATION.—Draw a square like the dotted squares of Fig. 215. Study the two drawings, decide where the centers of the arcs are and complete the designs.

PROBLEM VII.—To draw a sixfoil.

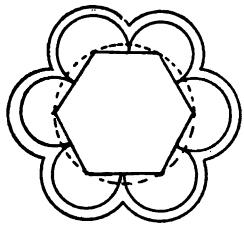


FIGURE 216

EXPLANATION.—Draw a circle, like the dotted one in Fig. 216, with a radius as long as one side of the regular hexagon is to be. Draw the hexagon (see Problem XI, p. 104). Bisect a side of the hexagon, and, using the vertices of the hexagon as centers, complete the sixfoil, as shown.

Outside arcs may be added if desired.

PROBLEM VIII.—To construct a right triangle.

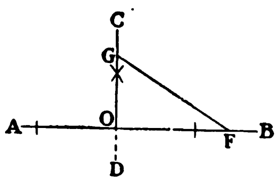


FIGURE 217

The symbol ⊥ means "perpendicular," "perpendicular to," or "is perpendicular to."

CONSTRUCTION.—Draw the line $CD \perp AB$, Fig. 217. Then draw the line GF connecting any point, as G, of the line CD with any point of AB, as F.

DEFINITIONS.—The longest side, that is, the side opposite the right angle of a right triangle, is called the *hypothenuse.*

What side of the triangle, GOF, is the hypothenuse? What angle is the right angle?

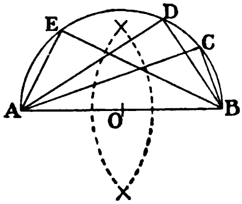


FIGURE 218

PROBLEM IX.—To construct a right triangle having a given hypothenuse.

CONSTRUCTION.—Let the line AB, Fig. 218, denote the given hypothenuse. Bisect AB as at O and with O as a center and OA as a radius, draw a semicircle. Connect any point of the semi-circumference, as E, D, or C, with A and with B. Any such triangle is a right triangle and the line AB is its hypothenuse.

Point out the right angle of each of the three triangles of Fig. 218.

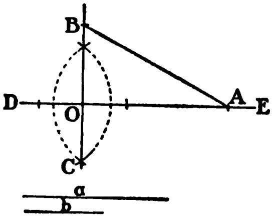


FIGURE 219

PROBLEM X.—To construct a right triangle, having given the two sides which include the right angle.

CONSTRUCTION.—Let the two given sides be a and b, Fig. 219.

Draw $BC \perp DE$, and make $OA = a$ and $OB = b$. Connect A with B. BOA is the required triangle.

How may an isosceles right triangle be constructed?

PROBLEM XI.—To find the relation of the squares of the sides of an isosceles right triangle.

CONSTRUCTION.—Construct an isosceles right triangle and on each of its three sides draw a square. Draw the dotted lines and cut the side squares as shown in Fig. 220. Fit the pieces over the large square on the hypothenuse. If the area of each side square were 9 sq. in. what would be the area of the large square?

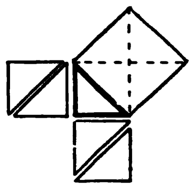


FIGURE 220

PROBLEM XII.—To find the relation of the squares of the three sides of any right triangle.

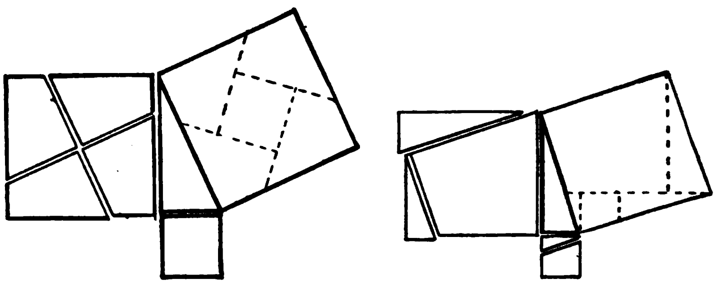


FIGURE 221

CONSTRUCTION.—Construct any right triangle and then construct a square on each of its three sides. Cut the side squares as shown.* Fit the pieces over the square drawn on the hypothenuse, as indicated by the dotted lines in Fig. 221. What single square has an area that equals the sum of the areas of the squares on the two shorter sides of any right triangle?

*The center of a square is the intersection of its diagonals.

1. Denoting the length of either short side of Fig. 220 by a, what denotes the area of the square drawn upon this side?

2. Denoting the length of the hypothenuse of Fig 220 by h, what denotes the area of the square drawn on the hypothenuse?

3. Write an equation from Fig. 220, showing the relation between a^2 and h^2.

4. Denote the lengths of the three sides of any right triangle (Fig. 221) by a, b, and h (h being the hypothenuse). What will denote the areas of each of the squares on the three sides?

5. Write an equation showing the relation of the squares of the sides of any right triangle.

PROBLEMS

1. The sides of a right triangle are 3″ and 4″; what is the length of the hypothenuse?

SUGGESTION.—$h^2 = 3^2 + 4^2 = 9 + 16 = 25$; what is the value of h?

2. The hypothenuse of a right triangle is 10″, and one of the sides is 6″; what is the other side?

SUGGESTION.—$10^2 = a^2 + 6^2$; find the value of a?

3. The sides of a right triangle are denoted by a, b, and the hypothenuse by h; find the unknown side in each of the following right triangles:

(1) $a = 9$, $b = 12$;	(4) $a = 32$, $b = 24$;
(2) $a = 12$, $h = 20$;	(5) $b = 21$, $h = 35$;
(3) $b = 15$, $h = 25$;	(6) $b = 27$, $h = 45$.

Before the hypothenuse of a right triangle, whose sides are 34 and 26, can be computed, it is necessary to know how to find the square roots of given numbers.

§188. Squares and Square Roots.

DEFINITION.—The product obtained by using any number *twice as a factor* is called the *square* of that number Thus, 36 is the square of 6, because 6, used twice as a factor gives 36 ($6 \times 6 = 36$). The square of a number as 6 is often written thus, 6^2. What does the small 2 show?

The following squares should be committed to memory:

SQUARES OF UNITS	SQUARES OF TENS	SQUARES OF HUNDREDS
$1^2 = 1$;	$10^2 = 100$;	$100^2 = 10000$;
$2^2 = 4$;	$20^2 = 400$;	$200^2 = 40000$;
$3^2 = 9$;	$30^2 = 900$;	$300^2 = 90000$;
$4^2 = 16$;	$40^2 = 1600$;	$400^2 = 160000$;
$5^2 = 25$;	$50^2 = 2500$;	$500^2 = 250000$;
$6^2 = 36$;	$60^2 = 3600$;	$600^2 = 360000$;
$7^2 = 49$;	$70^2 = 4900$;	$700^2 = 490000$;
$8^2 = 64$;	$80^2 = 6400$;	$800^2 = 640000$;
$9^2 = 81$;	$90^2 = 8100$;	$900^2 = 810000$.

1. Write all the pairs of numbers which, multiplied together, give the product 36; the product 16; the product 64; the product 49.

NOTE.—Write the pairs of factors of 36 thus: 1 and 36; 2 and 18; 3 and 12; 4 and 9; 6 and 6. Proceed similarly with the rest.

DEFINITION.—The square root of a number is *one of the two equal factors of it.* The sign of square root is $\surd$, called the *radical sign.* Thus, $\sqrt{25}$ means the square root of 25, which is 5.

2. Give the square roots of the following numbers:

9; 16; 49; 64; 81; 400; 2500; 3600; 160000; 490000; 810000.

To find the square root of a number not in the table above, it is necessary first to learn how the square of a number is formed from the number.

From the table answer the following questions:

3. How many digits are there in the square of any number of units? of tens? of hundreds?

4. Find the square of each of the following decimals: .1, .3, .5, .7, .9, .01, .03, .04, .06, .07, .09, .001, .003, .005, .006, .007, .009.

5. How many decimal places are there in the square of any number of tenths? of hundredths? of thousandths?

6. If, then, the square of any number of units contains only units and tens, what places of any number that is a square must contain the square of the units of its square root?

7. What places must contain the square of the tens? of the hundreds? of the tenths? of the hundredths? of the thousandths?

8. If, then, we separate a number, whose square root is desired, into two-digit groups, beginning at the decimal point and proceeding both toward the left and toward the right, what one of these groups must contain the square of the units of the square root? the square of the tens? of the hundreds? of the tenths? of the hundredths?

9 Find the square of 46.

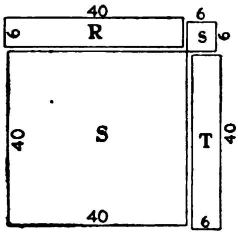


FIGURE 222

COMMON METHOD

$$\begin{array}{r} 46 \\ \underline{46} \\ 276 \\ \underline{184} \\ 2116 \end{array}$$

$46^2 = 2116.$

MEANING OF COMMON METHOD

$46 = 40 + 6;$

$46^2 = (40 + 6)^2 = (40 + 6)(40 + 6)$
$= 40 \times 40 + 40 \times 6 + 6 \times 40 + 6 \times 6$
$= 40^2 + 2 \times (40 \times 6) + 6^2 = 1600 + 480 + 36 = 2116.$

Show the meaning of the parts of this sum in Fig. 222.

Thus it is seen that the square of a two-figure number is the sum of (1) the square of the tens, (2) twice the product of the tens by the units, and (3) the square of the units.

This shows that if any two-figure number be denoted by $t + u$, where t denotes the tens and u the units, the square is formed thus:

$$\begin{array}{r} 40+6 \\ \underline{40+6} \\ 240+36 \\ \underline{1600+240} \\ 1600+480+36 \end{array} \qquad \begin{array}{r} t+u \\ \underline{t+u} \\ tu+u^2 \\ \underline{t^2+tu} \\ t^2+2tu+u^2 = (t+u)^2 \end{array}$$

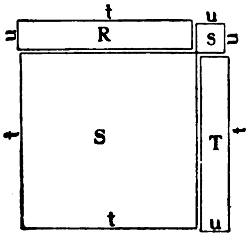


FIGURE 223

Show the meaning of the equation by Fig. 223.

10. Find the square root of 2116.

CONVENIENT FORM

$$\begin{array}{rrl} & \underline{46} & = \text{required root;} \\ & \overline{2116} & \text{denoted by } t^2 + 2tu + u^2; \\ & \underline{1600} & = \text{greatest square (of the table) in 2116;} \\ 2t = 80 & 516 & \text{contains } 2tu + u^2, \text{ where } t = 40; \\ 2t + u = 86 & \underline{516} & \text{denoted by } 2tu + u^2, \text{ where } u = 6; \end{array}$$

Check: $46 \times 46 = 2116.$

SHORTENED FORM

$$\begin{array}{rl} & 21'16 \mid \underline{46} = \text{required root} \\ & \underline{16} \\ 80 & \\ \underline{6} & \\ 86 & 5'16 \\ & \underline{516} \end{array}$$

Check: $46 \times 46 = 2116.$

11 Find the square roots of the following numbers:

(1) 484; (3) 1156; (5) 1296; (7) 7569; (9) 9604;
(2) 729; (4) 1225; (6) 5625; (8) 9409; (10) 110224.

12. Find the square root of 2079.36.

CONVENIENT FORM

	20′79.36′	45.6
	16′00	
80	4′79.	
5		
85	4′25.	
90.	54.36	
.6		
90.6	54.36	
	0	

Ans. $\sqrt{2079.36} = 45.6$

Check: $45.6 \times 45.6 = 2079.36$.

EXPLANATION.—Separate the number into two-digit groups. Beginning on the extreme left, subtract the greatest square of tens in 20 hundreds, viz., 16 hundreds ($=40^2$), and write 4 tens on the right as the first root digit. Double the 4 tens and use the result 80 as a *trial* divisor. 80 is contained in the remainder, 479, 5 times. Write 5 as the 2d root digit and also add it to the 80, giving 85 as the *complete* divisor. Double the part of the root found, 45, giving 90 for the next *trial* divisor and complete the steps as before.

13. Find the square root of 1578 to 3 decimal places.

CONVENIENT FORM

	15′78.00′00′00′	39.724 +
	9	
60		
9	678.	
69	621.	
78.0	57.00	
.7		
78.7	55.09	
79.40	1.9100	
.02		
79.42	1.5884	
79.440	.321600	
.004		
79.444	.317776	
	.003824 remainder.	

EXPLANATION.—Annex zeros and proceed as before.

Check: **$39.724 \times 39.724 = 1577.996176$.**
rem. $=$ .003824.
1578.

In actual practice the decimal point is needed only in the root.

14. Find the square roots of the following numbers:

(1) 1900.96; (4) 61.1524; (7) .5476;
(2) 4719.69; (5) 75.8641; (8) .458329;
(3) 5055.21; (6) 79.9236; (9) 1.216609.

15. Find to 3 decimal places the square roots of the following numbers:

(1) 5;	(4) .85;	(7) 1683;	(10) 1.85;
(2) 7;	(5) .25;	(8) 6875;	(11) 26.79;
(3) 15;	(6) 125;	(9) 7328;	(12) 64.893.

16. Find by multiplication the values of the following expressions:

(1) $(\frac{1}{2})^2$;	(3) $(\frac{4}{5})^2$;	(5) $(\frac{8}{15})^2$;	(7) $(\frac{a}{b})^2$;
(2) $(\frac{2}{3})^2$;	(4) $(\frac{12}{13})^2$;	(6) $(\frac{21}{32})^2$;	(8) $(\frac{x}{y})^2$.

17. Make a rule for finding the square root of a common fraction.

18. Find, without reducing the common fractions to decimals, the values of the following expressions and prove them by multiplication:

(1) $\sqrt{\frac{1}{4}}$;	(4) $\sqrt{\frac{144}{169}}$;	(7) $\sqrt{\frac{81}{289}}$;
(2) $\sqrt{\frac{4}{9}}$;	(5) $\sqrt{\frac{64}{121}}$;	(8) $\sqrt{\frac{400}{729}}$;
(3) $\sqrt{\frac{16}{25}}$;	(6) $\sqrt{\frac{49}{144}}$;	(9) $\sqrt{\frac{a^2}{b^2}}$.

19. The sides of a right triangle are respectively 34″ and 26″ long, how long is the hypothenuse?

20. The center pole of a circus tent is 35′ high, and a guy rope is stretched from the top of the pole to a stake 56′ from the bottom. How long is the rope, supposing the ground level and the rope straight, allowing 4′ for tying?

21. 50′ of the top of a tree standing on level ground is broken by the wind and remains fastened to the stump. If the top strikes the ground 30′ from the stump, how high was the tree?

22. A horse is staked out by a rope 40′ long to the top of a stake 15″ high. Over what area can the horse graze?

23. The vertical mast of a hoisting derrick 35′ high is held in position by four guy ropes staked to the ground at the four corner points of a square. The stakes are 75′ from the bottom of the mast. How much will the rope for the 4 guys cost at 2.5¢ a foot, 20′ being allowed for knots?

24. What is the area of the square whose corners are at the stakes?

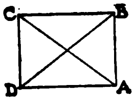


FIGURE 224

25. Find the length of the diagonals of a rectangle 48′× 64′ long.

26. A railroad runs diagonally across a rectangular farm 120 rd. × 160 rd. If each rail is as long as a diagonal of the farm, how many yards are there in the rails?

27. The center pole of a circus tent is held by 6 ropes tied to stakes at the corners of a regular hexagon (Fig. 44, p. 104). The ground is level, the pole is 48′ high and a side of the hexagon is 64′. How many feet of rope are needed, if the knots take 180′?

28. The sides of an equilateral triangle are 20′ long. A straight line drawn from any vertex to the middle of the opposite side is perpendicular to this side and bisects it. What is the area of the triangle?

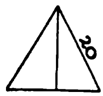


FIGURE 225

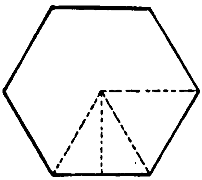


FIGURE 226

29. One side of a regular hexagon is 36′. Find the area of the hexagon?

30. A straight line drawn from the vertex of an isosceles triangle to the middle of the base is perpendicular to the base. Find the area of a regular pentagon whose sides are 24″, if the radius of the circular drawn around the pentagon is 18″.

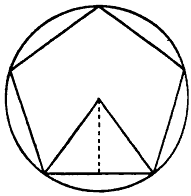


FIGURE 227

§189. Cubes and Cube Roots.

1. What is the volume of a cube whose edge is 5″? 6″? 7″? 11″? 18″? 21″? 24″?

DEFINITION.—The *cube of a number* is the product obtained by using the number *3 times as a factor*. Thus the cube of 4 is $4 \times 4 \times 4 = 4^3 = 64$.

Table of cubes of numbers to be learned:

CUBES OF UNITS	CUBES OF TENS
$1^3 = 1$;	$10^3 = 1000$;
$2^3 = 8$;	$20^3 = 8000$;
$3^3 = 27$;	$30^3 = 27000$;
$4^3 = 64$;	$40^3 = 64000$;
$5^3 = 125$;	$50^3 = 125000$;
$6^3 = 216$;	$60^3 = 216000$;
$7^3 = 343$;	$70^3 = 343000$;
$8^3 = 512$;	$80^3 = 512000$;
$9^3 = 729$;	$90^3 = 729000$.

Definition.—The cube root of a number is one of its *3 equal factors*. Cube root is indicated by the sign $\sqrt[3]{\ }$. Thus, $\sqrt[3]{729}$ means one of the three equal factors of 729, which is 9. The sign $\sqrt[3]{\ }$ is called a radical sign.

2. Between what two whole numbers are the following cube roots:

$\sqrt[3]{16}$? $\sqrt[3]{35}$? $\sqrt[3]{78}$? $\sqrt[3]{450}$? $\sqrt[3]{675}$? $\sqrt[3]{895}$? $\sqrt[3]{58000}$? $\sqrt[3]{480000}$?

3. Find by multiplication the values of the following cubes:

$(2.5)^3$; $(6.7)^3$; $(12.5)^3$; $(\frac{1}{2})^3$; $(\frac{2}{3})^3$; $(\frac{4}{5})^3$; $(\frac{7}{12})^3$; $(33\frac{1}{3})^3$; $(18\frac{3}{4})^3$.

4. Make a rule for finding the cube of any common fraction.

5. Prove the following relations by multiplication:

$$\sqrt[3]{2197} = 13;\ \sqrt[3]{4096} = 16;\ \sqrt[3]{\tfrac{125}{512}} = \tfrac{5}{8};\ \sqrt[3]{\tfrac{729}{1331}} = \tfrac{9}{11};$$
$$\sqrt[3]{\tfrac{3375}{4913}} = \tfrac{15}{17};\ \sqrt[3]{27a^3} = 3a.$$

6. Make a rule for finding the cube root of any common fraction.

7. Find the cube roots of the following:

$$\tfrac{1}{8};\ \tfrac{27}{64};\ \tfrac{125}{343};\ \tfrac{729}{8000};\ \tfrac{343}{1000};\ \tfrac{729}{125000}.$$

§190. Triangles Having the Same Shape (Similar Triangles).

1. Write the numerical values of the following ratios from Fig. 228:

(1) $\dfrac{ac}{AC}$; (2) $\dfrac{bc}{BC}$; (3) $\dfrac{ab}{AB}$; (4) $\dfrac{bc}{ac}$; (5) $\dfrac{BC}{AC}$;

(6) $\dfrac{ab}{ac}$*; (7) $\dfrac{AB}{AC}$; (8) $\dfrac{bc}{ab}$; (9) $\dfrac{BC}{AB}$.

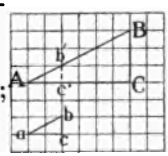


FIGURE 228

* The length of ab is $\sqrt{4+1} = \sqrt{5}$. Why? See Prob. XII, p. 320.

2. In Fig. 229 the triangles have the same shape. Write the values of the ratios:

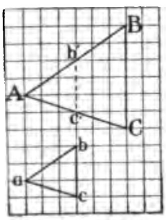


FIGURE 229

(1) $\dfrac{bc}{BC}$; (2) $\dfrac{ab}{AB}$; (3) $\dfrac{ac}{AC}$.

3. In Fig. 230 the triangles have the same shape, and $AC = 7 \times ac$. Find the sides of the triangle ABC, if $ac = \frac{7}{40}''$; $ab = \frac{1}{4}''$, and $bc = \frac{11}{40}''$.

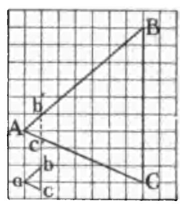


FIGURE 230

4. Supposing that *ac* ($= \frac{1}{4}$ *AC*) represents 7′ (Fig. 230), *ab*, 10′ and *bc*, 11′, what lengths do *AC*, *AB*, and *BC* represent?

5. In Figs. 231 and 232, *AB* = 4*ab*; if *AB* represents 1 mi., *BC*, 3 mi., and *AC*, $3\frac{1}{2}$ mi., what distances do *ab*, *bc*, and *ac* represent?

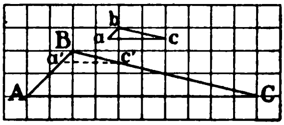


FIGURE 231

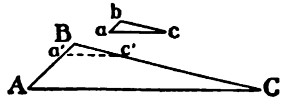


FIGURE 232

6. Draw a triangle having sides of 1″, $\frac{3}{4}$″, and $\frac{1}{2}$″, and another having sides of 3″, $2\frac{1}{4}$″, and $1\frac{1}{2}$″. Call the angles opposite the sides 1″, $\frac{3}{4}$″, and $\frac{1}{2}$″, *a*, *b*, and *c*, respectively, and the angles opposite the sides 3″, $2\frac{1}{4}$″, and $1\frac{1}{2}$″, *A*, *B*, and *C*, respectively. Do these triangles have the same shape? Carefully cut out the triangles and place angle *a* over angle *A*; then angle *b* over angle *B*; and, last, angle *c* over *C*? What do you find to be true in each case?

DEFINITION.—In triangles having the same shape, angles lying opposite *proportional sides* in different triangles are called *corresponding angles.*

7. Read the pairs of corresponding angles in Figs. 228, 229, 230, and 232.

8. In triangles having the same shape, how do corresponding angles compare in size (see Problem 6)?

9. In Fig. 228 find the ratio of the side lying opposite the angle *B* in triangle *ABC* to the side lying opposite the corresponding angle in triangle *abc*. Find a similar ratio between any other pair of such sides. How do the ratios compare?

10. Answer similar questions for the triangles of Figs. 229, 230, and 232.

DEFINITION.—In triangles having the same shape, sides, as *AB* and *ab* (Fig. 232) or *BC* and *bc*, that lie opposite the equal angles are called *corresponding sides.*

11. In triangles having the same shape state a law of relation of corresponding sides.

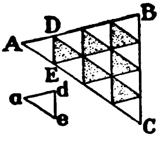


FIGURE 233

12. Triangles ABC and ade (Fig. 233) have the same shape. How do their corresponding angles compare? What relation holds for their corresponding sides, i.e., how do these ratios compare: $AB : ad$? $BC : de$? $AC : ae$? With a protractor measure each pair of corresponding angles. How do they compare in size?

13. What is the ratio of the areas of triangles ABC and ade (Fig. 233)? How may this ratio be found from the ratio of a pair of corresponding sides, as AB and ad?

14. Draw a pair of triangles, one having sides of 1″, $1\frac{1}{2}$″, and $1\frac{3}{4}$″, and the other having sides of 4″, 6″, and 7″. Give the ratio of each pair of corresponding sides; the ratio of the areas of the triangles? How may the ratio of the areas be computed from the ratio of a pair of corresponding sides?

15. State a law connecting the ratio of the areas of triangles having the same shape with the ratio of any pair of corresponding sides.

§191. Uses of Similar Triangles.

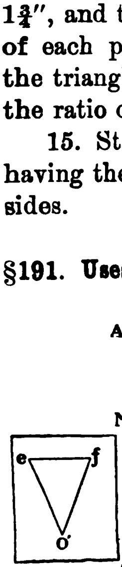


FIGURE 234

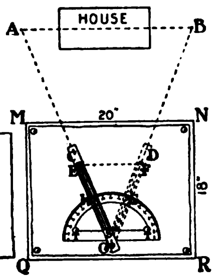


FIGURE 235

In Fig. 235 $MNQR$ represents an 18″ x 20″ board, provided with a moving arm, OE, and a protractor. The arm is made of light wood. A slot ES is cut away and a thread is stretched on its under side and fastened at the ends. The ends of the thread are sunk into the wood so as to permit the thread to move smoothly over the protractor as the arm CO is turned round O. A common pin stuck vertically at C and at O completes the apparatus. The pin at O should be driven through far enough to fasten the arm to the board at O as a pivot.. The protractor is held by thumb tacks at T, T. The board may be placed on a window sill, or on a post in a fixed position, while measurements are being made. It is desired to measure the distance from A to B, A and B being on opposite sides of a large building.

Use the proportion form of statement in all problems, calling the length of the unknown line x. Then find the value of x.

Solve some problems like this from your own measures. Buildings, steeples, hills, trees, furnish good problems.

1. The board was placed on a window sill and the arm set in such position, FO, that when the eye sighted along the pins, O and D, these pins were in line with B. The mark on the protractor at G was 52°. The board being held firmly in place, the arm was swung around O until the pins at O and at C were sighted into line with A and the reading on the protractor at H was 114°. How many degrees were there in the angle EOF?

2. The distances from A and from B to the window at O were measured and found to be 484′ and 520′. A triangle $O'ef$ (shown in Fig. 234) was drawn having angle $eO'f = 62°$, and $O'e = 4.84''$, $O'f = 5.20''$. The side ef was then drawn, measured, and found to be 5.18″. How long is the line AB?

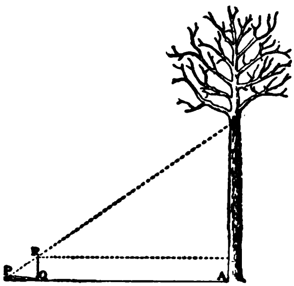


FIGURE 236

3. A woodman desires to fell only such trees as will furnish two 10-ft. cuts between the stump and the first limb. He wishes to allow 4 ft. for height of stump and waste in cutting at the bottom and top ends. To test a standing tree, he places a 4-ft. stick QR vertically in the ground 33 ft. from the tree, lies down on his back with his feet against the stake, and sights over the top of the stake to the first limb B. The distance from his eye P to the soles of his feet Q being $5\frac{1}{2}$ feet, should he fell the tree?

4. In Fig. 236, if $PQ = 6$ ft., $RQ = 4\frac{1}{2}$ ft., and $RA = 60$ ft., how long is AB?

5. How long is AB if PQ and RQ are the length, as in problem 4, and $RA = 90$ feet?

6. A woodman steps off a distance of 9 steps from a tree, faces the tree, and holds his axe-handle at arm's length in front of his eye, as the ruler is held in Fig. 125, p. 189. His arm is 28 in. long, and he finds that 2 ft. of the axe-handle just cover the distance from the ground to the first limb of the tree. If his step is 3 ft. long, how high is the first limb?

7. If $AB = 35$ rd., $AK = 80$ rd., and $BC = 56$ rd., how long is KD (Fig. 237)?

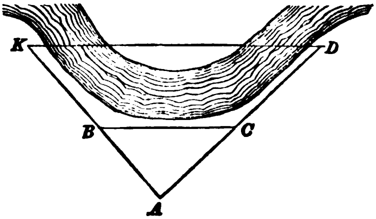


FIGURE 237

8. Make measures on objects in your vicinity and solve such problems as are suggested by those given.

9. How long is x (Fig. 238)?

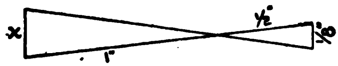


FIGURE 238

10. The thumb is held 2 in. from the end of the pencil, and the pencil is held 2 ft. in front of the eye and parallel to the line to be measured. When the end of the pencil is sighted into line with the corner b of the table, the

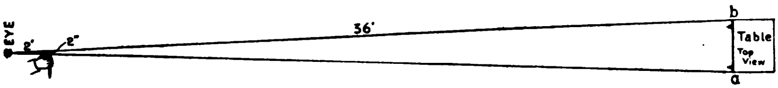


FIGURE 239

end of the thumb is in line with the corner a. How long is the end, ab, of the table, if the eye is 36 ft. from the table?

11. The eye sights past a point A over the point P on the fence (Fig. 240), and sees the upper edge of the moon in line with A and P. Then moving the eye $2\frac{1}{8}$ in. up the stake, the lower edge of the moon is in line with B and P. If the stake is 20 ft. from the fence and the moon is 240,000 miles away, how long is the moon's diameter in miles?

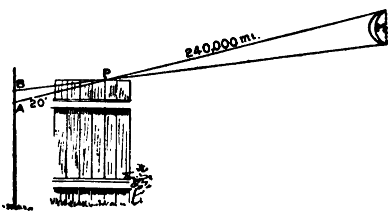


FIGURE 240

12. Make measures like these yourself.

13. The rays of light from the sun passing through a pin-hole in the screen at H (Fig. 241) give a small circular image of the

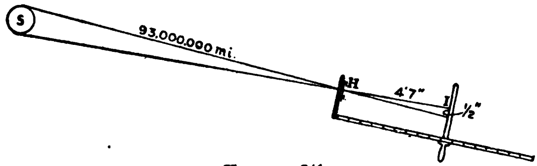


FIGURE 241

sun at I. With distances and diameter of image as shown in the cut, what is the diameter of the sun?

14. A small hole, H, in a window screen, gave a round image of the sun, 1.1 in. in diameter on a sheet of paper held 10 ft. from the pin-hole. If the sun is 93,000,000 mi. away, what is the sun's diameter?

15. Fig. 242 shows a crude apparatus for finding distances that cannot be measured directly. It consists of a board about 25″ long and 6″ wide with end pieces about 6″ high. The wood is cut

FIGURE 242

away from the end pieces so that the eye may sight through between the straight edges of two visiting cards at E, past two parallel threads at T, set carefully $\frac{1}{8}$″ apart and 25″ in front of E.

If S is in line with the upper thread of the instrument and D with the lower thread, how far is it from the stake at SD to the instrument, if $SD = 8'6''$?

SUGGESTION. — $\frac{1}{8} : 25 = 8\frac{1}{2} : x$. Then, $\frac{1}{8}x = 8\frac{1}{2} \times 25$, or $\frac{x}{8} = 212\frac{1}{2}$. Find x.

16. A pupil sighted through E at a stake held by another pupil at SD. When the pupil at E sighted over the top thread, the

pupil had to put his hand at S to line in with the slit at E and upper thread at T. The pupil at E then beckoned him to slide his hand down the pole until it lined in with the lower thread at T. The distance SD was 7.52′. How far was it from the instrument to the flagpole SD?

17. The distance from E to the bottom of the building, Fig. 243

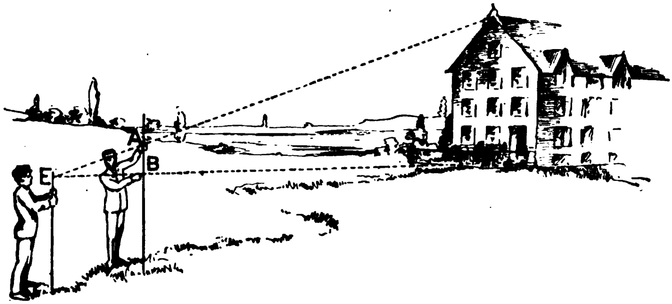


FIGURE 243

is 288′, the distance $EB = 8'$ and $AB = 18''$; how high is the building?

18. In Fig. 244 it is desired to find the distance across the lake from C to D. A surveyor stuck a flagpole at a place from which he could see both C and D. He measured the distances from D and from C to the pole he is holding, and found them to be 4563′

FIGURE 244

and 5481′ respectively. He then set a pole at B, $\frac{1}{10}$ of the distance from the first pole to D, and another pole at A, $\frac{1}{10}$ of the distance from the first pole to C. The distance from A to B was measured and found to be 84.64′. How far is it from C to D?

19. It is desired to make a scale drawing of the tract of ground *ABCDEF* (Fig. 245). The parts of the apparatus to be used, as shown at the right, are a square board about 16″ x 16″, with a

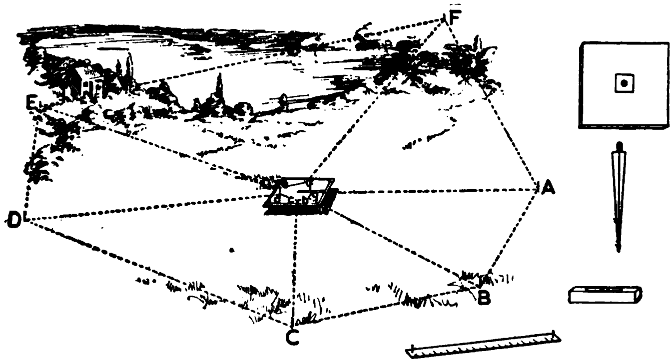


FIGURE 245

block containing a 1″ hole to fit over the top pin of a sharp stake to be stuck in the ground to support the board. A small level (which may be a phial of water) and a foot rule with a vertical pin at each end for sights to be used for sighting complete the apparatus.

The board is set up in the field as shown, a sheet of paper is pinned on it with thumb tacks, and the foot rule is placed upon the paper. A third pin is stuck near the center of the board at a point *o* (not shown in Fig. 245). Holding the edge of the foot rule against this center pin, sighting along the pins toward a pole at *A* the observer turns the front of the foot rule until the two pins of the ruler are in line with *A*. He holds the ruler and draws a line along its edge on the paper.

He now holds the edge of the ruler against the center pin, and carefully sights the two ruler pins into line with *B*, and, holding the ruler in place, draws a line on the paper toward *B*. He proceeds in the same way with each of the points *C*, *D*, *E*, and *F*, being careful not to turn the board around on the stake pin.

20. The lines from the stake supporting the board to *A*, to *B*, to *C*, and so on to *F* were measured and found as follows: to *A*, 460′; to *B*, 452′; to *C*, 378′; to *D*, 527′; to *E*, 535′ and to *F*, 832′. Using a scale of 1″: 100′, the distance to *A* (460′) was laid

off from the center pin on the line drawn toward A giving a (on the board), the distance to B (452′) was laid off from the center pin on the line drawn toward B giving b (on the board), and so on around to F. Calling the center pin o, how long is oa? ob? oc? od? oe? of?

21. With a ruler the points a, b, c, d, e and f were connected as shown in the figure. The lines were $ab = 2.8''$; $bc = 4.5''$; $cd = 5.12''$; $de = 2.95''$; $ef = 6.25''$; $fa = 4.48''$. How long are the lines AB, BC, CD, DE, EF, and FA?

The board may be supported by a light camera tripod or by a home-made tripod and the board may be held to the flat top of the tripod by a thumb nut. (See Fig 246.)

22. The distances from the apparatus to the corners of the field (Fig. 246) were: to A, 678′; to B, 612′; to C, 683′; to D, 738′;

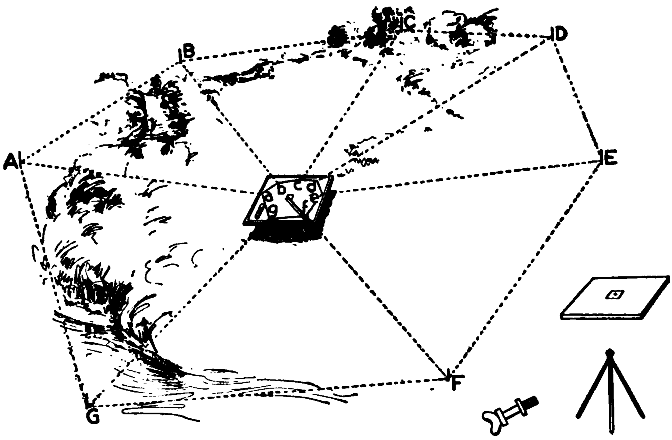


FIGURE 246

to E, 698′; to F, 625′; to G, 679′. Using a scale of 1″ : 200′, how long should the distances be made from the center pin to a? to b? to c? to d? to e? to f? to g?

23. With a ruler a, b, c, d, e, f, g, and a were then connected and lines measured. The measures were: $ab = 3.81''$; $bc = 4.12''$; $cd = 2.86''$; $de = 2.75''$; $ef = 5.86''$; $fg = 6.18''$ and $ga = 5.98''$. How long are the lines AB? BC? CD? DE? EF? FG? GA?

24. Having made the scale drawings, perpendiculars may be drawn from the center pin to each of the sides *ab*, *bc* and so on. From the measured lengths of these perpendiculars and the bases *AB*, *BC*, and so on of the triangular parts of the figures, the areas of the triangles *aob*, *boc*, and so on (calling *o* the point where the center pin stands) may be computed. The sum of these areas gives the total area of the figure *abcdefga*.

25. Suppose the area of triangle *aob* were 4.94 sq. in. what would be the area of *AOB* (*O* being the point on the ground just under *o*), if the scale were 1″: 200′? (See Problems 13 and 15, §190).

26. If the areas of the triangles *aob*, *boc*, *cod*, *doe*, *eof*, and *foa* (Fig. 245 prob. 7) in square inches were: 6.073, 7.740, 9.172, 7.490, 16.725 and 7.563, respectively, and the scale were 1″: 100′; what were the areas of the triangles *AOB*, *BOC*, *COD*, *DOE*, *EOF*, and *FOA*?

APPLICATIONS OF PERCENTAGE

§192. Insurance.

1. What will it cost to insure \$680 worth of household furniture, at the rate of $1\frac{3}{4}\%$?

DEFINITION.—The amount paid for insurance is called the *premium*.

2. An art gallery, valued at \$500,000, is insured at the rate of $1\frac{1}{4}\%$. What is the premium?

DEFINITIONS.—The written agreement between an insurance company and the insured is called a *policy*.

The amount for which the property is insured is the *face of the policy*.

3. A vessel, valued at \$16,000, was insured for $\frac{7}{8}$ of its value. Find the face of the policy.

4. A growing crop was insured at 5%. The premium was \$140. What was the face of the policy?

5. If it costs \$420 to insure a house for $\frac{3}{4}$ of its value, at $3\frac{1}{2}\%$, what is the house worth?

6. A stock of hardware was insured for \$7000, insurance costing \$110.75. What was the rate of insurance?

7. If a shipment of grain is worth \$840, and the premium amounts to \$17.60, what is the rate of insurance?

8. A machinist insured his tools, valued at \$360, for $\frac{7}{8}$ of their value, paying \$8.19. What rate did he pay?

9. A farmer paid a premium of \$10.50 for insuring his stock at $1\frac{1}{2}\%$. For what amount was the stock insured?

10. A man paid \$67.50 for the insurance of a steam launch at $1\frac{1}{2}\%$. For what amount was the launch insured?

11. What is the rate paid for insuring a bridge, valued at \$15,000, for $\frac{2}{3}$ of its value, the premium being \$240?

12. A library, worth \$28,000, was insured for $\frac{6}{7}$ of its value, the premium being \$720. What was the rate of insurance?

13. The contents of a grain elevator were insured at a rate of $\frac{3}{8}\%$. What was the amount of the policy, the premium paid being \$272.40?

14. The premium paid for insuring a quantity of lumber, for two-thirds of its entire value, at 3%, was \$36. What was the value of the lumber?

15. A piece of property, valued at \$27,200, was insured for $\frac{5}{8}$ of its value. The premium was \$212.50. What was the rate?

16. A tank of oil, holding 2592 gal., worth 18¢ per gallon, was insured at 4%. Find the premium.

17. A man insured a cargo worth \$2400 at $3\frac{1}{2}\%$. $\frac{5}{8}$ of the cargo was lost at sea. What amount of insurance should the man obtain? What premium had he paid?

18. What is the rate of insurance paid for insuring 26,000 bu. of grain worth 77¢, for $\frac{5}{8}$ of its value, if the premium is \$500.50?

19. What sums must be paid for policies to insure a factory, valued at \$21,000, at $1\frac{2}{3}\%$; a house, worth \$28,000, at $\frac{1}{4}\%$; and a barn, valued at \$5600, at $1\frac{3}{8}\%$?

20. A steamboat had a cargo, valued at \$2000, which was destroyed by fire. The insurance was \$1500. What per cent of the value of the cargo was covered by insurance? What was the premium, at $4\frac{1}{2}\%$?

§193. Taxes.

DEFINITION.—A *tax* is a sum of money levied by the proper officers to defray the expenses of national, state, county, and city governments, and for public schools and public improvements.

1. A town wishes to raise $23,500 to build a public hall. If the collector's commission is $2\frac{1}{2}\%$, what is the total amount to be collected?

2. If the taxable property of a town is valued at $923,846 and the rate of taxation is $3\frac{2}{3}\%$, what is the whole amount of the tax?

3. The taxable property of a town is $869,472, and the rate is $8\frac{1}{2}$ mills on the dollar. What is the amount of the tax?

4. What amount of money must be collected to raise a net tax of $643,000, allowing 6% for collection?

5. The tax in a city is $49,682; the rate is 15 mills on the dollar. What is the assessed valuation?

DEFINITION—*Assessed valuation* means the estimated value of the property that is assessed.

6. What is the rate of taxation when property assessed at $8940 pays a tax of $160.92?

7. The net amount collected as a tax was $2896.42. The collector's commission was $3\frac{1}{2}\%$. What was the whole amount collected?

8. The assessed valuation of the taxable property of a town was $1,218,694. The tax to be raised was $18,280.41. How many mills on the dollar was the rate of taxation?

9. A tax collector received $175.18 as his 2% commission on a certain sum collected. The assessed valuation of the taxable property was $922,000. What was the tax of a man, whose property was valued at $18,000?

10. The real property (houses, lands, etc.) of a certain town is valued at $4,560,800, and the personal property (movable property) at $945,900. If $12,000 is to be raised by taxation, how much must a man pay, whose property is valued at $9,840?

11. A net tax of $8965 is to be raised in a certain city. The

assessed valuation of the taxable property is $5,689,243. The collector's commission is $1\frac{1}{2}\%$. What will be the tax of a man, whose property is valued at $56,000?

12. Property on a city street is assessed 2% on its valuation. How much more will a man pay who owns 100 front feet, valued at $125 per foot, than one who owns property valued at $7500?

13. If the assessed valuation of the taxable property in a certain city is $1,069,210, and the whole amount of tax collected is $13,899.73, what is the rate of taxation and the net amount after deducting the collector's commission of $1\frac{7}{8}\%$?

14. The taxable property in a town is $872,990. The rate of taxation is .015. What is the net amount collected after deducting the collector's commission of $2\frac{1}{2}\%$?

15. The assessed valuation of the taxable property in a certain city is $4,968,390. The tax collected amounts to $94,399.41. How many mills on the dollar was the rate of taxation?

16. A certain town wishes to raise by taxes $20,500 to build a schoolhouse. What tax must be levied to cover this and the cost of collection at 4%?

§194. Trade Discount.

1. A hardware dealer bought the goods named in the following bill:

(1) 2 doz. braces, @ 45¢ each;
(2) 50 lb. $\frac{3}{4}''$ bolts, @ $3\frac{1}{2}$¢;
(3) 12 bales barb wire, @ $2.15;
(4) 25 kegs nails, @ $2.50;
(5) 8 cooking stoves, @ $28.75;
(6) 12 baseburners, @ $32.50;
(7) 150 lb. screws, @ $4.75 per cwt.

The bill was subject to discounts of 10%, 7%, and also 5% 30 da. or 6% 10 da. What was the total cost if the bill was paid in 30 da.? in 10 days?

2. Find the amount needed to settle the following bill, sub-

ject to the successive discounts, 10%, 8%, and 5% off for 30 da. or 6% for cash, the bill being paid in cash:

(1) 2 chests carpenter's tools, @ \$48.50;
(2) 2 doz. augurs, @ 25¢ each;
(3) 2 doz. carpenter's rules, @ \$1.80;
(4) 480 lb. hinges, @ \$4.75 per C;
(5) 3 doz. tin buckets, @ 22¢ each;
(6) 4 doz. gardening rakes, @ 28¢ each;
(7) 14 scoop shovels, @ 85¢;
(8) 6 wash boilers, @ 78¢;
(9) 12 washtubs, @ 52¢;
(10) 10 doz. clotheslines, @ \$1.10.

3. What amount will be needed to settle the bill of problem 2 in 30 days?

4. What amount will settle the following bill of house furnishings, the bill being subject to the discounts, 20%, 10%, 8%, and 5% 30 da., 6% cash, the bill being paid in 30 da.?

(1) 8 dining tables, @ \$30;
(2) 12 sets dining-room chairs, @ \$18.50 a set;
(3) 10 bookcases, @ \$12.50;
(4) 8 rocking chairs, @ \$12.75;
(5) 15 center tables, @ \$15;
(6) 12 Wilton rugs, @ \$25;
(7) 15 Axminster rugs, @ \$23;
(8) 250 yd. ingrain carpet, @ 45¢.

5. What amount paid in cash will be needed to settle the bill?

6. What amount in cash will settle this bill of plumber's supplies:

(1) 40′ lead pipe, @ 15¢, discounts 20%, 10%, 7%, 2% cash;
(2) 100′ iron tubing, @ $3\frac{1}{2}$¢, discounts 20%, 10%, 7%, 2% cash;
(3) 8 bathtubs, @ \$20, discounts 20%, 7%, 2% cash;
(4) 8 lavatory outfits, @ \$10, discounts 20%, 7%, 2% cash,
(5) 6 water meters, @ \$5, discounts 20%, 12%, 2% cash;
(6) 4 pumps, @ \$25, discounts 20%, 7%, 2% cash;
(7) 5 pump valves, @ \$2.85, discounts 20%, 7%, 2% cash?

§195. Stocks and Bonds.

A Stock is a written agreement (called also a *stock certificate*), made by a company to pay the holder a certain part of the earnings of the company. The sum paid is reckoned at a certain number of dollars per *share*, or at a certain rate per cent of the face value of a share. A share usually has a face value of $100, $500, $50, $1000, etc. A share of mining stock often has a much smaller face value.

When a stock company pays to the holders of its stock $2 on every $100 of its capital stock, or 2% on its stock, the company is said to be paying a *$2 dividend*, or a *2% dividend*.

When stock is paying a high rate of dividend it may sell in the market *above par*, or for more than its face value. If the rate of dividend is low, the stock may sell for less than its face value, or *below par*. When it sells for its face value it is sold *at par*.
city government, or by a company, to pay the holder a stated sum of money with interest on it at a stated rate. The sum of money to be repaid is called the *face* of the bond.

Stocks and Bonds are usually purchased through an agent, called a *broker*, who makes a business of buying and selling stocks and bonds. He charges a certain per cent (usually $\frac{1}{8}$%) of the face value for buying, and an equal rate for selling. This charge is called *brokerage*.

In the following problems, regard the face value of the stock or bond as $100 and the brokerage as $\frac{1}{8}$%, unless otherwise stated.

Following is a list of newspaper quotations on the N. Y. stock market on certain stocks and bonds:

STOCKS

	Open	High	Low	Noon
Amal. Copper	65¾	66¼	65⅝	65⅞
American Sugar	129¾	130	129⅝	129¾
B. & O. com	101¼	101¼	100¾	101
B. & O. pfd	95½	95½	95½	95½
Chi. & Alton com	36¼	37	36¼	36¾
Chi. & Alton pfd	71½	72¼	71½	72
Ill. Central R. R.	148	148⅝	148	148⅝
Peoples Gas	105⅝	105⅞	105½	105⅞
Pressed Steel com	65½	65½	65¼	65½
Pressed Steel pfd	94½	94½	94½	94½
Rubber Goods com	24½	24½	24½	24½
U. S. Steel com	37¾	37¾	37½	37½
U. S. Steel pfd	87⅜	87½	87¼	87¼
St. L. and S. F. com.	78½	80⅜	78	80¼
St. L. and S. F. pfd.	79⅞	80½	79⅞	80½

BONDS

Closing bid and asked prices for government bonds were as follows:

	Bid	Asked
New 2s	108⅝	109⅛
Coupons	108⅝	109⅛
New 3s	108¼	109
New 3s coupon	108⅜	109
New 3s small	108	109
Registered 4s	112½	113
Coupon 4s	112½	113
Registered 4s new	139¼	139¾
Coupon 4s new	139¼	139¾
Registered 5s	107⅛	107⅝
Coupon 5s	107⅛	107⅝

BOND TRANSACTIONS

No. Sales		
27000	Atchison Gen. 4s	100⅛@100¼
12000	B. and O. gold 4s	101 @101⅛
1000	B. and O. 3½s	94½
1000	C. B. and Q. 4s	106⅛
9000	C. and A. 3½s	76½@ 76¾
25000	U. P 4s	102¼

No. Sales		
5000	U. S. Leather 4s	85¼
3000	B. & O. coupon 4s	106¼@106⅜
82000	Cen. Pac. R. R. 3½s	88¼@ 88⅞
10000	Wabash R. R 5.s	115¼@115½
10000	Manhattan 4s.	105½

1. A certain express company has a capital stock of \$500,000, divided among its stockholders in shares of the par value of \$100 each. In 6 mo. the net profits amount to \$10,500, which the company distributes among its stockholders as a dividend. What is the rate of dividend? How much does a holder of 200 shares receive as dividend?

2. What must a man have paid for 400 shares Amal. Copper stock on the date of the table, including brokerage, if he bought at the opening price? at the highest price? at the lowest? at the noon price?

3. Answer similar questions for U. S. Steel pfd. (preferred).

DEFINITIONS.—*Preferred stocks* are stocks which pay a fixed dividend (say of 7%) before any dividends are paid on common stocks; *Common stocks* pay dividends dependent on the net earnings of the company after expenses and dividends on preferred stock have been paid.

4. How much did a man gain or lose by buying 2000 Am. Sugar at the opening price and selling at the highest price, paying brokerage of $\frac{1}{8}$% for buying and also for selling?

5. Answer similar questions for B. & O. com.; for B. & O. pfd.

6. How much does a man make or lose who buys 2500 shares Ill. Cen. R. R. stock at the lowest quotation of the table and sells at the highest, paying brokerage of $\frac{1}{8}$ for buying and $\frac{1}{8}$ for selling?

7. If a man invests in St. L. and S. F. pfd. stock, paying 7% annual dividend, at the lowest quotation of the table, what interest does he receive on his investment?

SUGGESTION.—The investor must pay \$79$\frac{7}{8}$ + \$$\frac{1}{8}$ per share (of \$100), and he receives \$7 per year as interest.

8. Answer a similar question on C. & A. pfd.

9. What interest does a man receive on an investment in Government New 2s if he invests at the price "bid," brokerage $\frac{1}{8}$? at the price "asked"?

NOTE.—Government 2s, 3s etc., are government bonds paying 2%, 3%, etc., annually.

10. Answer similar questions for New 3s; for New 3s coupon; for Registered 4s; for Registered 4s new.

11. What rate of interest does the investor who bought the 1000 B. & O. 3$\frac{1}{2}$s receive?

12. Answer similar questions for the investor who bought the 9000 C. & A. $3\frac{1}{2}$s, if he paid the lowest quoted price; the highest.

13. Which pays the higher rate of interest on the investment, and by how much, U. P. 4s as quoted, or Wabash R. R. 5s at the lowest quotation? at the highest?

14. Which is the more profitable investment, and by how much, B. & O. coupon 4's at the lowest quotation, or a straight loan at 4%?

15. Answer other similar questions on the table.

16. An investor in Chi. & Alton com. stock at the highest quotation received two 2% and one 3% dividend, and sold the stock 18 mo. later at $38\frac{1}{2}$. What rate of interest did he receive on his investment? How much profit did he make if he bought 1000 shares?

17. Answer similar questions on the table.

§196. Compound Interest.

With certain classes of notes, if the interest is not paid when due, it is added to the principal and this amount becomes a new principal, which draws interest. Savings banks add the interest on savings deposits at each interest-paying period and pay interest on the entire amount. This interest on interest is called *compound interest.* Bankers also collect their interest on loans and then reloan the interest, and in this way virtually receive compound interest on their money.

Compound interest is usually payable annually, or semi-annually.

1. Find the compound interest on a note of $200 at 6% for 5 yr., interest payable annually.

SOLUTION.—

Interest on $200	for 1 yr. at 6%	$= 200 \times \$\ .06$	$= \$\ 12.00$
Amount of $200	for 1 yr. at 6%	$= 200 \times \$\ 1.06$	$= \$212.00$
Amount of $212	for 1 yr. at 6%	$= 200 \times \$(1.06)^2$	$= \$224.72$
Amount of $224.72	for 1 yr. at 6%	$= 200 \times \$(1.06)^3$	$= \$238.23$
Amount of $238.23	for 1 yr. at 6%	$= 200 \times \$(1.06)^4$	$= \$252.52$
Amount of $252.52	for 1 yr. at 6%	$= 200 \times \$(1.06)^5$	$= \$267.68$

The compound interest=$267.68—$200=$67.68.

NOTE.—The $267.68 is called the *compound amount.* The expressions $(1.06)^3$, $(1.06)^4$. and $(1.06)^5$ mean $1.06 \times 1.06 \times 1.06$, $1.06 \times 1.06 \times 1.06 \times 1.06$, and $1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06$. and are read "1.06 cube," "1.06 to the 4th power," and "1.06 to the 5th power."

2. Find the simple interest on a note for the same face at the same rate and for the same time as in problem 1. By how much does the compound interest exceed the simple interest?

3. Find the compound interest on \$180 at 6% for 3 yr., interest compounded semi-annually.

SOLUTION.—

Amount on \$180 for ½ yr. at 6% = 180 × \$ 1.03 = \$185.40
Amount on \$185.40 for ½ yr. at 6% = 180 × $\$(1.03)^2$ = \$190.96
Amount on \$190.96 for ½ yr. at 6% = 180 × $\$(1.03)^3$ = \$196.69
Amount on \$196.69 for ½ yr. at 6% = 180 × $\$(1.03)^4$ = \$202.59
Amount on \$202.59 for ½ yr. at 6% = 180 × $\$(1.03)^5$ = \$208.67
Amount on \$208.67 for ½ yr. at 6% = 180 × $\$(1.03)^6$ = \$214.93

Compound interest = \$214.93 — \$180 = \$34.93.

4. If P denotes any principal at r% for n yr., interest being compounded annually, show that if A denotes the compound amount,

$$A = P\left(1 + \frac{r}{100}\right)^n.$$

5. Show that if the interest is compounded semi-annually,

$$A = P\left(1 + \frac{r}{200}\right)^{2n}.$$

the letters meaning the same as in Problem 4.

6. Show that if I denotes the interest, compounded annually,

$$I = P\left(1 + \frac{r}{100}\right)^n - P.$$

7. If the interest is compounded semi-annually, show that

$$I = P\left(1 + \frac{r}{200}\right)^{2n} - P.$$

8. A man has a deposit of \$100 in a savings bank paying 4%, interest compounded semi-annually, for 3 yr. How much is due the depositor at the end of that time?

9. How much is due on a savings deposit of \$250, in a savings bank paying 3% semi-annually, the deposit having been in the bank 3½ years?

10. A man paid compound interest at 6%, interest compounded annually, on a note of $150 for 7 yr. How much was required to pay the note?

NOTE.—Many notes have coupon notes attached for the amount of interest due at each interest-paying period. These coupon notes usually bear a higher rate of interest than does the note itself.

11. A note for $500, bearing 6% interest, has 3 interest coupons attached, the coupons, when due, bearing 7% interest, payable annually. If the note runs 4 yr., no coupons being paid in the meantime, how much money will be required to pay the entire debt at the end of this time?

12. A note of $450, bearing interest at 7%, interest payable semi-annually, coupons bearing 8% interest, amounts to what sum at the end of 4½ yr., no coupons being paid until the end of the time?

USE OF LETTERS TO REPRESENT NUMBERS

§197. Problems.

1. Joseph had 45¢ and he earned 15¢ more selling oranges. How many cents had he then? (Answer by indicating the operation you perform, thus: 45¢ + 15¢.)

2. William had x marbles and Harold gave him y marbles. How many marbles had he then?

3. A cow gave x lb. of milk at the morning milking and y lb. at the evening milking. How many pounds did she give at both?

4. James earned 80¢ selling papers on Saturday and spent 45¢ of his earnings. How many cents did he save on Saturday? (Answer by indicating the operation you use.)

5. During July a boy earned m dollars and spent s dollars. How many dollars did he save?

6. Helen bought 15 pencils at 3¢ apiece. How much did she pay for all?

7. Elizabeth took x music lessons during February at y dollars per lesson. What was the cost of her lessons for the month?

8. A thermometer rose A° on one day and 5 times as much the day following. How much did it rise on the following day?

9. The area of a rectangular lot is 63 sq. rd. and the length of one side is 9 rd. How long is the other side?

10. The area of a rectangle is x square inches and the length of one side is y inches. How long is the other side?

11. How many lots each of b ft. frontage can be made from a frontage of a feet?

12. An orange boy earns a cents on Wednesday, three times as many cents on Thursday, and as many cents on Friday as on Wednesday and Thursday together. On all three days he earns 80¢. How much does he earn on Thursday? on Friday?

NOTE.—In all such problems use the equation. We have $a + 3a + 4a = 80$, or $8a = 80$. If $8a = 80$, $a = 10$. $3a = 30$, Thursday earnings, and $4a = 40$, Friday earnings.

13. The altitude of a rectangle is x in. and the base is $3x$ in. How long is the perimeter? What is the area of the rectangle?

14. $4a$ means $4 \times a$. Compare the values of $a \times 4 \times a$ and $4 \times a \times a$. How is $a \times a$ written? How is $4 \times a \times a$ written? *Ans.* $4a^2$.

15. The mercury stood at x degrees at 2 p.m. and fell 3° during the next hour. The reading at 3 p.m. was 25°. What was the reading at 2 p.m.?

16. The mercury stood at 12° at 8 a.m. During the next two hours it rose x degrees. What was the thermometer reading at 10 a.m.? If it had fallen y degrees, what would have been the reading at 10 a.m.?

17. The thermometer read 28°, the mercury fell x degrees, then $3x$ degrees, and then rose $6x$ degrees, when the reading was 32°. What was the number of degrees in each of the three changes?

18. James paid \$$x$ for a hat and twice as much for a coat. He paid \$4.50 for both. What did he pay for each?

19. I paid \$30 for a bicycle and sold it for \$$x$. How much did I gain?

20. I paid $\frac{1}{2}$ of what I gained for a coat. How much did I pay for the coat?

21. Louis had x apples and ate y of them. How many did he have left?

22. Henry had $7a$ papers and sold $4a$ of them. How many did he have left?

23. I walked x miles due south one day and y miles due north the next day. How far was I then from my starting point?

24. A man rows a boat downstream at a rate that would carry his boat a miles per hour through still water, and the current alone would carry him down b miles per hour. How far will he go in 1 hour?

25. A man walks from rear to front through a railway coach 3 mi. per hour, and the coach is running at the same time a mi. per hour. How fast does the man pass the telegraph poles along the track?

26. How fast does he pass them if he walks from front to rear?

27. Mary had a pencils and sold them at 5¢ apiece. How many cents did she receive for them?

28. James sold x oranges at a cents apiece. How many cents did he receive for them?

29. A dealer sold a wagons for $40a$ dollars. What was the price per wagon?

30. A farmer paid x dollars for 15 A. of land. How much did he pay per acre?

31. The area of a rectangle is m sq. rd. and it is l rd. long. How wide is the rectangle?

32. A township is x mi. square and contains 36 sq. mi. What is the value of x? What does x represent?

33. Helen bought x dolls at 10¢ apiece and y yd. of muslin at 8¢ a yard. How many cents did she pay for both?

34. James had m cents and earned c cents more. He invested all his money in papers at 3¢ apiece. How many papers did he buy?

35. He sold his papers at 5¢ apiece. How much did he receive for the papers? How much did he gain?

36. The base of a triangle is x ft. and the altitude is y ft. What is the area in square inches?

37. The area of a triangle is x sq. ft. and the base is 6 ft. long. What is the altitude?

38. A parallelogram has a base $(x+y)$ in. long, and is 6 in. high. What is the area? Express this answer in two ways and make an equation by writing the two expressions equal. Why are the two expressions equal?

§198. Uses of the Equation.

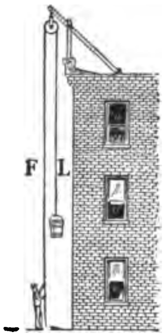


FIGURE 247

1. If each brick weighs 7 lb. and there are 10 bricks in the bucket (Fig. 247), with how many pounds of force does the bucket pull downward on the rope, the bucket itself weighing 5 pounds?

2. If we denote the number of pounds of force with which the man pulls downward on the rope to balance the bucket by p, write an equation showing the number of pounds in p, the 5-lb. bucket being loaded with 10 bricks.

3. A horse is raising a 10-ft. steel I-beam weighing 30 lb. for each foot of length. If the force the horse must exert to hold the beam suspended in the air be denoted by F, write an equation showing the number of pounds in F.

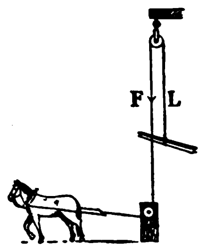


FIGURE 248

4. A wagon weighing 1800 lb. is loaded with 3 T. of coal. When it is being drawn over ordinary pavement it pulls backward on the traces of the team with a force of $\frac{1}{50}$ as many pounds as there are pounds in the entire weight of the coal and wagon. If the force exerted by the moving team is F lb., write an equation showing the number of pounds in F.

5. Write an equation showing how many pounds each horse draws if both draw with equal force.

These problems show how the equation may be used in solving simple problems of mechanics; and we need to learn the laws upon which the use of the equation is based.

§199. Principles for Using the Equation.

Review p. 97,

1. Two weights, one of x lb. and the other of y lb., are tied to strings which pass over pulleys at A and B (Fig. 249). The strings are knotted at C. If x is greater than y, how will C move? Under what condition will C remain at rest? What relation is shown to exist between x and y by a movement of C toward the right?

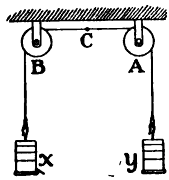


FIGURE 249

C's standing stationary indicates a *balance in value* between the forces y lb., drawing toward the right, and x lb., drawing toward the left. This fact is expressed in symbols by writing x lb. $= y$ lb., or, better, by $x = y$, simply.

2. If a 10-lb. weight be hung to the weight y, how many 5-lb. weights must be hung to x to secure a *balance of value?*

NOTE.—This is shown by writing $y + 10 = x + 5 + 5$, read "y plus 10 equals x plus 5 plus 5."

3. How many pounds in all were added to x?

4. If 50 lb. were added on the right, how many 5-lb. weights must be added on the left before the sign of equality (=) can be written between the numbers? If $7a$ lb. were added on the right, how many a-lb. weights would be needed on the left for balance?

5. How many pounds would be needed on the right if b lb. are added on the left? $b + c$ lb.? $p + 35$ pounds?

6. Write the equation form of statement for each case of Problems 4 and 5.

7. If a greater number of pounds were added to y lb. than to x lb., say 15 lb. to y lb. and 10 lb. to x lb., what would occur in the apparatus shown in Fig. 249?

This *destroying the balance*, or *equality*, is stated briefly in signs by writing $y + 15 > x + 10$, which is read "y plus 15 is *greater than* x plus 10."

If the balance, or equality, is destroyed by adding a heavier weight to x than to y, the fact is stated in *symbols* thus: $y + 10 < x + 15$; read "y plus 10 is *less than* x plus 15."

Notice in each case that the *vertex* of the horizontal V *always points toward the smaller number.*

DEFINITIONS.—An expression in which the sign $<$ or $>$ stands between two numbers is called an *expression of inequality.*

We now have signs for writing briefly the three possible relations which may exist between any two numbers, as a and b. They are called *relation signs.*

8. Read and give the meanings of (1) $a > b$, (2) $a = b$, (3) $a < b$, (4) $x + 20 < x + 15 + 10$.

9. If $y = x$, and a lb. are added to y lb. and b lb. to x lb., how will the knot C move for case (1) Problem 8? for case (2)? for case (3)? State in symbols the fact shown by the knot C in each case after adding the weights, using the correct relation signs.

10. If any given weight, a lb., be added to y, what must be true of the weight b lb. if, when added to x, we may write $y + a = x + b$?

11. If z lb. be added to both the y lb. and the x lb., when $y = x$, what equation may we write?

PRINCIPLE I (FOR ADDITION OF EQUATIONS).—*If the same number, or equal numbers, be added to both sides of an equation, the sums are equal.*

ILLUSTRATION.—If $y = x$, and $a = b$, then we may write

$$y + a = x + a,$$
and $$y + b = x + b,$$
and $$y + a = x + b,$$
and $$y + b = x + a.$$

12. If 8 lb. be removed from the left scale pan (Fig. 249), how many 4-lb. weights must be removed from the right pan to *restore the balance?*

NOTE.—This is written in signs $y - 8 = x - 4 - 4$, and is read "$y - 8$ equals x minus 4 minus 4."

13. How many pounds in all were removed from the right scale pan?

14. If $y = x$, and if a lb. be removed from the left, and b from the right, to what must the value of b be equal to enable us to write $y - a = x - b$?

PRINCIPLE II (FOR THE SUBTRACTION OF EQUATIONS).—*If the same number, or equal numbers, are subtracted from equal numbers, the differences are equal.*

ILLUSTRATION.—If $y = x$ and $a = b$, then we may write:

$$y - a = x - a,$$
and $$y - b = x - b,$$
and $$y - a = x - b,$$
and $$y - b = x - a.$$

15. If in Fig. 249 y be doubled, what corresponding change in x will restore the balance?

NOTE.—The double of y is written $2y$ and read "two y."

16. What equation states that there is a balance?

17. If $y = x$, and if 4 weights, each equal to y, are put on the right of the apparatus in Fig. 249, how many weights, each equal to x, must be put on the left to keep the balance?

18. If a weights, each equal to y, are on the right, how many weights, each equal to x, must go on the left to secure balance?

NOTE.—The equation is $ay = ax$.

19. If $a = b$, and a weights, each equal to y, are put on the right, and b weights, each equal to x, are put on the left, what will be shown by the scales if $y = x$. (Answer with an equation.)

PRINCIPLE III (FOR THE MULTIPLICATION OF EQUATIONS).—*If equal numbers are multiplied by the same number, or by equal numbers, the products are equal.*

ILLUSTRATION.—If $y = x$ and $a = b$, then

$$\begin{aligned} ay &= ax, \\ \text{and } by &= bx, \\ \text{and } ay &= bx, \\ \text{and } ax &= by. \end{aligned}$$

20. If $y = x$, and half of the weight on the left be taken off, what fractional part of the weight on the right must be taken off to restore the balance?

21. If only $\frac{1}{3}$ of y be kept on the right (Fig. 249), what fractional part of x must remain on the left?

22. If only $\frac{y}{a}$ lb. be kept on the right (Fig. 249), what fractional part of x lb. must remain on the left for balance?

23. Suppose $y = x$ and $a = b$, and that $\frac{y}{a}$ lb. are on the right, and $\frac{x}{b}$ lb. are on the left; what equation would be shown to be true by the apparatus?

NOTE.—The equation is $\frac{y}{a} = \frac{x}{b}$, read "$y$ divided by a equals x divided by b."

PRINCIPLE IV (FOR THE DIVISION OF EQUATIONS).—*If equal numbers are divided by the same number, or by equal numbers, the quotients are equal.*

ILLUSTRATION.—If $y = x$ and $a = b$, then

$$\frac{y}{a} = \frac{x}{a},$$

$$\text{and } \frac{y}{b} = \frac{x}{b},$$

$$\text{and } \frac{y}{a} = \frac{x}{b},$$

$$\text{and } \frac{y}{b} = \frac{x}{a}.$$

In Fig. 183, p. 293, $a\ (x + y)$ represents the area of the whole rectangle, while ax and ay represent the areas of its two parts. As the two parts of the large rectangle, taken together, must equal the whole rectangle, we may write:

$$a(x + y) = ax + ay$$

In Fig. 184, p. 294, the area of the whole rectangle is given by $(a + b)\ (x + y)$, while the areas of the several parts are ax, ay, bx, and by. Since the parts, taken together, make up the whole rectangle, we may write:

$$(a + b)(x + y) = ax + ay + bx + by$$

These two equations illustrate the meaning of a fifth principle of the equation, viz.:

PRINCIPLE V.—*Any whole equals the sum of all its parts.*

These five fundamental principles or laws, exemplified by the scales, p. 97. and the pulley device (Fig. 249), p. 349, *must not be violated* in using the equation.

§200. Problems.

1. Find the value of the letter x, y, or z, in each of the first nine problems:

(1) $3x = 15$ lb.;	(4) $9z = 36$¢;	(7) $11x = 55$;
(2) $8x = 24$ ft.;	(5) $\frac{3}{4}x = \$3$;	(8) $\frac{7}{8}y = 21$;
(3) $7y = 18$ mi.;	(6) $\frac{2}{3}y = 14$ sq. in.;	(9) $ax = 3a$.

2. If $3x = 9$, to what is $2x$ equal? $7x$? $\frac{1}{2}x$?

3. If $7x = 21$, to what is $5x$ equal? $\frac{2}{3}x$? $6\frac{1}{3}x$?

4. If $x + 3 = 8$, what is x? $3x$? $5x$? x^2?
5. If $2x + 7 = 15$, what is x? $3x$? $7x$? x^2? $\sqrt{x}$?
6. If $x - 3 = 6$, what is x? $3x$? x^2? $\sqrt{x}$?
7. $2x + 3x + 6x = 33$; find x.
8. $4x - 2x + x = 64$; find x.
9. $7x - x + 2x = 32$; find x.
10. $xy = 16$, and $y = 8$; what is x? If $x = 8$, what is y?
11. $ax = 35$, and $a = 5$; what is x? If $x = 5$, what is a?
12. $ab = 10$; what is b if $a = 1$? 2? 3? 4? 10? 20?
13. By what principle may we write

$$c(x + y + z) = cx + cy + cz,$$

x, y, and z denoting the bases of 3 rectangles whose altitudes are each equal to c? (Answer by sketching the proper figure and pointing out the rectangles whose areas represent each of the products in the equation.)

14. If $a = 8$ and $b = 5$, find the values of the following expressions and tell what ones are equal:

(1) $(a + b)$; (4) $(a + b)^2$; (7) $a^2 - 2ab + b^2$; (10) $a^2 + b^2$;
(2) $(a - b)$; (5) $(a - b)^2$; (8) $a^2 + 2ab + b^2$; (11) $a(a^2 + b^2)$;
(3) $2ab$; (6) $a^2 + b^2$; (9) $(a + b)(a - b)$; (12) $b(a - b)$.

15. If $x = 9$ and $y = 4$, tell which of the following express true relations and write the correct *relation sign* in each case: (See p. 350.)

(1) $x > y$;
(2) $x = y$;
(3) $x < y$;
(4) $y < x$;
(5) $x + y = 13$;
(6) $x - y = 5$;
(7) $(x + y)^2 = 169$;
(8) $(x - y)^2 = 25$;
(9) $x^2 - y^2 > 25$;
(10) $x^2 + y^2 = 169$;
(11) $(x + y)^2 = (x^2 + y)^2$;
(12) $(x - y)^2 = x^2 - y^2$;
(13) $(x + y)^2 = x^2 + 2xy + y^2$;
(14) $(x - y)^2 = x^2 - 2xy + y^2$;
(15) $(x - y)^2 = x^2 + 2xy + y^2$;
(16) $(x + y)(x - y) < x^2 - y^2$;
(17) $(x + y)(x - y) = x^2 + 2xy + y^2$;
(18) $(x - y)(x + y) = (x^2 + y^2)$;
(19) $(x - y)(x + y) < (x^2 - y^2)$;
(20) $x(x + y) = x^2 + xy$.

§201. Statements in Words and in Symbols.

1. Write the symbolic statements for these verbal phrases and statements. Let x stand for the number when there is but one number to be symbolized in the problem.

(1) A certain number increased by 15.

(2) Twice a number diminished by 8.

(3) Seven times a number increased by three times the number.

(4) The square of a number, divided by 8.

(5) The sum of the square and the first power of a number.

(6) Eight times a number, divided by three.

(7) One-third of 10 times a number.

(8) Three times a certain number, diminished by one, equals 20.

(9) Eighteen times the square of a number equals 72.

(10) Twenty-five times a number, increased by 5, equals 30 times the number, diminished by 15.

(11) One-eighth of the sum of a certain number and 18.

(12) Six times the difference between a certain number and 3 equals 18.

(13) The product of the sum and the difference of x and 3 equals 18 (x being greater than 3).

(14) The difference between x and 18 is greater than 18; is less than 25; is equal to 20 ($x > 18$ in each case).

2. State in words what these expressions mean. For example, (1) means "double a certain number, diminished by 9":

(1) $2x-9$;	(6) $(x+1)(x+1)$;	(11) $2x+6=12$;
(2) $16x+x$;	(7) $x(x-4)$;	(12) $7x-2+16$;
(3) $28x+17$;	(8) $3(9-x)$;	(13) $5x+7=42$;
(4) x^2+x;	(9) $12(x^2-1)$;	(14) $(x-4)(x+4)=20$;
(5) x^2-x;	(10) $(x-1)(x-1)$;	(15) $(a+b)(a-b)=a^2-b^2$.

3. Translate into symbols these verbal phrases and statements, using a and b, or x and y, for the two numbers:

(1) The sum of two numbers equals 25.

(2) The difference of two numbers equals 15.

(3) The sum of the squares of two numbers is less than 27.

(4) The square of the sum of two numbers equals 100.

(5) The difference of the squares of two numbers equals 9.

(6) The sum of the squares of two numbers equals seven times the difference of the numbers.

(7) The product of two numbers equals their sum.

(8) The quotient of two numbers equals their difference.

(9) A certain number increased by 1 equals another number diminished by 3.

4. Translate into words these symbolic expressions. For example, (1) means "one-ninth of the difference between 6 times a certain number and its square":

(1) $\frac{6x - x^2}{9}$;	(6) $\frac{8x - 5}{3} = \frac{7y + 4}{5}$;	(11) $\sqrt{mn} = \sqrt{m}\,\sqrt{n}$;
(2) $\frac{x + 3}{7}$;	(7) $\frac{x}{y} + 1 = \frac{x + 1}{y + 1}$;	(12) $\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x - y}} = 1$
(3) $\frac{3 - x}{x}$;	(8) $\frac{a^2 - b^2}{a - b} = a + b$;	(13) $a^2b^2 = (ab)^2$;
(4) $\frac{a + b}{a - b}$;	(9) $\frac{x}{y} = \frac{y}{x}$;	(14) $\frac{a^2}{b^2} + 1 = \left(\frac{a}{b}\right)^2 + 1$;
(5) $\frac{x^2}{x + 2}$;	(10) $\frac{x^3}{y^3} = \left(\frac{x}{y}\right)^3$;	(15) $\sqrt{a} + 1 = \sqrt{a + 1}$.

5. Find the number which may be put in place of the letter in each of these equations to furnish true equations:

(1) $2x + 3 = 7$;

SOLUTION.—

$$\begin{array}{l} 2x + 3 = 7 \\ 3 = 3 \\ \hline 2x = 4 \quad \text{by Principle II,} \\ \frac{2x}{2} = \frac{4}{2} \\ x = 2 \quad \text{by Principle IV.} \end{array}$$

Check: $2x + 3 = 2 \times 2 + 3 = 4 + 3 = 7$, which is correct.

(2) $3x - 1 = 5$;	(4) $\frac{1}{6}x + 1 = 2$;	(6) $\frac{6x}{7} + \frac{2}{7} = \frac{14}{7}$;
(3) $8x - 6 = 18$;	(5) $\frac{3x}{4} - 2 = 1$;	(7) $\frac{6x}{7} - \frac{2}{3} = \frac{15}{3}$.

NOTE.—First multiply both sides of (7) by 21.

6. Find the value of the numbers of problem 1 (8), (9), and (10).

§202. **Problems.**—FOR EITHER ARITHMETIC OR ALGEBRA.

1. The mercury column in a thermometer rose a certain number of degrees one day, and 3 times as many degrees the next day. It rose 12° during the 2 days. How many degrees did it rise each day?

ARITHMETICAL SOLUTION.—

A certain number denotes the rise the first day.
3 times this number denotes the rise the second day.
Hence 4 times a certain number denotes the rise in two days.
4 times a certain number equals 12° (by the given problem).
Once the number equals 3°, the rise the first day (Principle IV).
3 times the number equals 9°, the rise the second day (Principle III).

Check. $3° + 9° = 12°$, the rise in two days.

ALGEBRAIC SOLUTION.—

Let x denote the first day's rise.
Then, $3x$ denotes the second day's rise.
$x + 3x$ denotes the rise in 2 days.
$4x = 12°$.
$x = 3°$, the first day's rise (Principle IV).
$3x = 9°$, the second day's rise (Principle III).

Check: $3° + 9° = 12°$.

2. A man bought 4 times as many hogs as cows, and after selling 5 hogs he had 23 hogs left. How many cows did he buy?

ARITHMETICAL SOLUTION.—

A certain number represents the number of cows bought.
4 times this number represents the number of hogs bought.
4 times this number minus 5 denotes the number of hogs left.
Then 4 times this number, minus 5, equals 23 (by the problem).
4 times this number equals 23 plus 5 (Principle I).
4 times this number = 28.
This number = 7, the number of cows (Principle IV).

Check: $4 \times 7 - 5 = 23$.

ALGEBRAIC SOLUTION.—

Let x denote the number of cows bought.
Then, $4x$ denotes the number of hogs bought.
$4x - 5$ denotes the number of hogs left.
Then $4x - 5 = 23$ (by the problem).
$4x = 28$ (Principle I).
$x = 7$ (Principle IV). *Ans.* 7 cows.

Check: $4 \times 7 - 5 = 23$.

3. Two masses were placed on one scale pan of a balance and found to weigh 18 lb. One of the masses was then placed in each pan, and it required 4 lb. *additional* on the light pan to balance the scales. What was the weight of each mass?

ARITHMETICAL SOLUTION.—

A certain number of pounds denotes the weight of the heavier mass.
Another number of pounds denotes the weight of the lighter mass.
The first number plus the second number denotes the combined weight (18 pounds).
The first number minus the second denotes the difference of the weights, or the additional weight, which equals 4 pounds.
2 times the first number equals 18 plus 4 equals 22.
The first number equals 11.
11 plus the second number = 18. (Principle IV).
The second number = 7. (Principle II).
The weights are, then, 7 lb. and 11 lb.

Check: 11 lb. + 7 lb. = 18 lb.
11 lb. — 7 lb. = 4 lb.

ALGEBRAIC SOLUTION.—

Let x denote the number of pounds in the weight of the heavier mass.
Let y denote the number of pounds in the weight of the lighter mass.
Then, $x+y$ denotes the number of pounds in the combined weight, $=18$
$x-y$ denotes the number of pounds in the additional weight, $=4$.

$$\begin{aligned} x+y&=18\\ x-y&=\ \ 4\\ \hline 2x\qquad&=22 \text{ (Principle I).}\\ x\qquad&=11 \text{ (Principle IV).}\\ 11+y&=18\\ y=18-11&=7 \text{ (Principle II).}\end{aligned}$$

Check: $11+7=18$
$11-7=\ \ 4$

Solve the following problems by both the algebraic and the arithmetical method, and state which of the solutions is the shorter:

4. A man cut for me 3 times as many ash trees as oak trees, and as many hickory trees as ash trees and oak trees together. In all he cut 32 trees. How many of each kind did he cut?

5. A man bought 4 times as many 2¢ stamps as 5¢ stamps, and ½ as many 10¢ stamps as 2¢ stamps. For all he paid $4.95. How many of each kind did he buy?

NOTE.—Let x denote the number of 5¢ stamps purchased.

6. The side of a rectangular field is twice its breadth and the distance around the field is 240 rd. How many acres does the field contain?

7. I bought a number of books one day, 4 times as many the next day, and 3 books the third day. In all I bought 33 books. How many did I buy each day?

8. A newsboy sold a certain number of papers on Wednesday, twice as many on Thursday, and 3 more on Friday than on Thursday. In all he sold 48 papers. How many did he sell each day?

NOTE.—Let x denote the number of papers sold on Wednesday.

9. Texas lacks 17,470 sq. mi. of being large enough to make 5 states the size of Illinois. How large is Illinois if Texas contains 265,780 square miles?

10. Maude has 78¢, which lacks 18¢ of being 6 times as much money as James has. How many cents has James?

11. It is twice as far from Chicago to Niles by a certain route as from Niles to Albion, and from Chicago to Albion is 190 mi. How far is it from Chicago to Niles?

12. Rochester, N. Y., is 17 mi. more than twice as far from Chicago as is Detroit. Rochester is 587 mi. from Chicago. How far is Detroit from Chicago?

13. Niagara Falls is 8 mi. less than 9 times as far from Chicago, as is Michigan City. Niagara Falls is 514 mi. from Chicago. How far is it from Chicago to Michigan City?

14. It is 4 times as far, less 16 mi., from Chicago to N. Y. City as from Chicago to Ann Arbor. If it is 976 mi. from Chicago to N. Y. City, how far is it to Ann Arbor?

15. Champaign is 5 mi. more than $\frac{1}{3}$ as far from Chicago as is Cairo. If it is 128 mi. from Chicago to Champaign, how far is it from Chicago to Cairo?

16. A father and his son together earn $75 a month, and the father earns 4 times as much as the son. How much does each earn?

17. William has 3 times as many marbles as Joseph, and both together have 24. How many marbles has each?

18. A line 32 in. long is to be divided into 2 parts, one of which is 8 in. longer than the other. How long is each part?

19. A tree 75 ft. high was broken by the wind so that the part standing was 15 ft. longer than the part broken off. How long was each part?

20. The area of a rectangular field is 6400 sq. rd. and its length is 160 rd.; what is its width?

21. The area of a rectangular sidewalk is 120 sq. rd. and the width is $1\frac{1}{2}$ yd.; how long is the walk?

22. A man received \$180 for 72 days' work; what was his daily wage?

23. Find the side of a square field equal in area to a rectangle 80 rd. long by 20 rd. wide.

24. A man received \$3000 for 80 A. of land; how much did he receive per acre?

25. The circumference of a circle is 2200 rd.; what is the radius? (Equation: $6\frac{2}{7}r = 2200$.)

26. The area of a circle is $201\frac{1}{7}$ m.^{2}; how long is the radius? the circumference?

27. Find the radius and the circumference of a circle whose area is 1 square inch.

§203. Formal Work.

Find the value of the unknown quantity, x or y, in each of the following:

1. $6x + 4 = 22.$
2. $3x - 10 = 14.$
3. $4y - 2y + 2 = 16.$
4. $8y + y + 3 = 21.$
5. $\frac{3}{4}x - 2 = 16.$
6. $\frac{5}{6}x + 5 = 35.$
7. $\dfrac{x+4}{3} = 8.$

NOTE.—First multiply both members by 3.

8. $\dfrac{2x - 14}{5} = 2.$
9. $\dfrac{9x + 3}{6} = 5.$
10. $6(x + 2) = 72.$
11. $\dfrac{7x + 18}{3} = 13.$
12. $\dfrac{30}{x} = 15.$

NOTE.—First multiply both members of 12 by x.

13. $\dfrac{25}{2x} = 5.$
14. $\dfrac{x + 3x + 12}{11} = 4.$
15. $\dfrac{7x - 2x + x}{5} = 18.$
16. $21 - x = 33 - 2x.$

NOTE.—First add $2x$ to both members, then subtract 21 from both members.

17. $12x + \dfrac{x}{4} = x + 90.$
18. $7x - \dfrac{2x}{3} = 2x + 39.$
19. $\dfrac{12x + 27}{21} = 3.$
20. $\dfrac{36 - 5x}{3} = 7.$

21. In the following equations W denotes the weight of the rope, or cable, in lb. per yd.; L the *working load* in tons; S, the *breaking load* in tons, and D, the *diameter* of the rope, or cable, in inches.

For hemp cable, $W = .577D^2$; $L = .109D^2$; $S = .654D^2$.
For tarred hemp cable, $W = 1.036D^2$; $L = .247D^2$; $S = 1.480D^2$.
For manila rope, $W = .765D^2$; $L = .329D^2$; $S = 1.877D^2$.
For iron wire rope, $W = 3.847D^2$; $L = 2.862D^2$; $S = 17.012D^2$.
For steel wire rope, $W = 3.946D^2$; $L = 4.441D^2$; $S = 27.630D^2$.

Find the weight per yd., the working load and the breaking load of a hemp cable of 1″ diameter; of $2\frac{1}{2}$″ diameter.

22. Solve similar problems for tarred hemp rope; for manila rope.

23. Find W, L, and S for an iron wire rope for which $D = 1\frac{1}{4}$″; $D = 2\frac{1}{8}$″.

24. For the same values of D, find W, L, and S for steel wire rope.

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§204. Equations Containing Two Unknown Numbers.

In these problems two numbers will be unknown. The facts of each problem will furnish two equations. From these facts write both equations. Then change one or both equations, or combine them, in accordance with Principles I-V, pp. 350-2.

PROBLEMS

1. The sum of two numbers is 10 and their difference is 4. Find the numbers.

SOLUTION.—Let x stand for the larger number.
Let y stand for the smaller number.
Then
(1) $x + y = 10$ (why?)
and
(2) $x - y = 4$ (why?)
Add Equations (1) and (2) member to member and write—
(3) $2x = 14$. By which Principle?
Then
(4) $x = 7$.
Which Principle enables us to get Equation (4) from Equation (3)?
Putting this value, 7, of x, in place of x in Equation (1), we have
(5) $7 + y = 10$
Subtracting 7 from both sides of Equation (5), we get
(6) $y = 3$

By which Principle do we have the right to subtract 7 from both sides of Equation (5) and then write the two differences equal?

The two desired numbers are then 7 and 3.

N.B.—The problem shows how to find any two numbers when their sum and their difference are known.

2. The sum of two numbers is 18 and their difference is 6. Find the numbers.

3. A piece of wire 8 ft. long is cut into two pieces, one of which is $2\frac{1}{2}$ ft. longer than the other. How long is each piece?

4. A 16-oz. mixture of acid and water contains 9 oz. more water than acid. What is the weight of each part of the mixture?

5. In a school of 40 children there are 12 more girls than boys. How many girls are in the school? How many boys?

6. The side of a rectangular lot is 75 ft. longer than the end and it is 350 ft. around the lot. Find the length, width and area of the lot.

7. The sum of two weights is 40 lb. and their difference is 8 lb. Find the weights.

8. I am thinking of two numbers. Their sum is 23 and their difference is 8. What are the numbers?

9. The top end of a May-pole 20 ft. high broke so that the part left standing was 5 ft. longer than the part broken off. How long was each piece?

10. A mixture of 18 oz. of vinegar and water contained twice as many ounces of vinegar as water. How many ounces of each liquid was in the mixture?

SOLUTION.—Equation (1) $x + y = 18$. Explain how this equation is obtained.

Equation (2) $x = 2y$. How obtained?

Write $2y$ in place of x in Equation (1), and have $3y = 18$. Find y.
From Equation (2) then find x.

Put the numbers found for x and y in Equation (1) in place of x and y and notice whether the result is a true equation. This is called a "check."

11. A mixture of water and acid weighing 28 oz. contains 3 times as much water as acid. How many ounces of each does the mixture contain?

12. A silver dollar weighs 412.5 grains and it contains 9 times as much pure silver as alloy. Find the weight of pure silver and of alloy in a silver dollar.

13 A piece of bronze weighing 15 lb. is made of copper and tin. It contains 4 times as much copper as tin. How much of each metal is there in the piece?

14. A bronze gun, weighing 25 tons, is made of copper and tin in the proportion of 9 times as much copper as tin. How much of each metal is there in the gun?

15. How many cubic inches of pure acid and of water are needed to make a gallon (231 cu. in.) of acid solution $12\frac{1}{2}\%$ pure, i.e. containing $12\frac{1}{2}\%$ of pure acid and $87\frac{1}{2}\%$ of water?

16. The sum of one number and twice another is 11. The difference of the two numbers is 5. Find the numbers.

17. The sum of two numbers is 13. Twice the first number minus 3 times the second is 1. Find the numbers.

18. The sum of a number and half another number is $8\frac{1}{2}$. Twice the first number minus 3 times the second is 5. Find the numbers.

19. Divide 24 in. into two parts, one of which is 6 in. greater than the other.

20. Divide 32 into two parts, one of which is 3 times the other.

21. Divide 28 into two parts, one of which is $\frac{3}{4}$ of the other.

22. Two numbers are to each other as 3 to 5 and their sum is 64. Find the numbers.

23. If 1 be added to the numerator of a certain fraction it becomes $\frac{1}{2}$; but if 1 be added to the denominator, the fraction becomes $\frac{1}{3}$. Find the fraction.

24. Three times one number added to 2 times another number equals 23. Also 2 times the first number minus the second number equals 6. Find the numbers.

SOLUTION.—The equations are $3x + 2y = 23$
and $2x - y = 6$

Multiply both sides of the second equation by 2 and get $4x - 2y = 12$.
Now add this to the first equation and have $7x = 35$.
From this find x, then substitute the value of x in the second equation and find y.
Check with the first equation.

25. Eight times a number minus another number equals 4. Three times the first number plus 5 times the second equals 66. Find the numbers.

SUGGESTION.—Multiply the first equation through by 5 and add the second equation.

26. Two numbers are to each other as 5 to 7 and their sum is 6. Find the numbers.

27. Two numbers are as 2 to 7 and their difference is 25. Find the numbers.

28. One man said to another, "Give me one of your sheep and I'll then have as many as you." The other replied, "No; you give me one of yours and I'll have twice as many as you." How many sheep had each?

Find the numbers the letters stand for in the following and check your work.

29. $\begin{cases} x + y = 17 \\ x - y = 7 \end{cases}$

30. $\begin{cases} x + y = 17 \\ x - 2y = 8 \end{cases}$

31. $\begin{cases} 2x + y = 25 \\ x + y = 15 \end{cases}$

32. $\begin{cases} 2x + 3y = 30 \\ 2x - y = 10 \end{cases}$

33. $\begin{cases} 2x - y = 18 \\ 3x - 2y = 20 \end{cases}$

34. $\begin{cases} 6x + y = 8 \\ x - y = 3 \end{cases}$

35. $\begin{cases} 8a + 2b = 37 \\ 6a + 2b = 17 \end{cases}$

36. $\begin{cases} 3c + d = 21 \\ c - 2d = 2 \end{cases}$

USES OF THE EQUATION

§205. Thermometers.

Fig. 250 shows the way the thermometer tube is marked off and numbered on the three thermometers most used by civilized countries. The Fahrenheit, or common, scale is on the left; the Centigrade scale, used everywhere in scientific work, is on the right; and the Reaumur (Rō′mŭr) scale, much used in Germany, is between the other two.

On all scales the fundamental points are the *freezing-point* where the top of the mercury column stands when the bulb is in water containing melting ice, and the *boiling-point*, where the top of the mercury column stands when the bulb is in water commencing to boil. For any temperature the *reading* is the number which belongs to the mark on the scale at the top of the mercury column.

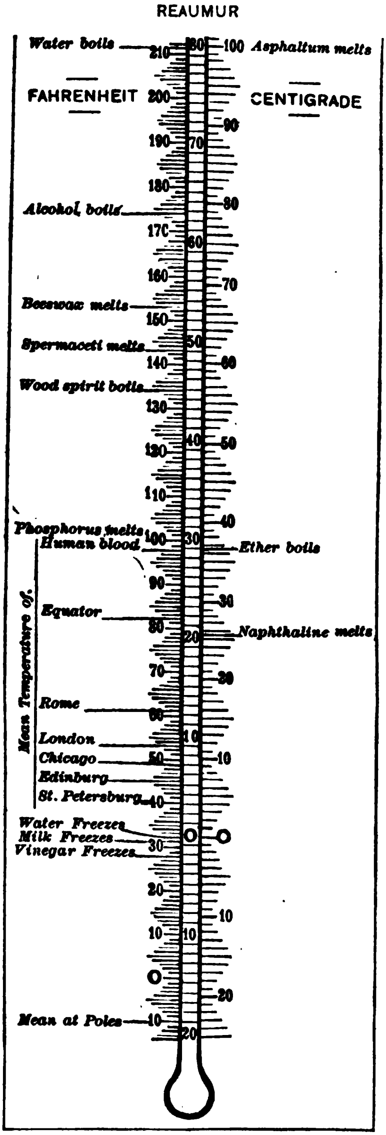

FIGURE 250

1. What is the reading for "Water boils" on the Fahrenheit scale? on the Centigrade scale? on the Reaumur scale?

2. What are the three scale readings for "Water freezes"?

3. Into how many equal parts is the space between the boiling-point and the freezing-point divided on the Reaumur scale? on the Centigrade? on the Fahrenheit?

4. The lengths of those small spaces are marked off on the tubes above the boiling-point and below the freezing-point and numbered, as shown in Fig. 250. Notice how the numbers run on each of the scales between the freezing and the boiling points; below the freezing point. How should the numbering continue above the boiling point?

5. What do we call a change of temperature sufficient to expand, or contract, the mercury column an amount equal to one of the short spaces on the Fahrenheit scale? on the Centigrade scale? on the Reaumur scale?

These are written 1°F., 1°C., and 1°R., and are read "1 degree Fahrenheit," "1 degree Centigrade," and "1 degree Reaumur."

6. If the readings above 0° are written +1°, +2°, +3°, +5°, etc., how should we write the readings below 0°?

7. What do the + and − then tell us about the readings?

8. Point out these readings in the figure: +2° F.; +36°C.; +29°R.; −14°C.; −20°R.; −38°F.; −10°F.

9. Give the readings on each scale for the mean temperature at the Equator; at Rome; London; Chicago; Edinburgh; St. Petersburg; at the Poles.

10. Which degree is the longest, the 1°F., the 1°C., or the 1°R.? Which is the shortest?

11. How many Fahrenheit degrees are equal to 80°R.? How many Centigrade degrees equal 180°F.? How many Reaumur degrees equal 100°C.?

12. On a sunny day find how many Fahrenheit degrees warmer it is in the sun than in the shade? Give the *C* and *R* readings corresponding to the two *F* readings and also the *C* and *R* differences.

§206. Applied Algebra (Graduation of Thermometers).

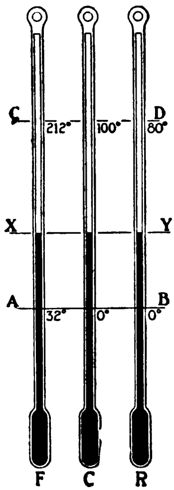

FIGURE 251

The problems of §202, p. 356, show how the use of the equation simplifies some kinds of arithmetic problems. The work given here will illustrate how some problems that would be quite difficult by arithmetic can be easily handled by means of the equation.

EXPERIMENTS. — To convert thermometer readings from one scale to another:

Three thermometer tubes, exactly alike, were filled with mercury to the same height. Their bulbs were immersed in a vessel containing melting ice. Points were marked on the tubes at the tops of the mercury columns; beside the point on the first tube was written 32, and beside those on the second and third tubes 0 was written.

All three of the bulbs were then immersed in water commencing to boil, and the tops of the mercury columns were marked; beside the mark on the first 212 was written; beside that on the second, 100; and beside the mark on the third, 80. Let *AB* of Fig. 251. denote the straight line passing through the tops of the three mercury columns when the bulbs are in

melting ice. Let CD denote the line passing through the tops of these columns when the bulbs are in boiling water.

Suppose the space lying between the line AB and the line CD on the first tube to be divided into 180 equal spaces and call each of these small spaces one degree Fahrenheit (written 1°F.). Denote its length (in in. or in cm.) by f.

Suppose this same space on the second tube between the lines AB and CD to be divided into 100 equal spaces, and call one of these smaller spaces one degree Centigrade (written 1°C.). Denote its length (in in. or in cm.) by c.

Divide the space on the third tube between the lines AB and CD into 80 equal spaces, and call one of these spaces one degree Reaumur (written 1°R.). Denote the length of this space (in in. or in cm.) by r.

Notice carefully that no two of the lengths f, c, and r are equal. Denote the length of the space from AB to CD by S.

1. S equals how many times f? c? r?

2. Since $180\,f$ equals S, and $100\,c$ equals S, how must $180\,f$ compare with $100\,c$? (Answer this by writing an equation.)

This problem exemplifies a new principle of much importance in using equations:

PRINCIPLE VI.—*Numbers that are equal to the same number are equal to each other.*

3. Since $180\,f$ equals S, and $80\,r$ also equals S, how must $180\,f$ compare with $80\,r$?

4. If $180\,f = 100\,c$, f equals how many times c?

5. If $180\,f = 80\,r$, f equals how many times r?

6. From these two equations, giving the value of f, write another equation by the aid of Principle VI.

7. Explain the meaning of the following equations:

$$(1)\ f = \tfrac{5}{9}\,c;\ (2)\ f = \tfrac{4}{9}\,r, \text{ and } (3)\ c = \tfrac{4}{5}\,r.$$

Now suppose spaces equal to f marked off on the first tube above 212 and below 32, spaces equal to c marked off on the second tube above 100 and below 0, and spaces equal to r marked off on the third tube above 80 and below 0. The first tube is then graduated to the Fahrenheit scale, the second, to the Centigrade scale, and the third, to the Reaumur scale.

§207. Equivalent Readings on the Three Thermometers.

On all thermometers, when the tops of the mercury columns are above 0, the readings are called *positive*, and are marked +; when below zero they are called *negative*, and are marked −.

If the thermometers are exposed to the same temperature between that of melting ice and of boiling water, the tops of the mercury columns will stand in a straight line, as XY. Let the mark that stands by this line at the top of the column on the first be called F; on the second top, C; and on the third, R.

1. Denoting by S the distance from the line AB to the line XY show by Fig. 251 that (1) $S = f\ (F-32)$; (2) $S = Cc$; and (3) $S = Rr$.

2. Show by Principle VI that (4) $f\,(F-32) = cC$; (5) $f\,(F-32) = rR$; and (6) $cC = rR$.

3. Show from equations (1), (2), and (3), §206, problem 7, that (7) $c = \frac{9}{5}f$; (8) $r = \frac{9}{4}f$ and (9) $r = \frac{5}{4}c$, by using Principles III and IV.

4. In equation (4) problem 2 substitute $c = \frac{9}{5}f$ from (7) problem 3 and by Principles IV and II show that (I) $F = \frac{9}{5}C + 32$.

5. In a similar way show that the following equations are true:

(II) $F = \frac{9}{4}R + 32$;
(III) $C = \frac{5}{9}\ (F-32)$;
(IV) $C = \frac{5}{4}R$;
(V) $R = \frac{4}{9}\ (F-32)$;
(VI) $R = \frac{4}{5}C$.

NOTE.—F, C, and R may be regarded as standing respectively for any corresponding readings on the Fahrenheit, Centigrade, and Reaumur thermometers.

§208. Problems.

1. Convert the following into their F. and R. equivalents by the aid of the proper equations (I) to (VI).

MELTING TEMPERATURES ON CENTIGRADE SCALE.

Ice	0.0°	Zinc	412.0°
Benzol	+ 4.4°	Antimony	432.0°
Tallow	43.0°	Silver	1000.0°
Paraffin	46.0°	Copper	1100.0°
Wax	62.0°	Gold	1200.0°
Sulphur	115.0°	Cast iron	1200.0°
Tin	230.0°	Cast steel	1375.0°
Bismuth	250.0°	Wrought iron	1600.0°
Cadmium	320.0°	Platinum	1775.0°
Lead	326.0°	Iridium	1950.0°

2. Convert these Fahrenheit boiling temperatures into their C. and R. equivalents:

Ether	$+95°$	Alcohol..........	$+172.4°$
Carbon Disulphide	$+114.8°$	Water	$+212°$
Sulphuric Acid ...	$+14°$	Mercury.........	$+674.6°$
Chloroform	$+141.8°$	Zinc	$+1904°$

3. Convert any observed Fahrenheit readings to their Centigrade and Reaumur equivalent readings.

4. The heat of the body of a healthy man is 37.2°C. What are the Fahrenheit and Reaumur equivalents?

5. Solid carbonic acid dissolves in ether and the solution is a liquid of –90°C. What is the Fahrenheit equivalent?

SOLUTION.—In equation (I) if $C = -90°$, $\frac{9}{5}C = -162°$. Then 162° measured downward and 32° measured upward, or, $-162° + 32° = -130°$.

6. For every vapor and gas there is a so-called *critical* temperature above which it remains in the gaseous condition, no matter how high the pressure upon it. The following are the critical temperatures of some vapors and gases: Ether vapor, +196°C.; carbonic acid, +31°C.; etheline, +9°C.; oxygen, –118°C.; nitrogen, –145°C., and hydrogen, –174°C. What are the Fahrenheit equivalents?

§209. Laws of Thermometer Represented Graphically.

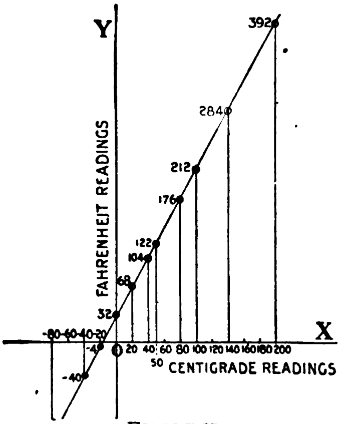

FIGURE 252

When it is necessary to change a great many readings from one scale to another it is convenient to plot the equations. To illustrate this, take the equation $F = \frac{9}{5}C + 32$. Draw two perpendicular lines, as OX and OY, Fig. 252. Let all Centigrade readings be measured off horizontally and the corresponding Fahrenheit readings vertically to any convenient scales.

The equation shows that if $C = 0°$, $F = 32°$; if $C = 20°$, $F = 68°$; and so on. In this way we fill out the following table:

C.	F.	C.	F.
0°	32°	100°	212°
20°	68°	200°	392°
40°	104°	−20°	−4°
50°	122°	−40°	−40°
75°	167°	−80°	−112°

NOTE.—The numbers preceded by the sign — are readings below zero, and such numbers for C. must be measured off from 0 toward the left, and for F. they must be measured off downward.

Study the points on Fig. 252 and note whether they seem to lie on a straight line. With a ruler draw a straight line through these points.

For minus (—) values of C, as for $C = -20$, proceed thus: $F = \frac{9}{5} \times (-20) + 32 = 9 \times (-\frac{20}{5}) + 32 = 9 \times (-4) + 32 = -36 + 32$. But 36° measured downward and then 32° measured upward is the same as 4° measured downward, or $F = -4°$, if $C = -20°$; and similarly for other minus (—) values of C.

1. Measure off the value $C = 60°$, then measure vertically upward to the line drawn through the points, thus obtaining the corresponding Fahrenheit reading. What is F. for C. = 60°?

2. Similarly, find from the drawing the values of F. corresponding to these values of C.: 120°; 160°; 180°; 220°; −60°; −100°; 110°.

3. Make a similar table and plot of one or more of the other five equations of problem 5, p. 367.

METHODS OF SHORTENING AND CHECKING CALCULATIONS

§210. Illustrations.

1. Additions, subtractions, multiplications, and divisions are conveniently checked by casting out the 9's, as explained on pp. 28, 54, and 75.

2. To check against gross errors (blunders), first *think* through the problem, making rough mental calculations with numbers that are approximately correct, and decide *about what the result must be.*

3. Check by performing reverse operations; that is, check addition by adding columns in reverse order; check subtraction by adding the subtrahend to the remainder; check division or square and cube roots by multiplication, and so forth.

The second rule may be illustrated by a few problems:

1. Find the value of $15\frac{4}{5}$ A. of land at $\$87\frac{1}{2}$.

Think thus: 16 A. @ \$90 would be worth \$1440; at \$85, 16 A. would be worth \$80 less, or \$1360. At $\$87\frac{1}{2}$, $15\frac{4}{5}$ A. of land is worth $\frac{1}{5}$ of \$85 less than $\$1440 - \frac{1}{2}$ of \$80 (=\$1400), or about \$1383. The exact computation gives \$1382.50.

Or thus: $87\frac{1}{2} = \frac{7}{8}$ of 100. Therefore $15\frac{4}{5} \times 87\frac{1}{2} = 1580 \times \frac{7}{8} = 7 \times 197.5 = \$1382.5$. *Ans.* \$1382.5.

2. On June 13, 1903, wheat is quoted in Chicago at $76\frac{3}{8}$¢ per bushel. Find the cost of 150 bu. at this price.

3. 250 shares of Illinois Central R. R. stock sold at $\$135\frac{1}{4}$ a share. For how much did they sell?

4. The diameter of a circular rod is $1\frac{3}{4}''$. How many inches in the circumference of a right section of the rod? How many square inches are there in the area of a right section of the rod? (For an approximation use $\pi = 3\frac{1}{7}$.)

5. The outside diameter of a circular hollow iron tube is $2\frac{1}{2}''$ and the inside diameter is $2''$. How many cubic inches are there in a $12'$ length of the tube?

6. A distance was measured with a chain 98.75 ft. long and was found to contain 38.75 lengths of the chain. The chain was supposed to be $100'$ long. How great an error was made in measuring the line by using the supposed length?

§211. Shortening and Checking Addition.

1. The noon temperatures on 7 successive days were 66°, 54°, 44°, 62°, 66°, 79°, 88°. Find the average for the week.

	66	
	54	
	44	164
226	62	
	66	292
391	79	
	88	
	7) 459	
	65.6° *Ans.*	

MENTAL WORK. $70 + 50 + 44 = 164$; $164 + 60 = 224$; $224 + 2 = 226$; $226 + 70 = 296$; $296 - 4 = 292$; $292 + 80 = 372$; $372 - 1 = 371$; $371 + 90 = 461$; $461 - 2 = 459$.

This is called two-column addition. A little practice will make this method useful for 1, 2, or 3 columns of figures. It may be used to advantage to check additions made in the ordinary way.

Another method in use by expert accountants is to group the figures into sums of 10, 20, 30, and so on. Thus, in the given problem, $8 + 2$, $6 + 4$, $6 + 4$, and 9 make 39. Then $7 + 3$, $8 + 6 + 6$, $6 + 4$, 5, are 45. Sum, 459.

2. Write a few two and three column addition problems and practice these methods until you can use them rapidly.

§212. Making-up Method of Subtraction.

1. A paying-teller in a bank had $5485 in his cash drawer in the morning, and during the day he paid out the following amounts: $37.50; $165.75; $10.25; $3.50; $2.88; $1.76; $65.17; $968.23; $3.67. How much money remained in the drawer?

CONVENIENT FORM.

Total,	$5485.00
	37.50
	165.75
	10.25
	3.50
	2.88
	1.76
	65.17
	968.23
	3.67
	$4226.29, balance

MENTAL WORK.—Add the first column, thinking thus: 10, 23, 31, 41. 41 and 9 make the next larger number than 41 ending in 0 (the first figure in the total.) Write the 9 in the result and add the 5 into the second column. Then 11, 21, 34, 48. $48 + 2 = 50$, the next number larger than 48 which ends in 0 (the second figure of the total). Write 2 in the result and add 5 into the third column. Then, 21, 31, 39. $39 + 6 = 45$. Write 6 and add 4 into fourth column. Then 10, 17, 26. $26 + 2 = 28$. Write 2 and add 2 to next column. Then, 12. $12 + 2 = 14$, and finally, $1 + 4 = 5$. Write the 4.

The advantage of this method is that it foots all the numbers and subtracts their sum, at once, as the numbers stand in the account book, from the total, giving the balance directly.

2. A bank customer's deposit at the beginning of the month was $398.75. During the month he drew out the following amounts: $16.75; $1.75; $5.25; $12.87; $128.32; $40.45; $2.18; $9.16; $1.57; $11.38; $12.62. Find the customer's balance at the end of the month.

§213. Shortened Multiplication.

1. Multiply 73 by 67.

$$\begin{array}{l|l} 73 = 70 + 3 & 73 \times 67 = (70 + 3)(70 - 3) = 70^2 + \cancel{70} \times \cancel{3} - \cancel{3} \times \cancel{70} - 3^2 = \\ 67 = 70 - 3 & \qquad 70^2 - 3^2 = 4900 - 9 = 4891. \end{array}$$

Algebraic form: $(a + b)(a - b) = a^2 - b^2$.

This applies to finding the product of any two numbers the sum of whose units is 10 and whose tens digits differ by 1.

RULE.—*Find the difference between the square of the tens and the square of the units in the larger number.*

2. Find these products by the rule:

(1) 34×26.	(4) 58×42.	(7) 71×69.
(2) 22×18.	(5) 65×55.	(8) 99×81.
(3) 43×37.	(6) 98×82.	(9) 95×85.

3. Find the square of 38.

$$38^2 = (30 + 8)^2 = 30^2 + 2 \times 8 \times 30 + 8^2 = 1444$$

Algebraic form: $(a + b)^2 = a^2 + 2ab + b^2$.

Show the correctness of the following rule:

RULE.—*Square the tens, double the product of the tens by the units and square the units, then add the three results. The sum is the square of the number.*

4. Square these numbers by the rule:

(1) 36.	(3) 64.	(5) 19.	(7) 125.
(2) 47.	(4) 58.	(6) 95.	(8) 148.

NOTE.—Call the 12 in (7) 12 tens; also call the 14 in (8) 14 tens.
36 might be written $40 - 4$. Then $36^2 = (40 - 4)^2$, $(40 - 4)^2 = 40^2 - 2 \times 4 \times 40 + 4^2$. Algebraic form: $(a - b)^2 = a^2 - 2ab + b^2$. Make a rule for squaring 36 in this form.

When long decimals are to be multiplied and the product is required to only a few decimal places, contracted multiplication is of great advantage. The next problem illustrates this sort of multiplication.

5. The radius of a circle is 238.36 ft. What is the length of the circumference to the second decimal place, or to the nearest .01 foot?

NOTE.—The length of the diameter is 476.72 feet.

COMMON FORM

476.	72
3.	1416
28	6032
47	672
1906	88
4767	2
143016	
1497.66	3552

SHORTENED FORM

476.72	
6141.3	digits reversed
1430.16	
47.67	
19.07	
48	
29	
1497.67	

EXPLANATION.—In the shortened form the digits of the multiplier are written in reverse order, and the units digit is always written under that decimal place in the multiplicand which is to be the last one retained in the product.

Multiply by units digit first, then by tens, and so on; in each case begin the multiplication by any digit with the digit just above it in the multiplicand. Begin the writing of each partial product in the same vertical line on the right.

NOTE.—It is necessary on beginning to multiply by any digit to glance at the product by the preceding digit of the multiplicand to see how many units are to be added into the product by the digit just above. Thus, the multiplication by 4 would begin with 6, but 4 times the preceding digit (7) is 28, and this being nearly 3, the product 4 × 6 would be increased by 3, giving 27.

Expert computers use the shortened form altogether.

6. Find the following products to the second decimal place by the method of shortened multiplication:

(1) 36.428 × 3.1416. (3) 7.8843 × 1.0863.
(2) 186.086 × 108.336. (4) 168.7431 × 28.329.

7. Find these products to .001 by shortened multiplication

(1) 36.1872 × 6.8734. (3) 629.3865 × 3.1416.
(2) 128.63 × 3.8629. (4) 1284.683 × 3.1416.

§214. Shortened Division.

1. Divide 648.7863 by 68.372 to the nearest .01.

CONVENIENT FORM

..	9.49—	Quotient
68.37)	648.7863	
	615 33	
	33 45	
	27 35	
	6 10	
	6 15	

EXPLANATION.—Find the units digit of the quotient in the usual way. Then cut off one digit from the right of the divisor and find the next digit of the quotient, then cut off another digit from the divisor, etc. A dot is sometimes placed over each digit in the divisor as it is set aside.

2. Find the following quotients to two decimal places:

(1) 1786.786 ÷ 3.1416.
(2) 632.068 ÷ 8.6249.
(3) 1206.3862 ÷ 28.3762.
(4) 865.28476 ÷ 361.2946.

§215. Shortened Square Root: Square Root by Subtraction.

```
          261
        68432.93628
        4
   46 | 284
      | 276
  521 | 832
      | 521
  522) 311.9 (586
       2670
        449
        417
         32
Square root = 261.586
```

1. Find the square root of 68432.93628 correct to the third decimal figure.

EXPLANATION.—By placing a dot over every alternate digit in the given number, counting both ways from the decimal point, we obtain as many dots as there are to be figures in the square root. Find half the figures by the ordinary method of square root, and the rest by contracted division as shown. The result will very nearly equal the square root.

2. Find, by the shortened method, to two decimals the square roots of the following:

(1) 6432.1864. (2) 38629.72468. (3) 8764.932651.

Square root may be found by subtraction, after noticing that $1 = 1^2$; $1 + 3 = 2^2$; $1 + 3 + 5 = 3^2$; $1 + 3 + 5 + 7 = 4^2$; and so on.

It is seen that the square root of the sum of the odd numbers in order from 1 upward is equal to the number of odd numbers added. An example will illustrate the use of this principle:

3. Extract the square root of 104976.

```
     104976 (324=square root
     1
     9
     3
     6
     5          3 subtractions
61 | 149
   |  61
      88
      63
641 | 2576    2 subtractions
    |  641
      1935
       643
      1292
       645
       647
       647    4 subtractions
```

EXPLANATION.—Place a dot over each alternate digit beginning on the right, thus separating the number into groups of 2 digits each. From the first group on the left subtract 1, then 3, then 5, and so on as shown, until the remainder is less than the next odd number. The number of subtractions is the first root digit. In the present case it is 3.

Bring down the next two-digit group. Double the root digit found and annex 1 to it, and subtract, then replace the 1 by 3 and subtract, etc. The number of subtractions indicates the next root digit.

Study the remaining steps and learn how to proceed further. This method gives the exact square root and may be used to check results found in the ordinary way. Some computing machines are based on this method of obtaining the square root.

4. Find the square roots, by subtraction, of the following:

(1) 1156. (3) 174.24. (5) 54,756.

(2) 44,944. (4) 49,284. (6) 289,444.

SYNOPSIS OF DEFINITIONS

PAGE

6. The *average* of two or more numbers is their sum divided by the number of them. (*Cf.* pp. 85, 132.)
8. A *board foot* is a board one foot long, one foot wide, and not more than one inch thick.
10. *Cash rent* of farming land is a stated amount of cash per acre.
    Grain rent of farming land is a stated part of all the crop.
    The *tenant* farmer is one who raises his crop on another man's farm.
12. *Normal* here means average. (*Cf.* p. 13.)
    The *mean* of two numbers is half their sum.
17. The *digits* (or *figures*) are the ten characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
    The *name value* of a digit is the value depending only upon the name of the digit.
    The *place value* of a digit is the value depending only upon the place of the digit.
22. *Addition* is combining numbers into a single number.
    The *sum* or *amount* is the result of the addition.
    The *addends* are the numbers to be combined or added.
26. The *displacement* is the number of tons of water a floating vessel pushes aside.
33. *Subtraction* means either of two things:

    (1) The way of finding the difference, or remainder, of two numbers.

    (2) The way of finding either one of two addends when their sum and the other addend are known.

    With the first meaning, the number from which we subtract is the *minuend*. The number to be subtracted is the *subtrahend*. The result is called the *difference* or *remainder*.

    With the second meaning, the known sum is the *minuend*. The known addend is the *subtrahend*. The unknown addend is the *difference* or *remainder*.
43. *Literal numbers* are those that are denoted by letters.
44. *Multiplication* of whole numbers is a short way of finding the sum of equal addends when the number of addends and one of them are given.

    The given addend is the *multiplicand*.

    The number of equal addends is the *multiplier*.

    The result or sum is the *product*.

    When a problem is expressed in an equation, the equation is called the *statement* of the problem.

The number which may stand in place of x in an equation is called the *value of x*.

A *factor* of a given whole number is one of two or more whole numbers, which, multiplied together, produce the given number. (*Cf.* pp. 51, 142.)

52. A number which has factors other than itself and 1 is called a *composite* number. (*Cf.* p. 143.)

A number which has no factors other than itself and 1 is a *prime* number. (*Cf.* p. 143.)

57. *One inch of rainfall* means one cubic inch of water for each square inch of horizontal exposed surface.

The number of cubic units (cu. in., cu. ft., cu. yd., etc.) a vessel holds, when full, is called its *capacity*.

61. *Division* of whole numbers is a short way of subtracting one number from another a certain number of times in succession. (*Cf.* p. 62.)

62. *Division* is a way of finding one of two numbers when their product and the other number are given.

The product is called the *dividend*.

The given number is the *divisor*.

The required number is the *quotient*.

We may say also that the *dividend* is the number to be divided, the *divisor* is the number by which the dividend is measured or divided, and the *quotient* is the measure. (*Cf.* p. 61.)

78. A *one-brick wall* is a wall one brick thick, the bricks lying on the largest surfaces, the sides being exposed.

85. *To average* means to find the average.

86. The *range of temperature* is the difference between the highest and the lowest temperatures.

97. Such an expression as $y = 10$ is called an *equation*.

The number on the left of the sign of equality is called the *first member*, or the *left side*, of the equation. The number on the right is called the *second member*, or the *right side*, of the equation.

98. The pencil point of the compasses is called the *pencil-foot* or the *pen-foot*. The other point is called the *pin-foot*.

A *circle* is a curve such as is drawn with compasses.

An *arc* of a circle is a part of a circle.

The *center* of a circle is the point where the pin-foot of the compasses was placed to draw the circle.

A *diameter* of a circle is a straight line joining two points of the circle and passing through the center. (*Cf.* p. 107.)

A *radius* of a circle is a straight line joining its center and a point of the curve.

Circles whose centers are at the same point are called *concentric* circles.

101. Dividing a line into two equal parts is called *bisecting* the line.
102. An *equilateral* triangle is an equal-sided triangle.
An *isosceles* triangle has at least two sides equal. (*Cf.* p. 103.)
That side of an isosceles triangle not equal to either of the two equal sides is called the *base* of the isosceles triangle.
103. A *scalene* triangle has no two sides equal.
104. A regular *hexagon* is a regular six-sided figure.
105. The sum of all the bounding lines of a figure is called the *perimeter* of the figure.
Money is the common measure of the value of all articles that are bought and sold.
107. One of a whole number of equal parts of a given magnitude is called a *fractional unit.* (*Cf.* p. 138.)
109. A representation of an object, showing the various parts as folded back and spread out on a flat surface, is called a *development.*
110. A *parallelogram* is a quadrilateral whose opposite sides are parallel. (*Cf.* pp. 282, 283.)
A *rectangle* is a parallelogram whose angles are (equal) right angles. (*Cf.* pp. 282, 283.)
A *square* is both a rectangle and a rhombus.
111. The *altitude of a parallelogram* is the distance square across.
The *altitude of a triangle* is the shortest distance to the base from the opposite corner. With all except isosceles triangles any side may be regarded as the *base.*
118. The *mean solar day* is the average time interval during which the rotation of the earth carries the meridian of a place eastward from the sun back around to the sun again. It is the average length of the interval from noon to the next noon. (*Cf.* p. 217.)
119. A *section* of land is a tract one mile square and containing 640 acres.
120. A *township* is a tract of land six miles square.
124. *Per cent* means hundredth or hundredths.
125. *Interest* is money paid for the use of money. (*Cf.* p. 252.)
132. The *average rate of running* is the distance divided by the time.
135. The *ratio* of one number (or quantity) to a second number (or quantity) is the quotient of the first number (or quantity) divided by the second.
136. The ratio of one magnitude to a second magnitude is called also the *measure* of the first by the second.
The result of measuring one number by another is called the *numerical measure* of the first by the second.
An equation of ratios is called a *proportion.*
138. The *fractional unit* of a fraction is one of the equal parts expressed by the fraction. (*Cf.* p. 107.)

139. The number above the fraction line is called the *numerator* (meaning *numberer*).

The number below the fraction line is called the *denominator* (meaning *namer*).

The numerator and the denominator are together called the *terms* of a fraction.

141. A fraction is said to be in its *lowest terms* when the numerator and the denominator are the smallest possible whole numbers without changing the value of the fraction

The *greatest common divisor* (G. C. D.) of two numbers is their greatest exact common divisor.

142. Two numbers that have no common factor, except 1, are said to be *prime to each other*.

A factor of a number is an exact divisor of the number, or a divisor that is contained without a remainder. (*Cf.* p. 44.)

143. A number that has no factors except itself and 1 is called a *prime* number. (*Cf.* p. 52.)

A number that has factors besides itself and 1 is called a *composite* number. (*Cf.* p. 52.)

Any number that can be exactly divided by the number 2 is called an *even* number. All other whole numbers are called *odd* numbers.

147. A denominator which is common to two or more fractions is called a *common denominator*. (*Cf.* p. 140.)

When a common denominator is the least number that can be found which may be used as a common denominator of the fractions, it is called the *least common denominator* (L. C. D.).

148. A number that can be exactly divided by another number is called a *multiple* of the latter number.

A number that can be exactly divided by two or more numbers is called a *common multiple* of those numbers.

149. The *least common multiple* (L. C. M.) of two or more numbers is the least whole number that is exactly divisible by each of the numbers.

151. A *proper* fraction is a fraction whose numerator is less than its denominator.

An *improper* fraction is a fraction whose numerator is equal to, or greater than, its denominator.

A *mixed* number is a number that is composed of an integer and a fraction.

160. To *multiply* a whole number by a *fraction* means to divide the multiplicand into as many equal parts as there are units in the denominator, and to take as many of these equal parts as there are units in the numerator of the multiplier.

170. Dividing 1 by any number, whole or fractional, is called *inverting* the number.

The *reciprocal* of any number is the number inverted.

172. Fractions containing fractions in one or both terms are called *complex* fractions.

The outside terms of a complex fraction (such as $\frac{\frac{6}{7}}{\frac{5}{11}}$) are called the *extremes*, and the inside terms are called the *means*.

176. A straight line connecting the mid-point of a side of a triangle with the opposite corner is called a *median* of the triangle.

The *vertex* of an angle is the corner. (The plural of *vertex* is *vertices* (vêr′-tĭ-sēs.)) (*Cf.* pp. 274, 288.)

178. To *trisect* a magnitude is to divide it into three equal parts.

179. A right-angled triangle is called a *right triangle.*

180. *Parallel* lines are lines running in the same direction.

182. A square is *inscribed* in a circle if the vertices of the square are all on the curve.

186. The first, second, third, and fourth numbers of a proportion are called the *first, second, third,* and *fourth terms* of the proportion.

The first and the fourth terms of a proportion are called the *extremes*, and the second and the third terms, the *means.*

The first two terms of a proportion are together called the *first couplet;* and the third and the fourth terms, the *second couplet.*

190. A dot, called the *decimal point*, or *point*, is used to show the units' digit. The point always stands (or is supposed to stand) just to the right of the units' digit or place.

191. The unit of the 1st place, or digit, to the right of the decimal point is called the *tenth;* of the 2d place, or digit, the *hundredth;* of the 3d, the *thousandth;* of the 4th, the *ten-thousandth;* and so on.

192. A *decimal fraction*, or *decimal*, is a fraction whose denominator is 10, 100, 1000, or some power of ten, in which the denominator is not written but is indicated by the position of the decimal point.

A *power* of 10 here means a number obtained by using 10 as a factor any whole number of times.

193. A *pure* decimal is a decimal whose value is less than one.

A *mixed* decimal is a decimal whose value is greater than one.

Numbers expressed in both decimals and common fractions are called *complex* decimals.

A *simple* decimal is expressed without the use of common fractions.

Finding the sum of decimal numbers is called *addition of decimals.*

195. Finding the difference of decimal numbers is called *subtraction of decimals.*

198. By the *number of decimal places* of a number is meant the number of digits (zero included) on the right of the decimal point.

204. The distance round a circle is sometimes called the *circumference* of the circle. (*Cf.* p. 107.)

207. The *specific gravity* of any solid or liquid substance is the ratio of its weight to the weight of an equal bulk of water.

209. Decimals that do not terminate are called *non-terminating* decimals.

Non-terminating decimals that repeat a digit or group of digits indefinitely are called *repetends*, or *circulating decimals*, or *circulates*.

211. A *right* section is a section made by cutting squarely across.

213. A *denominate* number is a number whose unit is concrete.

A *concrete* unit is a unit having a specific name.

A *compound* denominate number is a number expressed in two or more units of one kind.

215. A *perch* of stone is a square-cornered mass, $1' \times 1\frac{1}{2}' \times 16\frac{1}{2}' = 24\frac{3}{4}$ cubic feet.

A *cord* of firewood is a straight pile, $4' \times 4' \times 8' = 128$ cubic feet.

A *cord foot* is a straight pile of wood, $4' \times 4' \times 1' = 16$ cubic feet.

217. A *leap* year is a year of 366 days.

A *common* year is a year of 365 days.

225. Reduction from higher to lower denominations is called *reduction descending*.

Reduction from lower to higher denominations is called *reduction ascending*.

229. A *meter* is approximately one ten-millionth of the length of the part of a meridian of the earth, that reaches from the equator to the pole, called a quadrant of the earth's meridian.

231. An *are* is one square dekameter.

A *liter* is one cubic decimeter.

A *gram* is the weight of one cubic centimeter of distilled water at the temperature of its greatest density (39.1° Fahrenheit).

236. The number written before the sign "%" is called the *rate per cent*. The number, together with the sign "%," is called the *rate*. (*Cf.* p. 252.)

239. The result of finding a given per cent of any amount, or number, is called the *percentage*.

The amount, or number, on which the percentage is computed is called the *base*.

243. *Elevation* means height above mean sea level.

248. *Commission* is a sum of money paid by a person or firm, called the *principal*, to an agent for the transaction of business. It is usually reckoned as some per cent of the amount of money received or expended for the principal.

249. A shipment of goods sent to an agent to be sold is called a *consignment*.

A commercial (or a trade) *discount* is a certain rate per cent of reduction from the listed prices of articles. The discount is usually allowed for cash payments or for payment within a specified time. (*Cf.* p. 260.)

252. *Interest* is money charged for the use of money. It is reckoned at a certain rate per cent of the sum borrowed for each year it is borrowed. (*Cf.* p. 125.)

When money earns 3, 6, 7, or 10 cents on the dollar *annually* (each year) the *rate* is said to be 3%, 6%, 7%, or 10% *per annum* (by the year), and the *rate per cent* is said to be 3, 6, 7, or 10. (*Cf.* p. 236.)

The sum of money on which interest is computed is called the *principal*.

The principal plus the interest is called the *amount*.

259. A *promissory note* is a written promise, made by one person or party, called the *maker*, to pay another person or party, called the *payee*, a specified sum of money at a stated time.

The sum of money for which the note is drawn is called the *face value*, or the *face*, of the note.

The date on which the note falls due is called the *date of maturity*, and the *time to run* from any given date is the time yet to elapse before the note falls due.

260. *Discount* is a deduction from the amount due on a note at the date of maturity. (*Cf.* p. 249.)

The sum of money which, at the specified rate and in the time the note is to run before falling due, will, with interest, amount to the value of the note when due, is called the *present worth* of the note.

The difference between the value of the note, when due, and the present worth is called the *true discount*.

The *bank discount* of a note is the interest upon the value of the note when due, from the date of discount until the date of maturity.

261. When a note or bond is paid in part the fact is acknowledged by the holder by his writing the date of payment, the sum paid, and his signature on the back of the note or bond. This is called an *indorsement*.

264. The small wheels under the front of a locomotive engine are called *engine truck wheels*, or *leaders*. The large wheels are called *drivers*. The smaller wheels just behind the drivers are called *trailers*.

266. *Tractive* force is pulling (or drawing) force.

267. Replacing a letter (in an equation) by a number is called *substituting* the number for the letter.

Performing the operations indicated in an equation and obtaining the number-value of a letter is called *finding the value* of that letter.

274. The surfaces of the cube meet each other in edges, thus forming *lines*.

The corners are called *vertices*. A single corner is a *vertex*. (*Cf.* pp. 176, 288.)

The edges meet each other in corners of the cube, thus forming *points*.

278. When two lines meet, making the angles at their point of meeting (intersection) equal, the lines are said to be *perpendicular* to each other, and each is called a *perpendicular* to the other.

The angles thus formed are called *right angles*.

279. Lines which go through the same point are called *concurrent* lines.

282, 283. A *quadrilateral* is a four-sided figure.

282, 283. A *rhombus* is a parallelogram whose sides are equal.

283. A *rhomboid* is a parallelogram that is not a rectangle.

A *trapezoid* is a quadrilateral having at least one pair of opposite sides parallel.

285 A *quadrant* of arc is one quarter of a complete circle.

An *angle* is the amount of turning of a line about a point as a pivot. It may also be regarded as the difference of direction of two lines.

A *straight* angle is the sum of two right angles.

286. An *angular degree* is one of the ninety equal parts of a right angle.

A *degree of arc* is one of the ninety equal parts of a quadrant.

A *sextant* is one-sixth of a complete circle.

An *octant* is one-eighth of a complete circle.

287. A *perigon* is an angle equal to one complete revolution.

288. If two lines intersect each other, two angles lying opposite to each other are called *opposite* or *vertical* angles.

The lines which include an angle are called the *sides* of the angle

The point where the sides meet is called the *vertex* of the angle. (*Cf.* pp. 176, 274.)

289. An angle that is smaller than a right angle is called an *acute* angle.

An angle that is larger than a right angle and less than a straight angle is called an *obtuse* angle.

291. Two angles whose sum equals a right angle, or 90°, are called *complemental* angles. Two angles whose sum equals two right angles, or 180°, are called *supplemental* angles.

297. An imaginary curved line, called the *equator*, divides the surface of the earth into the northern and the southern hemispheres.

298. The imaginary curved lines running east and west are called *parallels;* those running north and south are called *meridians.* (*Cf.* pp. 299, 310).

299. *Longitude* is the distance in degrees, minutes, and seconds (of arc) measured on a parallel due eastward or westward from a chosen meridian called the *prime meridian.* (*Cf.* pp. 297, 300.)

Latitude is a similar distance measured northward or southward on a meridian.

305. The *date line* is the 180th meridian.

306. A four-sided figure (a *quadrilateral*) having at least one pair of parallel sides is called a *trapezoid.*

The *altitude of a trapezoid* is the distance square across between two parallel sides called the *bases of the trapezoid.*

307. The lengths of the bases and of the altitude of a trapezoid are called its *dimensions.*

A *square* of roofing is a ten-foot square (100 sq. ft.).

Shingles are said to be laid *so many inches to the weather* when the lower end of each course of shingles extends so many inches below the course next above it.

310. A *township* is a tract of land six miles square.

A *range* is a tier (or row) of townships running north and south.

311. A *section* is a tract of land one mile square.

Such of the north and the west rows of half-sections of a township as do not have exactly 320 acres each are called *lots*.

312. The *volume* of any figure is the number of cubical units within its bounding surfaces.

317. A straight line connecting two points of an arc is called a *chord* of the arc.

318. A circle is said to be *circumscribed* around a triangle and the triangle is said to be *inscribed* in the circle if all the vertices of the triangle are on the curve.

319. The longest side of a right triangle, that is, the side opposite the right angle, is called the *hypothenuse.*

321. The product obtained by using any number twice as a factor is called the *square* of that number.

322. The *square root* of a number is one of its two equal factors

326. The *cube* of a number is the product obtained by using the number three times as a factor.

327. The *cube root* of a number is one of its three equal factors.

328. In triangles having the same shape, angles lying opposite proportional sides in different triangles are called *corresponding angles.*

In triangles having the same shape, sides lying opposite equal angles are called *corresponding sides.*

336. The amount paid for insurance is called the *premium.*

The written agreement between an insurance company and the insured is called a *policy.*

The amount for which the property is insured is called the *face* of the policy.

338. A *tax* is a sum of money levied by the proper officers to defray the expenses of national, state, county, and city governments, and for public schools and public improvements.

Assessed valuation means the estimated value of the property that is assessed.

341. A *stock* (called also a *stock certificate*) is a written agreement made by a company to pay the holder a certain part of the earnings of the company.

When a stock company pays to the holders of its stock \$2 on each \$100 of its capital stock, or 2% on its stock, the company is said to be paying a \$2 *dividend*, or a 2% *dividend.*

At par means at its face value.

Above par means at more than its face value.

Below par means at less than its face value.

A *bond* is a written agreement made by a national, state, county, or city government, or by a company, to pay the holder interest at a stated rate on a stated sum of money, called the *face* of the bond.

A *broker* is a man that makes a business of buying and selling stocks and bonds for other people.

A broker's charge for his services is called *brokerage*.

342. *Preferred stocks* are stocks which pay a fixed dividend before any dividends are paid on common stocks.

Dividends are paid upon *common stocks* after expenses and dividends on preferred stock have been paid

Government 2*s* are government bonds paying 2% interest.

343. *Compound interest* is interest on interest.

The *compound amount* is the principal plus the compound interest.

345. *Coupon notes* are notes, attached to interest-bearing notes, for the amount of interest due each at interest-paying period.

349. An expression in which the sign $<$ or $>$ stands between two numbers is called an *expression of inequality*.

350. The signs $=$, $<$, and $>$ are called *relation* signs.

General Definitions

Quantity is limited magnitude.

Number is the result of the measurement of quantity.

Number may also be defined as the ratio of quantities of the same kind.

Arithmetic is the science of numbers and the art of using them.

GENERAL INDEX

EXPLANATORY.—The following index has been prepared for daily use by young students of various ages, and the principal purpose underlying its construction has been to enable all students to find *readily* and *quickly* the thing desired, that it may be in use not occasionally but continually. To this end the entries have been arranged not merely for the noun (or the principal noun) but also, in many cases, for other words of the phrase, thus making the index usable also by students to whom grammatical distinctions are not entirely familiar.

In reading, the phrase after the comma (where one occurs) is often to be read first; thus "addition, short methods for" is read "short methods for addition."

The references are to pages. Where two page references are separated by a dash the meaning is "and included pages;" thus 27-30 means pages 27, 28, 29, 30.

ANSWERS

TO

THE RATIONAL GRAMMAR SCHOOL ARITHMETIC

Page 5

3. $.07
4. $.18
6. $2.75 change
9. $904.56
10. $\frac{1}{16}$th part

Page 6

11. $62.40
12. 1897—$79.86
1898—$80.40
1899—$95.28
1900—$95.67
1901—$101.59
13. $21.73
14. No.
Breadstuffs, ... $6.49
Meat $2.33
Dairy and Garden $2.88
Other Food .64
Clothing .90
Metals 3.81
Misc. 4.68
15. $\frac{1351}{7886}$, $\frac{1328}{6827}$, $\frac{2000}{10159}$
16. $90.56
17. Bread $15.01
Meat $8.04
Dairy $13.67
Other Food $9.01
Clothing $15.57
Metals $14.54
Misc. $14.73
19.
1897—98—$.54
1898—99—$14.88
1899—1900—.39
1900—01—$5.92

Page 7

1. (a) 160 rods
(b) $\frac{1}{2}$ mile
(c) 640 rods of wire
3. 320 rods wire

Page 8

6. 640 posts
7. $160
8. $49.92
9. $71.25

Page 8

10. $122.07
12. 55
13. 550
15. $50.37
16. 5; 10; 12
17. 20; 5; 7$\frac{1}{2}$; 15; $\frac{5}{8}$; 6$\frac{2}{3}$.
18. 4; 8

Page 9

19. 6$\frac{2}{3}$; 10$\frac{2}{3}$
20. 13$\frac{1}{3}$; 16; 24
21. 6; 30
22. $5.94
23. $109.77
24 $181.29
25. $529.66

Page 10

1. $2,105.60; $13.16
2. $8.16; $1,305.60
3. $800
4. for $\frac{1}{2}$ of all crops
$192 more profitable
5. Corn more profitable by $384
6. Wheat $512 more
Corn $896 more
7. Grain $160 more
8. Cash rent $96 more
9. 3264 bu.
10. $1103.64
11. $729.64; $374

Page 11

1. 121.70; 304.25; 486.80 cu. in.
2. 91.275; 106.487; 129.306; 140.71
3. 60.85; 7.606; 64.653; 76.06; 79.86; 83.668
4. 121.70 cu. in.

Page 14

1. 35; 3890$\frac{4}{5}$; 11671$\frac{7}{9}$
2. 662,500 lbs.
3. 4668$\frac{3}{4}$ lbs.

Page 15

4. 21787$\frac{1}{2}$ lbs.
5. 184078.125 lbs
6. 15446$\frac{2}{3}$; 1634$\frac{1}{16}$
7. 49021.875 lbs.
8. 1683 lbs.

Page 16

1. 44.5; 47.8; 92.3
2. Nov. 7. P.M. 2.9

Page 18

1. 45,284; 6,934; 321,113; 5,005
2. 9,010; 10,009; 600,600; 5,050,003; 80,080

Page 20

1. 4,213,122,607; 45,450,327,001
2. 718,200,000,000; 190,402; 3,000,124; 15,009; 90; 186,000,000,000; 147,100,100

Page 23

1. 242
2. $12.10
3. $1.85
4. 35
5. $.60
6. 233 mi.
7. 12 yd
8. 21 yds.
9. 19 yds.
10. 44 spools.
11. 4$\frac{1}{8}$ doz.

Page 24

12. 34
13. 19 lbs.
14. 73 lbs.
15. 265 lbs.
16. 20 bu.
17. 12 bu.

Page 24

18. 35 doz.
19. 23 lbs.
2. 1557

Page 25

3. 1031
4. 66
5. 287
6. Chicago, 26 10.46 in.
Des Moines, 29 11.33 in.
Detroit, 32 9.62 in.
Kansas City, 28 9.21 in.
Omaha, 29 10.12 in.
St. Louis, 23 11.29 in.

Page 26

7. 2876 ft.
8. 3967
9. 88,304
10. 88,995
11. 3567 teachers and schools
172,273 scholars
12. Immigrants, 1901—264,870 male
Immigrants, 1901— 123,665 female
Total, 388,535
Immigrants, 1900—243,091 male
Immigrants 1900—117,058 female
Total, 360,149

Page 27

13. Week No.
1st., 1,798,798
2nd., 1,823,561
3rd., 1,814,340
4th., 1,820,085
5th., 605,388
Total, 7,862,172

Page 27

14. 148,890,876 lbs
15. 245,083,133
16. $61,180,966

Page 28

17. 2,050,000,000
18. 9,477,398

Page 29

19. 17,436,267
20. 26,913,665
21. 22,965,741
1. 5,592,217
2. 26,333,004
3. 4,091,349
4. 84,141,954

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5. 3,740,381

Page 31

6. 765,855
7. 620,290 N. Central
8. Rhode Island 1,250 Texas 265,780 New York 7,268,894 Nevada 42,335 Nevada more than twice size of New York.
9. 956,731 5,823-019
10. 15,341,220
11. 28,686,008
13. Population 5,592,217; area, 66,465 sq.mi. Population 15,-454,678 area 102,200 sq.mi.
14. 956,731; 2,676,-509
2. 50:5

Page 32

3. 6
4. 9
5. 6:15
6. a
7. 15z
8. 15 9's: 15 12's: 15 x: 15z
9. I. 17 a, II. 14 a, III. 73 a, IV. 1462 z, V. 125 y, VI. 148 b, VII. 23 m, VIII. 241 n. IX. 197 c, X. 191 d, XI. 1981 x, XII. 1877 x.

Page 33

1. 238 mi.

Page 34

3. 408
5. 437
7. 7 yd.
8. ½ pk.

Page 34

9. ⅛ yd.
1. 25½ gal: 23 gal: 18 gal: 15 gal: 12½ gal: 10 gal.
2. 7 bu.
3. 15,000

Page 35

4. 1215
5. (1) $7428.56 (2) $2675.12 (3) 299,189 yd.
6. (1) 5,037,273: (2) $1282.78 (3) 292½
7. Deposits $373.-80
8. 30,466
9. 3598 ft.
10. 68,526 sq. mi.
11. 47,259: 1955: 201,649: 71,322
12. 12,404: 254: 24,058: 14,998
13. 34,855: 1701: 177,591: 56,324
14. All less than previous week.
Cattle, 15,929 less
Calves, 1,859 less
Hogs, 71,777 less
Sheep, 33,206 less
Corres. week 1900
Cattle, 6,706 less
Calves, 133 more
Hogs, 10,545 more
Sheep, 16,276 more
Corres. week, 1899
Cattle, 14,537 more
Calves, 620 more
Hogs, 41,925 more
Sheep, 9,604 more
15. All less than previous week.
Cattle, 6,295 less
Calves, 70 less
Hogs, 5,428 less
Sheep, 9,431 less
Compared to corresp. week 1900
Cattle, 6,034 less
Calves, 98 less
Hogs, 483 more
Sheep, 666 more
Compared to Corresp. week 1899
Cattle, 2,848 more
Calves, 44 more
Hogs, 7,421 more
Sheep, 11,816 more
16. 104,100 cattle 393,900 hogs 104,800 sheep
17. Chicago greatest in cattle: 18,-000 more than Kansas City,

Page 35

30,500 more than Omaha, 36,600 more than St. Louis. Chicago greatest in Hogs: 112,600 more than Kansas City, 136,400 more than Omaha, 163,500 more than St. Louis. Chicago greatest in sheep: 56,400 more than Kansas City, 58,500 more than Omaha, 65,500 more than St. Louis,

Page 37

18. As compared with the previous day
Cattle decr. 40,500
Hogs decr. 87,800
Sheep decr. 68,800
Corresp. week 1900
Cattle decr. 7,400
Hogs incr. 54,100
Sheep incr. 19,800
Corresp. week 1899
Cattle incr. 12,400
Hogs incr. 187,800
Sheep incr. 23,500
Corresp. week 1898
Cattle decr. 24,300
Hogs incr. 28,400
Sheep incr. 9,200
Corresp. week 1897
Cattle decr. 38,900
Hogs incr. 19,800
Sheep decr. 800
1. 12,043 ft.
2. 13,258 ft.
3. 1,250,000 sq.mi.
4. 43,950 sq. mi.
5. Philippines smaller than Texas by 151,-370 sq. mi. Philippines larger than Illinois by 57,760 sq. mi. Philippines larger than R. I by 113,160 sq.m
6. 5,150,235
7. 658,678T.
8. $120,706,122
9. 123 years
10. 264,530 sq. mi.
11. 7,226,559

Page 38

12. 1,202,898
14. So. Atlantic by 113,870 sq. mi; No. Central by 145,565 sq. mi. Western by 567,245 sq. mi.
15. No. Atlantic, 10,603,415 more; No. Cent. 12,253,047 more So. Central 9,988,608 more
16. 563,748; 1,191,-093; Maine exceeds by 627,-345
18. 2,196,681; dif. in total pop. larger than this by 250,663; dif. in total pop. being 2,447,344
20. Maine 33,580
N.H. 35,058
Vt. 11,219
Mass. 567,303
R. I. 83,050
Conn. 162,162
N. Y. 1,271,041
N. J. 438,736
Penn. 1,044,101
Total, 3,646,250
24. W. N. Atlantic incr. 1,861,-506 over E. N. Atlantic. S. S. Atlantic incr. 376,696 over N. S. Atlantic. E. N. Central incr. 1,056,766 over W. N. Central. E. S. Central incr. 1679,088 over W. S. Central. W. Western gained 155,-803 more than E. Western. W. Western gained 416,576 more than Middle Western. E. Western gained 260,873 more than Middle Western
1. $1,871,568,787
2. Un. Kingdom, +$116,471,003
Ger. +94,352,391
Can., +65,809,640
Neth. +54,382,038
Mex., +21,400,198
Italy, +19,599,197
British Australia, +17,004,883
British Africa, +21,483,098

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Jap., +17,323,093
Bra., −3,928,245
Arg. +9,207,733
Rus. +1,104,510

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3. Ag. Implements +$1,832,143
Books, maps, etc., +517,837
Carriages and cars +833,448
Cop. ing. +166,855
Cot. clth. −1,114,683
Cot. mfs. +650,981
Cycles and parts of −241,087
Bld'r's hardware +357,616
Swg. machines, +112,954
Oth. Machinery, −328,378
Corn, −302,141
Wheat +1,220,799
W. flour −1,777,158
Coal, −1,514,679
Cotton +1,882,526
Fr. & n. +778,676
Furs and fur skins +471,630
Cotton Seed Oil +$214,619
Beef, salted or pkl. +32,783
Bacon, +192,408
Hams, +30,879

Page 42

1. Pop. 8,074,559 Pop. 5,974,091 Sch. Enroll., 1,213,519; no. tch. 27,123; school ex., $40,118, 540
2. Pop., 1900, 2,582,578; Pop., 1890, 2,003,922; Sch. Enroll., 367,287; no. teachers, 8,261; sch. expend . $9,756,432 Pop., 1900, 1,912,334; pop. 1890, 1,496,982; sch. Enroll. 310,325; sch. ex., $6,534,163 no. teachers, 7,356
3. 120,415
4. 242; 6,261; 267; 3,933
5. 9,084; 26,196; 32,161

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6. $2,138,158; $-730,333; $654,-029; $525,965; $693,334
7. More than half; less than half.
8. Pop., 1900, 13,-317,403; pop., 1890, 10,029,-377; sch. enroll. 2,019,428; no. teachers, 45,-663; sch. ex., $60,176,405 Incr. pop. 3,-288,026
1. 4
2. 7
3. 5 9s
4. 6 c's

Page 43

5. 9x, 18y; 6z; 29z; 586a; 142z
6. 120x sq. ft.
7. 5x cts.
8. 5; 35; 95; 26; 8; 5; 5; 5; 9; 12; 2; 100; 32; 10½; 72

Page 44

2. 126,720
3. $2.04
4. 768

Page 45

5. 258,300
6. $20,664
8. 512
9. 27,664: 1400: 56x
10. 135,597: 231y: 231 a

Page 47

1. Receipts, $512.18; Expenses, $128.23; profit, $383.95 per 40 acres; $9.598 per acre

Page 48

2. Receipts, $295; expenses, $106.80; profit per 20 acres, $188.20; profit per acre, $9.41
3. Receipts, $1803.20; exp., $704.40; profit for 30 acres, $1098.80; profit per acre, $36.62⅔

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4. Receipts, $638.75; exp., $166.69; profit 50 acres, $472.06; profit per acre, 9.44$\frac{6}{50}$
5. Receipts, $114; expen. 39.25; profit $74.75 for 10 acres; profit per acre, $7.475
6. Receipts, $368.00; ex., 122.25; profit $228.15 for 10 acre lot; profit $22.815 for 1 acre

Page 50

7. Expenditures, $766.75
8. Receipts, $3713.53; exp. $2034.37; profit, $1679.16 net earnings for the year
1. 464.4 grains
2. $0.80
3. $2500
4. $3.60
5. $45.00
6. $35
7. 12 lbs.
8. $4.20
9. Milk; $1.63

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10. 4320; 103,680
14. 16,200; 388,800
15. 16,122,240,000
16. $164.25

Page 57

1. 1 in.; 9 cu. in.
2. 9: 18: 45: 81
3. 144 cu. in: 288: 432: 864
4. 81
5. ½ inch, ⅓, ¼, ⅙, ⅛, ⅐
6. 144; 288; 72
7. 12; 24; 24;
8. 6½; 936; 11,232; 8424; 252,720; 2106; 254,826

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10. 373.33; 10,752; 233,333⅓
12. 48 sq. in.; 336 sq. in.; 2560 sq. in.; 182,250 sq. ft.; 9x; 9x yd; ab mi., xy
13. 27; 486

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14. 81 cu. in.; 81 cu. ft.; 81 cu. yds; 81 cu. rds.; 81; 9a
15. 147 cu. in.; 96 cu. in.; 96 cu. ft.; 180 cu. ft.; 27 cu. yd.; 180; 12x; 3xy; abc.
17. 30,000 lbs.; 18,000 lbs.
18. 10,330 lbs.

Page 59

19. 15,504
20. $12.48
21. 576; 108
22. 10,944 lbs.
23. 20,736 cu. ft.
1. 6; 8; 9; 23; 22; 17; 8; 16; 9; 10; 6; 2.
2. a+b
3. a−b
4. ab; abc; xyz
5. 36; 96; 54; 72; 288; 432; 1296
6. 8x; 7y; 25ab; axy

Page 60

7. ab; cx; 15abx
1. 8

Page 61

2. 7; 88
3. 9; 30
4. 30
5. 4
7. 13
8. 16
9. 15; 14; 16
10. 12; 13⅓; 20

Page 63

2. 5
3. 30
4. 12
5. $.90
6. $.65
7. $.27
8. 14 yds.
9. 30 mi. per hr.
10. 12 gals.
11. 25
12. 84
13. 55

Page 64

2. 1760; 3520
3. 40
4. 30
5. 40
6. 70 ft.
7. 24
8. 45
9. 36
10. 40,659$\frac{5}{6}$ per. mo.; 1355$\frac{59}{180}$ per day

Page 65

11. 37,381
12. $46,693\frac{1}{4}$; $51,923\frac{2}{3}$; $41,909\frac{1}{7}$; $21,544\frac{2}{3}$; 19,236
13. 20
14. St. Joseph 5
Memphis 7
Oakland 7
Hartford 20
Worcester 11
Peoria $5\frac{1}{2}$
15. 544; 786; 321; 17,953; 7170; 8411; 3071; 3830; 112,836
17. 8; 77; 700; 100; 10; 9; 32; 15

Page 66

3. $79\frac{5}{9}$
4. 243
5. 324
6. 279
7. 469
8. 428
9. 742
10. 468

Page 67

1. 36
2. $25.50
3. 14
4. $33
5. $205
6. 8
7. 75
8. $120
9. $245
10. 2 horses
11. 2 horses
12. 1 horse

Page 68

2. 245
3. 314
4. 231
5. 243

Page 69

1. 6 nearly 7
2. 3 almost 4
4. 182.+
5. 149.6
7. 10.2+; 1.58+
9. 186
10. 835.2 sq. mi.
11. 34—nearly 130
12. almost 270
13. 11.29 mi.

Page 70

14. 147.8
15. 23.08
16. 16.7
17. 13
18. $589.23+
19. $12,908.74
20. 269; 342; 235; 140; 189; 293; 15; 20.6

Page 75

2. 68
3. 150
4. 25
5. 26.25

Page 77

1. 62,500 sq. ft.
2. $2,000
4. $1,250
6. $900
11. $1.20; $18; $60
13. $14.40; $4.80; $72; $240

Page 78

16. $219
21. N. E. corner, 30 ft. from E. line & 70 ft. from N. line; N.W. corner, 34 ft. from W. line & 70 ft. from N. line; S. W. corner, 30 ft. from S. line & 34 ft. from W. line
22. $112.60

Page 79

1. $5.60

Page 80

7. 12,178
11. $162
12. $27
13. $22.50
14. $17.28
15. $210

Page 81

16. $51.54

Page 82

1. 128
2. $11\frac{13}{17}$; $12\frac{1}{2}$; $11\frac{13}{17}$; $14\frac{2}{7}$; $11\frac{13}{17}$; $4\frac{4}{9}$; $4\frac{1}{6}$
4. $58\frac{2}{3}$
5. $63\frac{1}{15}$
6. $143\frac{11}{15}$

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7. $190\frac{2}{3}$
8. 1100
9. 1188
10. $3,748\frac{4}{27}$
11. $18\frac{14}{27}$
12. $1527\frac{7}{9}$
13. 27
14. $133\frac{1}{3}$
15. $90
16. 135
17. 20
1. $139\frac{5}{64}$

Page 84

2. $3,775\frac{25}{64}$
3. $27\frac{188}{381}$
4. $42\frac{6}{7}$

Page 84

5. $1275\frac{28}{45}$
6. $29\frac{5}{7}$.
7. $19\frac{3}{7}$; $3118.83; $163.91
8. $81\frac{71}{81}$; $9454.8; $115.47
9. $363\frac{229}{367}$; $20,493.18; $56.32
10. $6\frac{288}{308}$; $8516.23; $1229.72

Page 85

1. $74\frac{337}{471}$; $5352.55; $71.54.

Page 86

1. 38; $38\frac{3}{7}$
2. $41\frac{1}{7}$; $37\frac{4}{7}$.
3. $\frac{2}{7}$; $3\frac{4}{7}$.
4. 9
5. 12.75
6. 36; $31\frac{4}{13}$
7. $4\frac{9}{13}$
9. 78°; 32°; 76°; 98°

Page 88

1. Adams, $49\frac{3}{4}$
Benson 51
Boyd $29\frac{1}{2}$
Claussen 51
Denning $52\frac{1}{2}$
Doan 48
2. $281\frac{1}{2}$ hrs
3. Adams, M. $2.25; T. $2.40; W. $2.40; Th. $2.55; F. $2.70; S. $$2.62\frac{1}{2}$
Benson, M. $2.40; T. $2.55; W. $$2.47\frac{1}{2}$; Th. $$2.62\frac{1}{2}$; F. $2.55; S. $2.70
Boyd, M. 0; T. $1.20; W. $1.80; Th. $1.80; F. $1.95; S. $2.10
Claussen, M. $2.40; T. $2.55; W. $2.55; Th. $2.55; F. $2.70; S. $2.55
Denning, M. $2.55; T. $2.70; W. $2.55; Th. $$2.47\frac{1}{2}$; F. $2.70; S. $2.70
Doan, M. $2.40; T. $2.40; W. $2.40; Th. $2.40; F. $2.40 $2.40; F. $2.40; S. $2.40
4. Adams $$14.92\frac{1}{2}$
Benson 15.30
Boyd 8.85
Claussen 15.30
Denning $15.67\frac{1}{2}$
Doan 14.40

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5. $84.45
6. Denning; Boyd.
7. $16.20
8. Adams $1\frac{3}{4}$;
Benson 3;
Denning $4\frac{1}{4}$;
Claussen 3
9. Adams $1\frac{3}{4}$
Benson 3
Claussen (3)
Denning $4\frac{1}{4}$
10. Adams $1.05
Benson 1.80
Claussen 1.80
Denning 2.55
Total $7.20
11. $18\frac{1}{2}$

Page 89

4. Dr. $2875.; Cr. $1232.75; On hand $1642.25

Page 90

5. Receipts $2.00
Expend. 1.25
Balance $.75
6. Receipts $105.15; Expend. 27.61; Balance, $77.54
7. Amt. Due, $129.75
8. $17.50
9. $5.86

Page 91

10. $48.75
11. $9.78
12. $43.43
13. $60.70
14. $36.75

Page 92

15. $2.98
1. 13c.
2. 13c.
3. 72c.
4. 4.21c.
5. 35c.

Page 93

6. $3.47
7. 1st. $276.01
2d. 203.77
3rd. 173.13
4th. 228.94
M. $24.205
T. $29.00\frac{1}{2}$
W. 27.195
Th. 39.05
Fri. 18.55
S. 82.46
Total 881.85
1. $1.57; $2.19; $1.50; $8.96; $6.65; $8.59
Total $29.56

Page 94

2. $153.64; $89.32

Page 95

2. Oct. $168.37
Nov. 173.16
Dec. 155.82
Jan. 166.93
Feb. 148.94
March 146.91
April 116.24
May 118.09
June 120.49
July 108.37
August 118.29
Sept. 111.51
Total $1642.12
3. Oct. $5.43
Nov. 5.77
Dec. 5.02
Jan. 5.38
Feb. 5.31
Mar. 4.73
4. $160.02
5. $166.91; $178.63

Page 96

6. April $3.87
May 3.65
June 4.02
July 3.49
Aug. 3.65
Sept. 3.71
7. $113.83
8. $136.92
9. $575.57; $64.92

Page 97

2. 4lb; y + 4lb = 14lb.
3. x = y
4. 20.
5. 35
6. 4lb.
7. 4.
8. 4.
9. 3.
10. 8.
11. 3.

Page 105

1. 10; 20; 4; 100
2. 2; 4; 5; 2½; 1¼
3. 21; 14; 7; 2
4. ½.
5. 10
6. 15; 3; 5; ½; ¼
7. $120; 24; 12; 1½; ½.
8. 1st; $400.
9. 50;16.

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1. 36c.
6. 696 sq. ft.

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8. 64 sq. yd; 64.
9. 23⅓ sq. yd.
10. 1228 sq. ft.

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11. 2400 sq. ft.; 24.
12. 19¼; 213⅓ sq. yds.

Page 110

13. 20; 30; 40; 80
14. $48,400.
15. $8,910.
16. 18 sq. ft.
17. 144 sq. ft.
18. 120,000 sq. ft.
60,000 sq. ft.
60,000 sq. ft.
19. 1080 sq. ft.

Page 111

23. ab/2
24. P = ½ ab
25. 120,000; 120,000; 70,000; 60,000
26. 50,000 sq.ft.
28. T = (A B + C E)/2
29. 60,000; 60,000; 50,000; 70,000; 60,000
30. 108 sq. ft.
31. 120,000 sq. ft.
32. 3128 sq. ft.
33. 168 sq. ft.

Page 112

34. 1. 252
2. 48
3. 80
4. 4860
5. 640
6. ½ mn rods
7. Pq/2 rods
8. 2y
1. 72 sq. in.; 72 sq. ft.; 60 sq. yds.; abc sq. yds.
2. 1360 sq. ft.
3. 1088 bu

Page 113

5. 35 5/9
6. 64.
7. 120.
8. 10.
9. 10.
10. 3924.
12. 567 lbs.
13. 1782 lb.
14. 180 cu. ft.
15. 1346.49+
16. 264; 1⅓.
17. 2176; 1¼.
18. 1080 cu. in., about ½
19. 1075 cu. in.; about ½; 2160 cu. in.; little over a bu.

Page 114

1. 80 lbs; 2160
2. 81 lbs.; 2187
3. 125; 15 5/8
5. 16/27; 27
6. 5 1/6; 3 3/21
7. 11 1/9; 1 1/28
8. 4 1/27; 3 10/13
9. 900; 1012½.

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10. 15c.
11. 4 6/11c.
12. 88c.
13. 88c.
14. 17,982 lbs.; $2174.47
15. 2,925 lbs.; $148.83
16. 146,200; $8,799.70
17. 8.966 tons; 73.1 tons
18. $248.04

Page 116

19. $301.436
20. $18.839
22. 110,500 bu.
1. 180°
2. 30°
3. 25°
4. 16°
5. 0°
6. −10°
7. 15°

Page 117

8. 5°; 11°; 32°; 2°; 14°; 9°; 9°; 5½°; 3½°
9. R5°; R1½°; R3½°; F4½°; F18°; F5°; R8½°; R21½°; R5½°; F33½°; R4½°; R90°
10. R84°; R180°; R159°; F35°; F33°; F38°; F16°; R11½°; F21½°
11. +69°
12. +9°
13. −7°
14. +3°
15. +4°; −1°; 0°; −8°

Page 118

1. 2
2. 12 hrs.
3. ½; 3/8; 1/24; 1/12; ¼
4. 1hr.
5. 1/24; 1/40; 1/96; 1/32; 1/48
6. 1hr.
7. 12
8. 1min.
9. 1/60 of whole turn.
10. 60; a second
11. 24; 168; 720; 8760

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12. 1440; 10,080; 43,200; 525,600.
13. Hourhand VI.; Min. handXII.; Sec. hand 60
14. Hour hand bet. V. and VII; min hand bet. 48 and 49; sec. hand at 46
1. 102,400 sq. rd.
2. 640 acres; 160 acres.
3. 160; 120; 80; 80; 40; 40; 20.
9. 2600 rd.
10. 480 rd.

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11. 6mi.; 6mi.;
12. 36 sq. mi.; 23,040
14. 144
15. 160x160

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6. .02; .06; .08; .12; .25; .33⅓; .87½; 1.
7. 1/50; 3/50; 2/25; 3/25; ¼; ⅓; ⅞
8. 4lb.; 9lb.; 12lb.
9. 4; 12; 22; 100
10. 2in.
11. 12ft.
17. $1
18. 2in. 52in.
19. 61.2 in. tall

Page 125

20. $1.25
21. .05; $5
22. $4.80
23. $500.
24. 100°
25. $9600.
26. $50
27. 25%; 75%
28. 25in.
29. 6%

Page 126

1. $8
2. .06
3. .03; .01; .005
4. .015; .025; .035; .07; .08; .17
5. 48c; $1.50; $2.88; $5.10; $7.44; $27
7. $54; $81; $121.50; $330.75
8. $1.68; $15.75; $26.25
10. $48; $72; $120; $240; $4; $12; $8

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11. 5%
13. $180; $198.
14. $24.50; $70.87½ $85.75

Page 128

3. Floor and Ceiling each 1,080,000 lbs. ea. end wall 432,000lbs ea. side wall 540,000 lbs.
4. 44496. lbs.
1. 39.6 mi. per hr.
2. 3.10 P. M
3. 378
4. 1 P. M
5. 4301 miles
6. 38 mi. per hr.
7. 23½mi. per hr.; 18½.
8. 41⅓; 34½.

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9. 7 miles
10. 50½ mi.
11. 13⅓
12. 5 sec.
13. ½ hr.
14. 679 ft.
15. 510.39
16. 2.05195
17. 2712 ft.; 1329.-45 tons

Page 130

18. 2.82 T (omitting engine and tender)
19. 3,192 tons
20. 1,666 tons
21. 2.51+ tons
22. $156464.40
23. 26.66⅔ tons
24. 11.75 tons

Page 131

1. 45½; 50½; 47; 46; 49⅔; 47½; 45⅔
2. 1⅔; 1½; ½; 2½; 1½; 1½
3. A2½; B½; A½; A2; B2; on time; A1.
5. 55.5; 24.5; 30.8; 50
6. 50 min.; 20; 25; 43.

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7. 1.11; 1.225; 1.-232; 1.165.
8. 1.11; 1.07; 1.16 1.20
9. 73.5; 66.9: 70.8: 72.4: 66.9: 70.5; 72.8.
10. 38.
11. 1070.

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1. $88\frac{7}{16}$; $8401.56
2. $50.40
3. $63.35
4. 18¾ acres; $1406.25
5. 37½ acres; $3281.25
6. ½; ½
7. $1500
8. ¼.
9. 160; 220; 560

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10. 3300
11. $82.46
12. 60.
13. 151.875 ounces
14. 4.1⅞ lbs.
15. $5.62½; $7.05
16. 2⅔; 45⅓; 160
17. 40½; 324; 72.

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1. ½; 2; ½; 2; ¼; 4; ¼; ¼; ⅛; 8; ½; ¼; $\frac{1}{16}$; 16.
2. ¼; ⅛; $\frac{1}{16}$; 3; 6; 16
3. E to F = 2; F to E = ½; E to G = 4; G to E = ¼; F to G = 2; G to F = ½
4. 2; ⅛; 3
5. 3; ⅓; 3; ⅓
6. 1; 1; 1; 1; 1.
8. ⅓; $\frac{1}{12}$; ⅓; $\frac{1}{12}$; 3; 12; 1

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10. 16; 12; $\frac{1}{12}$; $\frac{1}{16}$; 4; 32
11. $\frac{1}{3}$; 320; 9; ⅓
12. 2; ⅓;102, 400; 160; 640
13. 10; 20; ⅔; ⅓; ½; 2; 5
14. $\frac{9}{b}$; ½; $\frac{b}{a}$; 2; 3; ⅓

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1. 10c.
2. $.50
3. 8c.
4. 30c.
5. 8 ft.
6. 10
7. 12
8. 15; 24; 6
9. 7; 7½
10. 2; 8
11. A=20; C=42½

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1. 2; ½
3. 2; 4; 6; 12; 3;9; 18
4. 2; 3; 6; 12; 4;8
5. ½
8. ⅓

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9. 4; 2
10. $\frac{24}{24}$
11. 1. $\frac{3}{5}$; $\frac{6}{8}$; $\frac{9}{9}$; $\frac{12}{16}$; $\frac{32}{26}$
2. $\frac{1}{3}$; $\frac{2}{4}$; $\frac{3}{6}$; $\frac{6}{18}$; $\frac{12}{36}$
3. $\frac{2}{3}$; $\frac{4}{6}$; $\frac{6}{9}$;,$\frac{12}{18}$; $\frac{24}{36}$

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12. $\frac{1}{3}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{2}$; $\frac{1}{7}$; $\frac{1}{8}$; $\frac{1}{9}$; $\frac{1}{4}$; $\frac{1}{11}$; $\frac{1}{5}$; $\frac{1}{6}$; $\frac{1}{2}$; $\frac{1}{7}$; $\frac{1}{8}$; $\frac{1}{15}$; $\frac{1}{35}$; $\frac{1}{55}$; $\frac{1}{13}$
13. $\frac{1}{7}$; $\frac{1}{8}$; $\frac{1}{9}$; $\frac{1}{12}$; $\frac{1}{15}$; $\frac{1}{25}$; $\frac{1}{18}$; $\frac{1}{11}$; $\frac{1}{64}$
14. 2; 2; 3; 1; 4;5;8
17. $\frac{1}{3}$; $\frac{1}{7}$; $\frac{1}{12}$; $\frac{1}{18}$; $\frac{1}{24}$; $\frac{1}{125}$; $\frac{1}{75}$; $\frac{1}{19}$; $\frac{1}{a}$; $\frac{1}{x}$
18. $\frac{2}{5}$; $\frac{3}{5}$; $\frac{3}{6}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{4}{7}$; $\frac{5}{7}$; $\frac{7}{9}$; $\frac{8}{9}$; $\frac{1}{3}$; $\frac{3}{2}$; $\frac{6}{35}$; $\frac{8}{35}$
19. 4
20. 6; 8; 10; 12; 14; 16; 18; 20; 22; 50
21. 6; 9; 12; 15; 18; 21; 24; 27; 30; 36; 45; 60; 90
22. 10; 15; 20; 25; 30; 35; 40; 45; 50; 100; 150
23. $\frac{2}{6}$; $\frac{4}{6}$; $\frac{8}{6}$; $\frac{3}{6}$; $\frac{9}{6}$
24. $\frac{6}{12}$; $\frac{4}{12}$; $\frac{8}{12}$; $\frac{3}{12}$; $\frac{9}{12}$; $\frac{2}{12}$; $\frac{10}{12}$

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25. $\frac{5}{10}$; $\frac{6}{10}$; $\frac{5}{15}$; $\frac{3}{15}$; $\frac{9}{12}$; $\frac{10}{12}$; $\frac{14}{21}$; $\frac{18}{21}$; $\frac{18}{45}$; $\frac{28}{48}$; $\frac{77}{88}$; $\frac{40}{88}$
1. $\frac{2}{4}$; $\frac{4}{8}$; $\frac{8}{16}$
2. $\frac{2}{6}$; $\frac{3}{9}$; $\frac{4}{12}$; $\frac{6}{18}$
3. $\frac{2}{3}$; $\frac{2}{3}$; $\frac{2}{3}$; $\frac{2}{3}$; $\frac{2}{7}$; $\frac{2}{3}$
4. $\frac{4}{7}$; $\frac{5}{7}$; $\frac{4}{7}$; $\frac{6}{7}$
5. Same
6. $\frac{4}{10}$; $\frac{6}{15}$ $\frac{8}{20}$; $\frac{14}{35}$; $\frac{30}{75}$; $\frac{40}{100}$
7. 2; 2
8. 3; 7; 20; 25; 100

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10. $\frac{6}{16}$; $\frac{12}{18}$; $\frac{10}{18}$; $\frac{28}{50}$; $\frac{24}{48}$; $\frac{12}{24}$; $\frac{40}{100}$
12. $\frac{1}{7}$
14. $\frac{2}{3}$; $\frac{2}{5}$; $\frac{6}{7}$
18. $\frac{1}{2}$: $\frac{2}{3}$; $\frac{3}{4}$: $\frac{1}{2}$; $\frac{6}{7}$; $\frac{4}{7}$

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2. 1, 2, 3, 4, 6, 12
3. 12
4. 1, 3, 7, 21; 21
5. 1, 2, 3, 6; 6
6. 1, 2, 3, 4, 6, 12; 12; 1; 2, 7, 14; 14; 1, 2, 3, 4, 6, 12; 12

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8. a. 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97;
b. 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46, 48, 49, 50, 51, 52, 54, 55, 56, 58, 60, 62, 63, 64, 65, 66, 68, 69, 70, 72, 74, 76, 77, 78, 80, 81, 82, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100.
10. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47,49,
11. 1. 8; 2; 26; 38; 42;
2. 5; 9; 21; 15; 19; 27; 41;
3. 5; 2; 19; 41;
4. 8; 9; 21; 15; 26; 27; 38; 42;
12. 9, 21; 8, 16; 3, 7
14. 7x3; 2x2x2x3; 5x5; 3x3x3; 3x2x5
15. 5.
1. 2x2x3x3; 2x2x2x2x3
3. 2x3x9; 3x5x7; 2x2x2x2x2x3 ; 3x3x3x2x2x2; 5x5x3; 5x5x5x2

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8. 12; 14; 12
9. 1. 20; 2. 18; 3. 45; 4. 215; 5.36; 6. 22; 7. 63; 8. 163
10. 1. $\frac{8}{9}$; 2. $\frac{36}{49}$; 3. $\frac{8}{16}$; 4. $\frac{6}{7}$; 5. $\frac{9}{14}$; 6. $\frac{10}{11}$; 7. $\frac{15}{32}$; 8. $\frac{9}{21}$; 9. $\frac{4}{8}$; 10. $\frac{3}{7}$; 11. $\frac{1}{11}$; 12. $\frac{1}{7}$

Page 145

1. $\frac{2}{3}$
2. $\frac{3}{4}$
3. $\frac{3}{5}$
4. $\frac{1}{4}$
5. $\frac{2}{5}$
6. $\frac{1}{2}$
7. 2
8. $\frac{13}{23}$
9. $\frac{21}{38}$
10. $\frac{1}{3}$; $\frac{8}{11}$
11. $\frac{20}{33}$

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12. $\frac{4}{15}$
1. $\frac{3}{12}$; $\frac{2}{8}$; $\frac{4}{16}$; $\frac{5}{20}$; $\frac{16}{64}$; $\frac{25}{100}$

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2. $\frac{5}{6}$
3. $\frac{1}{3}$
4. $\frac{7}{10}$; $\frac{7}{8}$; $\frac{4}{9}$; $\frac{9}{14}$; $\frac{3}{16}$; $\frac{4}{15}$; $\frac{23}{22}$; $\frac{35}{32}$; $\frac{41}{84}$
5. $5\frac{13}{20}$
6. $6\frac{1}{10}$
7. $6\frac{1}{3}$
8. $1\frac{7}{40}$″; $\frac{1}{8}$″

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1. 70
2. 60
5. 12
7. 2
8. 20
11. 40; 24; 75; 120; 360

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2. 36; 120; 180; 1050; 168; 108
4. $\frac{282}{525}$; $\frac{153}{168}$; $\frac{59}{108}$; $\frac{211}{234}$

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4. $\frac{47}{48}$
5. $8\frac{23}{40}$
6. $25\frac{3}{125}$
7. $109\frac{23}{40}$

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2. William $\frac{1}{4}$ ft.
3. $1\frac{47}{112}$
4. $\frac{23}{32}$
6. $5\frac{29}{40}$; $8\frac{5}{33}$; $13\frac{7}{10}$; $30\frac{23}{88}$; $41\frac{1}{33}$; $227\frac{416}{1525}$
7. $44\frac{17}{80}$
8. $23\frac{17}{80}$
9. $2\frac{2}{3}$

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10. $103\frac{1}{2}$ gr.
11. $96\frac{9}{20}$ gr.
12. $\frac{13}{32}$; $\frac{3}{88}$; $\frac{5}{54}$; $\frac{a+b}{ab}$; $\frac{b-a}{ab}$; $\frac{21}{38}$; $\frac{17}{27}$; $1\frac{48}{143}$; $\frac{ab+ac}{cb}$; $\frac{my-mx}{xy}$; $1\frac{28}{63}$; $\frac{29}{132}$; $1\frac{23}{170}$; $\frac{ad+bc}{bd}$; $\frac{my-xn}{ny}$

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14. $1\frac{1}{36}$
15. $\frac{59}{80}$; $\frac{1}{80}$
16. $55\frac{3}{4}$; $16.70
17. $\frac{3}{35}$
18. $\frac{7}{10}$; $\frac{3}{10}$
19. $\frac{1}{8}$

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20. $\frac{3}{35}$
21. $\frac{33}{78}$
22. $\frac{217}{2538}$
23. 21.01\frac{3}{4}$
24. 300; $\frac{3}{11}$
1. 18
2. 32
3. 32
4. $\frac{20}{7}$
5. $\frac{37}{15}$
6. $\frac{5}{11}$
7. $\frac{15}{11}$
8. 12; 4; 3; 6; 5; 15; 4; 8; 6; 6; 10; 10

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9. $2\frac{3}{4}$; $2\frac{1}{4}$; $1\frac{3}{8}$; $7\frac{1}{2}$; 3; $\frac{3}{4}$; $3\frac{1}{3}$; $2\frac{2}{3}$; $5\frac{1}{4}$; $3\frac{1}{4}$; $12\frac{1}{2}$; $8\frac{1}{4}$
11. 5; 7; 7; 5; 19; 16

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1. $3\frac{3}{4}$
2. $1\frac{3}{4}$
3. $23\frac{3}{4}$
4. 5
5. $4\frac{1}{2}$
6. 8
7. $5\frac{1}{4}$
8. $7\frac{1}{5}$
9. $6\frac{1}{4}$
10. $20\frac{5}{8}$
11. $3\frac{3}{4}$
12. $6\frac{6}{11}$
13. $10
14. $7
15. $3\frac{3}{4}$; $1.20; $6.00
16. $5\frac{1}{4}$
17. $1\frac{1}{4}$

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1. 6; 20; 28; 10; 12; 18; 20; 16; $29\frac{19}{21}$
2. $28\frac{1}{8}$; $23\frac{5}{8}$; $89\frac{1}{4}$; $23\frac{19}{47}$; $23\frac{11}{31}$; $189\frac{7}{8}$

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1. $7
2. $70; $5
3. 154; $192\frac{1}{2}$
4. $977\frac{1}{2}$
5. 357

Page 162

1. 15 sq. in.; 48; 56 sq. ft. 160 sq. rds.; 320 sq. rds ab.
2. 4 sq. ft.; 14 sq. ft.; 27
3. $\frac{9}{16}$ sq. in.
4. $\frac{1}{2}$ sq. in.; $\frac{3}{14}$; $\frac{1}{2}$; $\frac{5}{18}$; $\frac{5}{72}$; $\frac{5}{16}$; $\frac{21}{32}$; $\frac{13}{36}$; $\frac{21}{35}$

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7. $\frac{3}{16}$; $\frac{3}{8}$; $\frac{4}{35}$; $\frac{1}{3}$; $\frac{3}{10}$; $\frac{1}{4}$; $\frac{3}{7}$; $\frac{9}{14}$; $\frac{1}{12}$; $\frac{5}{12}$; $\frac{3}{35}$; $\frac{19}{63}$; $\frac{4}{27}$; $\frac{1}{3}$; $\frac{1}{8}$; $\frac{23}{72}$

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3. $1277\frac{9}{25}$
4. $999\frac{13}{88}$
5. 462
6. $1390\frac{5}{18}$
7. $513\frac{5}{24}$
8. 4.12\frac{1}{2}$
9. 5.84\frac{3}{4}$
10. 10.22\frac{2}{3}$
11. 2.93\frac{1}{18}$
12. 9354\frac{15}{32}$

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1. 4; 5; 14; 16; 5; 12; 8; c; m
2. 18; 32; 108; 325; 723; 14,565
1. $\frac{4}{15}$
2. $\frac{7}{27}$
3. $\frac{1}{10}$
5. $\frac{1}{6}$
6. 25c.

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7. $\frac{3}{5}$
8. $\frac{1}{16}$
9. $\frac{5}{36}$; $5,000
10. $2\frac{3}{56}$
1. $6\frac{1}{7}$

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3. $1\frac{17}{32}$; $1\frac{4}{8}$; $1\frac{7}{18}$; $4\frac{13}{24}$; $5\frac{7}{8}$; $2\frac{1}{8}$; $9\frac{9}{80}$; $11\frac{1}{5}$; $2\frac{1}{21}$; $8\frac{1}{3}$
4. $12\frac{1}{2}$; $18.67\frac{2}{3}$; $16\frac{1}{2}$; $12.67\frac{4}{13}$

Page 171

1. 3\frac{3}{4}$
2. $\frac{1}{8}$
3. $39\frac{13}{63}$ lbs
4. $19\frac{3}{5}$
5. 66
6. $4\frac{4}{5}$
7. $\frac{7}{8}$ or $3\frac{1}{2}$
8. $11\frac{3}{5}$
9. $3\frac{3}{8}$
10. $4\frac{4}{5}$
11. $3\frac{1}{2}$
12. 10
13. $6\frac{3}{8}$; $9\frac{3}{7}$; $3\frac{3}{4}$; $8\frac{1}{5}$; $19\frac{3}{4}$
14. $21\frac{1}{3}$; $14\frac{2}{3}$; $9\frac{3}{5}$; $8\frac{8}{15}$
15. 9; $9\frac{1}{5}$; $7\frac{1}{2}$; $9\frac{9}{10}$; $6\frac{2}{15}$

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20. $\frac{10}{7}$
21. $3100
22. 1935
23. 105

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1. $1\frac{1}{2}$
2. $\frac{5}{136}$
3. $2\frac{11}{12}$
4. $\frac{15}{64}$
5. $1\frac{5}{8}$
6. $\frac{3}{5}$
7. $\frac{2}{21}$
8. $1\frac{19}{78}$
9. $3\frac{17}{27}$
10. $1\frac{3}{8}$
11. $4\frac{2}{27}$
12. $1\frac{371}{720}$
13. $1\frac{376}{1099}$
14. $1\frac{31}{525}$
15. $\frac{ad}{bc}$
1. 150 lbs.
2. R. 11 lbs.

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3. R 3x
4. 4 m lb.
5. R $6\frac{1}{2}$
6. R $35\frac{7}{8}$; R $5\frac{7}{8}$; R $51\frac{17}{40}$; L $45\frac{5}{16}$; L $65\frac{9}{16}$; L $597\frac{3}{20}$
7. F 4
8. B $44\frac{7}{24}$; F $213\frac{5}{24}$ T B $7\frac{73}{120}$; F $9\frac{7}{80}$; B $1\frac{13}{40}$; B 5 X
10. D $4\frac{11}{24}$; U $14\frac{5}{8}$; D $78\frac{1}{8}$; D $6\frac{3}{40}$ oz.

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12. $+3\frac{5}{8}$ lbs.; $+\frac{7}{8}$oz $+33\frac{3}{5}$ T; $+205\frac{7}{8}$lbs
1. $\frac{125}{88}$; $\frac{5}{104}$; $\frac{19}{80}$; $\frac{917}{864}$; $\frac{113}{800}$; $1\frac{383}{2880}$; $12\frac{26}{27}$; $5\frac{23}{48}$; $6\frac{31}{45}$; $260\frac{19}{60}$; $19\frac{11}{42}$; $2\frac{13}{24}$
2. $\frac{2}{3}$; $\frac{4}{9}$; $\frac{2}{3}$; 2; 4; $6\frac{16}{35}$; $\frac{3}{32}$; $2\frac{134}{203}$; $180\frac{4}{7}$; $1\frac{67}{500}$; $1\frac{113}{229}$; $\frac{6}{15}$; $1\frac{61}{178}$; $2\frac{214}{217}$; $\frac{35}{354}$; $\frac{383}{988}$
3. $\frac{3}{28}$; $\frac{11}{32}$; 25; $1\frac{756}{1085}$; $2\frac{1}{20}$; $\frac{43}{216}$; $\frac{2}{21}$; $1\frac{17}{84}$; $\frac{122}{385}$; $14\frac{2}{5}$; 0; $100\frac{20}{27}$; 10; $26\frac{10}{11}$; $71\frac{1}{84}$

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1. $4\frac{1}{2}$ ft.; $3\frac{3}{8}$ ft.; 6′; $2\frac{5}{8}$′; $2\frac{5}{8}$′; $1\frac{1}{8}$′; $1\frac{7}{8}$′
3. 20′; $12\frac{1}{2}$′; 10′; $3\frac{3}{4}$′; $7\frac{1}{2}$′; $3\frac{3}{4}$′; $6\frac{1}{4}$′
4. $298\frac{7}{16}$sq. ft.
6. 75″; 33″; 60″; 21″
7. 33″x75″; 30″x$7\frac{1}{2}$″ 6″ above center and at sides; 9″below

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9. 32″; 48″; 48″
11. 9.492 sq ft.
12. 24″ each; 28″;20″
14. 84″; 60″; 114″; 84″; 9″ and 3″
16. 11; 10; 4$\frac{1}{2}$

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1. 320′; 400′; 470.156 sq. rds.
2. 200; 80; 58$\frac{839}{1089}$
3. 200; 160; 117$\frac{887}{1089}$
4. 266$\frac{2}{3}$
5. 55′; 80′; 40′; 20′
6. 50′; 85′; 80′; 90′; 110′; 120′; 170′; 210′
7. 60′; 30′; 15′
8. 70$\frac{870}{1089}$; 13$\frac{37}{121}$;
9. 61$\frac{257}{363}$; 20$\frac{630}{1089}$; 111$\frac{721}{1089}$
10. 41$\frac{80}{1089}$ sq. rds.

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11. 130$\frac{10}{11}$ rds.

Page 187

1. 7$\frac{1}{2}$; 6; 18; 49; 4; 2b; 6; 6; 2ab; a
2. 32; 15; 72; 88; 4$\frac{2}{7}$; 24; 3$\frac{6}{7}$; 2; 2; 15; 25; 2; 2; $\frac{1}{5}$; $\frac{1}{2}$; 3$\frac{1}{2}$

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1. 75c.
2. 60c.
3. 90c.
4. 12
5. 60c.
6. 30
7. 2.40
8. 18
9. $49; $63; 38.50
10. 24; 32; 34
11. 65.96\frac{4}{7}$
12. 28
13. 2$\frac{1}{3}$
14. 2$\frac{3}{8}$
15. 7507$\frac{1}{2}$

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16. 272,160
17. 49
19. 240; 50; 40
20. 30 ft.
21. 40 ft.
22. 108$\frac{1}{3}$
22. 108$\frac{1}{3}$
23. 36
24. 9″; 6$\frac{3}{4}$″; 3$\frac{1}{2}$″;22$\frac{1}{2}$″
25. 9″ to mile
26. 1″ to 8$\frac{1}{3}$ft.; 1″ to 25′; 1″ to 6$\frac{2}{3}$′

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27. Water, 10$\frac{1}{5}$ ozs.; Fat, 8$\frac{2}{5}$; Prot., 7$\frac{4}{5}$; 6$\frac{3}{5}$; Water, 18$\frac{7}{10}$ozs.; Fat 15$\frac{2}{5}$; Prot., 12$\frac{1}{10}$; 12$\frac{1}{10}$; Water, 40$\frac{4}{5}$ozs.; Fat, 33$\frac{3}{5}$; Prot. 31$\frac{1}{5}$; 26$\frac{2}{5}$
28. Water, 68$\frac{4}{5}$ozs.; Fat, 20$\frac{4}{5}$; Prot., 21$\frac{1}{3}$; 17$\frac{1}{15}$; Water, 206$\frac{2}{5}$ozs., Fat, 62$\frac{2}{5}$; Prot., 64; 51$\frac{1}{5}$; Water, 258 ozs.; Fat, 78; Prot.,80; 64
29. Water, 15$\frac{3}{4}$ozs., Fat, 5$\frac{1}{4}$; Prot., 5; 4; Water, 7$\frac{7}{8}$ ozs.; Fat, 2$\frac{5}{8}$; Prot., 2$\frac{1}{2}$; 2; Water, 9$\frac{7}{20}$ ozs.; Fat, 3$\frac{3}{20}$; Prot., 2$\frac{13}{20}$; 2$\frac{1}{5}$; Water, 25$\frac{1}{5}$ozs.; Fat, 8$\frac{2}{5}$; Prot., 8; 6$\frac{2}{5}$
30. Water, 4$\frac{1}{3}$ozs.; Fat, 2$\frac{2}{15}$; Prot., 1$\frac{7}{10}$; 2$\frac{1}{2}$; Water, 10$\frac{5}{6}$ozs.; Fat, 5$\frac{1}{3}$; Prot.,4$\frac{1}{4}$;6$\frac{1}{4}$; Water, 6$\frac{14}{15}$; Fat, 3$\frac{31}{75}$; Prot., 2$\frac{18}{25}$; 4; Water, 32$\frac{1}{2}$ozs.; Fat, 16; Prot., 12$\frac{3}{4}$; 18$\frac{3}{4}$

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2. 36.04
3. 1891 25.54 in.
1892 36.56
1893 27.47
1894 27.47
1895 32.89
1896 33.14
1897 25.85
1898 33.77
1899 31.49
1900 28.65
1901 24.52
Total 327.35
4. Jan.21.89 1.99
Feb.25.15 2.88
Mar.26.50 2.41
Apr.18.75 1.70
May33.48 3.04
June39.25 3.57
July33.38 3.03
Aug.29.04 2.64
Sept.31.38 2.85
Oct.14.54 1.32
Nov.28.50 2.59
Dec.25.49 2.32
Total, 327.35
1. 223.76T
2 157.42$\frac{1}{2}$
3. 127.35
4. 1095$\frac{1}{2}$

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5. 159 792
6. 1829.1

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2. 16.875
3. .133 lbs.
4. .0817lb.
5. Wt. of water, .0975lb.; organic .031
6. Poplar water, .089; organic .0387; Ash water, .055; organic .0414
7. 1.077lbs

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8. 50.625 cu. in.
1. .04 1.48
.04 2.71
.06 3.51
.08 2.85
.10 3.20
.12 3.16
.15 7.36
.26 12.13
.31 15.04
.33 17.16
.34 20.56
.34 18.61
.45 16.10
.41 16.31
.35 17.67
.35 18.74
.35 18.73
2. .44 .44
.40 .38
.56 .54
.36 .34
.36 .34
.35 .32
.29 .18
.08 .03
.05 .03
.03 .02
.0 0
−.03 −.14
−.11 −.07
−.06 0
−.03 −.03
.0 0

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3. 6.37 5.14
7.42 6.62
11.15 11.81
9.82 9.47
16.04 16.08
20.59 16.39
20.16 15.39
11.03 8.12
4.87 2.75
7.92 4.52
.04 1.99
−.46 2.05
−3.89 −4.10
−4.80 −6.16
−3.73 −4.80
0 +.01

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4. 2yrs.; 17; bet. 40 and 50; bet. 40 and 50
5. 2yrs.; 13; bet. 40 and 50; bet. 50 and 60
7. Boys Girls
.036 .040
.052 −.020
−.020 .080
−.033 −.003
.043 −.046
.010 .039
−.135 .053
.132 .013
.115 .020
.549 .171

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1. 3,811,808; 381,180.8; 38,118.08; 3,811.808; 38.11808; 3,811.808
2. 1; 2; 3; 2; 4; 4; 4
3. 0; 1; 2; 3; 5; 3

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8. 143.547
9. 806.7525
1. Total wgt., 3236; H. pull in lbs., 242.7; tot. wgt., 3377.8; H. pull in lbs., 253.335; total wgt., 5256.35; H. pull in lbs., 394.226$\frac{1}{4}$; total wgt., 6258; H. pull in lbs.,469.35
2. Total wgt., 2240; H. pull in lbs., 280; total wgt., 1709.58; H. pull in lbs., 213.69$\frac{3}{4}$; total wgt., 5054.85; H. pull in lbs. 631.856$\frac{1}{4}$; total wgt., 5335.75; H. pull in lbs., 716.96$\frac{7}{8}$

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3. 1060.9, 273.7122; 4446.38, 1147.16604 ; 6669.25, 1720.6665

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4. (1.) 7855.65, 223.8860¼; 149.-25735, 261.-593145
(2.) 11530.86, 328.62951; 219.-08634, 383.-977638
(3.) 12600.65 359.118525; 239.41235, 419.-601645
1. $62.75
2. $62.75

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3. $37.25
4. (1.) 18011.444
18735.556
62795.833
67801.667
106885.636
(2.) 274230.136
5. 1. 14.29
2. 14.43
3. 14.48
4. 14.43
5. 14.35
6. 14.32
7. 14.45
8. 14.22
9. 14.33
10. 15.009

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5. .1; .01; .001
6. .5; .05
7. .10; 1; $\frac{1}{10}$; $\frac{1}{100}$
8. .01; .1; .01; .001
9. 6.8; .68; .068; 68.; 680; 6.8;

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11. 1. 23.8
2. 2.38
3. .00238
4. 2380.
5. 23800.
6. 238.
7. 2.38
8. 23.8
9. 23.8
1. $122, 473, 954.-50
2. 46,073; 1355$\frac{3}{34}$
3. $6.483
4. $298.70
5. $8.785
6. $23.126
7. $601.276

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8. 240.1; 201.9; 442.10
9. $12.85
1. 3.144
2. 3.138
3. 3.142; 3.151; 3.142; 3.160; 3.141; 3.193 av. cir., 30.809; av. dia., 9.77 ratio, 3.1546

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4. 3.119; 3.143; 3.142; 3.142; 3.142
5. Av. cir., 134.3; av. dia., 42.78; ratio, 3.139
7. 3.145; +.0034 diff.

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1. 18. 3.69
19. 3.69
20. 3.78
21. 3.82
22. 3.88
23. 3.87
24. 3.92
25. 3.92
26. 3.88
2. 18. 1.87
19. 1.85
20. 1.86
21. 1.85
22. 1.87
23. 2.04
24. 2.11
25. 2.10
26. 2.07
3. 18. 1.97
19. 1.99
20. 2.04
21. 2.06
22. 2.07
23. 2.04
24. 2.11
25. 2.10
26. 2.07
1. Alum 2.664
Zinc 6.984
Cast Iron 7.2
Tin 7.333
Wrt. Iron 7.68
Steel 7.840
Brass 8.381
Copper 8.832
Silver 10.481
Lead 11.350
Gold 19.214
Plat. 21.552
Wood
Cork .24
Spruce .5
Pine, Yel. .554
Cedar .561
Pine, Wht. .448
Walnut .670
Maple .750
Ash .844
Beech .852
Oak 1.04
Ebony 1.216

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1. Lignum V. 1.338
Liquid
Alcohol .800
Turpen. .870
Petr. .891
Olive Oil .912
Lin. Oil .952
S. Water 1.026
Milk 1.032
Acetic ac. 1.064
Mur. acid 1.2
Nitric ac. 1.52
Sulph. ac. 1.842
Mercury 14.08

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2. Gas
Hydrogen .069
Smoke (wd.) .090
Smoke (s. c.) .101
Smoke, 212° .469
Carb. ox. .967
Nitrogen .974
Air 1.
Oxygen 1.105
Carb. ac. 1.529
Chlorine 2.440
3. Glass (av.) 2.81
Chalk 2.79
Marble 2.71
Granite 2.66
Stone (com.) 2.53
Salt (com.) 2.13
Soil (com.) 1.99
Clay 1.95
Brick 1.89
Sand 1.82

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2. .75; .40; .80; .875; .625; .187; .156; .203
3. 1.50; 3.20; 2.-75; 6.875; 13.-312; 18.406
4. .14285; .23076; .47058; .39130; .43243; .26315
5. .333333; .666666 .166666; .833-333; .111111; .222222; .777-777; .363636; .545454; .8181-81; .373737; .4-07407; .740740
6. 1.35; 16.875; 5.286; 17.9625; 20.04; 1.008; .00875; .0875; .000625; 30.06-060

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7. .9166; .7692; .3684; .2727; 4.2222; .4666; .8589; .7107; .5947
8. .4702+
9. .30⅓

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1. 491.07$\frac{1}{7}$
122.767
1257$\frac{1}{7}$
11314$\frac{2}{7}$

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2. 1413.72
3. 268.9107$\frac{1}{7}$
4. 21480.945 lbs.
5. Diam. 1.7506; area, 2.407 sq. in.;
6. 7976.71875 lbs.
7. 1. 7951.4956 lb.
2. 47078.996 lb.
3. 31175.346 lb.
4. 37192.960 lb.
5. 42270.5 lb.
6. 31569.56
7. 73378.313
8. $a = \frac{c}{2} \times r$
9. C = 2HR
10. 763.41 sq. ft.

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1. 1. 99.89
2. 317.702
3. 1081.4
4. 366.731
5. 2917.7183
2. 1. 78.58
2. 97.04
3. 12.93
4. 36.159
5. 37.896
6. 801.32
7. 723.037
8. 2.797
9. 25.252
10. .017
11. 58.1211
12. .0003
3. 1. 3759.52
2. 677.8582
3. 756.0743
4. 8680.0868
5. 71.517
6. .71517
7. .06464
8. 1.2096
9. 16.210801
10. 12.096
11. 12.096
12. 79.105488
13. 79.105488
14. 79105.488
15. 2184.1974
16. .21841974
17. 52.904544
18. 52.904544

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4. 1. 4.5009
2. 4.591
3. 88
4. 320.152
5. 28.4
6. 38.6
7. 4620.
8. .0462
9. 462.
10. 462.
11. 462.
12. 231,000

5. 1. 26.666$\frac{2}{3}$
2. 876,388.888$\frac{8}{9}$
3. 1285.714
4. 122222.22
5. .333$\frac{1}{3}$
6. .333$\frac{1}{3}$
7. .666$\frac{2}{3}$
8. .444$\frac{4}{9}$
9. 4888.888
10. 4.333$\frac{1}{3}$
11. .855$\frac{5}{9}$
12. 12.312
13. 3636.363
14. 7575.757
15. 78.787
16. 7.878
17. 3.453
18. 11.441
19. .191
20. .636
21. 112.222
22. 33.333$\frac{1}{3}$
23. 186
24. 1.111$\frac{1}{9}$

Page 220

1. 230c.; 2300; 23000
2. \$2.48; \$24.80 2480
3. 49; 4.8
4. 10 pounds; 200 marks; 250 francs; 66$\frac{2}{3}$ rubles; 250 crowns; 250 liras
5. 20.54 pounds; 419.29 marks; 518.13 francs; 129.52 rubles; 492.61 crowns; 518.13 francs.
6. \$7.9625
7. \$323.08
1. (1) 288; (2) 1440; (3) 4080; (4) 4080; (5) 4608; (6) 26880
2. 10
3. 30
4. 1330 grains; $2\frac{37}{48}$ troy ozs.
5. 128 ounces; 96
6. 384; 774
7. 4800; 22400; 8008

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8. (1) 56, (2) 300, (3) 1391, (4) 1728, (5) 13,008, (6) 15,200
9. (1) 2 tons; (2) 1$\frac{1}{2}$ tons; (3) 8 tons; (4) 4$\frac{1}{2}$ tons; (5) 6$\frac{1}{2}$ tons; (6) 6$\frac{3}{4}$ tons

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10. (1) 3 long tons (2) 8$\frac{1}{2}$ l. t. (3) 5 l. t. (4) 42$\frac{1}{2}$ l. t. (5) $\frac{1}{40}$ l. t. (6) 1$\frac{1}{3}$ l. t.
11. 500lbs.; 1120lbs
1. 198 inches; 792 inches
2. 66 ft.
3. 678 inches; 456 inches
4. 32 rods, 2yds.
5. 60ft.
6. 237.6 bds.
7. 2840.2 bds.
8. 90
9. 335$\frac{5}{7}$
1. (1) 432 sq. in. (2) 440; (3) 1,296; (4) 5,616; (5) 46,704; (6) 6 588; (7) 4,536; (8) 6,516; (9) 39,204
2. (1) 4sq. rds. (2) 4A. (3) 6400 sq. rds. (4) $\frac{1}{2}$ A. (5) 272$\frac{1}{4}$ sq. ft. (6) 4840 sq. yds.
3. (1) 1 A. (2) 200 sq. ch. (3) 6400 sq. ch. (4) 230-40A. (5) 160 sq. rds. (6) 102,400 sq. rds.
4. 2rds.; 10 rds.; 40rds.; 160 rds.

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5. 10rds; 36sq rds more
6. 60 sq. rds more
7. 79 sq. yds less
8. 43,560
9. 201 sq.yds more
1. 1. 5pts.
2. 10 gills
3. 30 gills
4. 16 pts.
5. 48 qts.
6. 94 pts.

2. 1. 12 qts.
2. 6 qts.
3. 9$\frac{1}{2}$ gals.
4. 2 gals.
5. 8qts.,1pt.
6. 10 gals.,2qts.

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3. 57$\frac{3}{5}$cu in.; 28$\frac{4}{5}$ cu. in.
4. 37.8 cu. in
5. 1. 12pks.
2. 9pks.
3. 10pks.
4. 32dry gals
5. 2$\frac{1}{4}$bu.
6. 320pts.

6. $\frac{1}{16}$ lb.
7. 28bu.; 25bu.
8. 806$\frac{2}{5}$bu. 32.256 tons
9. About $\frac{4}{5}$ Winchester bushels.
10. 224 pts.; 2$\frac{1}{4}$ bu.
11. \$1.08 per bu.; 3$\frac{3}{8}$c. per qt.

Page 223

1. 1. 3600 sec.
2. 86,400 sec.
3. 604,800
4. $\frac{1}{60}$ hr.
5. 2hrs.
6. 7da.
7. 2wks.
8. 1wk. 1hr. 5 min. 52 sec.
9. 10,080min.

2. 52 wks. Wed.
5. \$1817$\frac{1}{2}$
6. \$2184
9. 12:37hrs 19 min 12:56hrs 17 min 13:13hrs 16 min 13:29hrs 16min 13:45 hrs

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10. 6:14hrs.; 6:23 hrs.; 9 min.
11. 6:26; 6:30; 4 min.; 6:36; 6:37 1min.; 6:46; 6:-43; 3 min.; 6:45 6:50; 5 min.
13. Jan. 1st. 1:39 April 1st. 1;35 July 1st. 2:5 Oct. 1st. 1:30
14. Jan. 1st. 1:39 April 1st. 1:34 July 1st. 2:5 Oct. 1st. 1:30
1. \$4.20
2. 70; 2000 years; 24000 months.
3. 1776
4. \$24
5. 129.60
6. \$432
1. 15552
2. 46$\frac{34}{433}$

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3. \$25
4. \$37.56
5. 1,244,160
6. 2804

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7. 7rms, 7 quires; 51rms., 9 qrs
8. 560 shillings; 6702; 26,880
9. 216; 864
10. 27,783
11. 82,266
12. £119, 6 s, 11 d.
13. 234; 2808
14. 2838
15. 78yds.; 2ft.;6in
16. 19,200; 160
17. 19,371
18. 12cwt, 10lbs

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19. 5781in
20. 160yds. 1ft. 9in
21. 277qts
22. 55815
23. 635
24. \$1925.92
25. 371$\frac{1}{4}$
26. 116.1
27. 54.87 copper; 18.29 nickel
28. 232.2
29. 18$\frac{2}{3}$
30. $\frac{4}{5}$; 1; $\frac{5}{4}$; 1
31. 8.616 oz
32. 1.32+
33. 35 pounds; 700 M
34. 283.8
35. $\frac{10}{11}$
36. 54ft. 10in.
37. 2mi., 44ch., 1rd 4li.

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38. 112rds., 4yds,. 10$\frac{4}{5}$in.
39. 24.2
40. 660
41. 198$\frac{2}{3}$
42. \$975
43. 5$\frac{3}{10}$A
44. 10,000
45. \$192.50
46. \$14,400
47. 6392.208sq rds
48. 27.720; 16$\frac{1}{21}$
49. 10.
50. .37$\frac{1}{2}$
51. \$4.61
52. \$45

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53. 31,500
54. 7.92
55. \$122828.28
56. \$530.30
57. 24589.2
58. \$22.75
59. \$1.80
60. 132$\frac{1}{2}$cu. in.
61. \$727.27
62. 1188.45
63. \$2117.5
64. \$992.30

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66. 1pt.,1.376gill
67. $13.86
68. 15$\frac{8}{3}$

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69. Gain,$.24
71. .921875
72. 478.12
73. $76.65
74. $\frac{1}{64}$
75. 13$\frac{1}{3}$c.
76. 2150.22
77. 10.8
78. $116.40

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2. 6m; 8dm; 7cm; 5mm
6. 2.112km
7. $.711
8. 48.
9. 51.52m
10. 4035m
11. 10,000; 1,000-000.
17. 215.28 sq. ft.

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20. 42m^2
21. 252.12m^2
22. $1664
23. 372 ares
24. 35m^2
25. 258.7846
26. 2000; 2Dl; .02 Kl; .6804 qtc Kl; .6804 qt. (liquid)
30. 204.6211b
31. 1000
32. 13$\frac{1}{4}$lbs.; 6000g.
33. 39.81456
34. 220.46; 1102.31
35. 38.58 capsu
36. 32.15 cents
37. $39.41
38. $17
39. 56

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40. 31.25
41. 131.7211726

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1. 24

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2. 60%
3. 40%
4. .83$\frac{1}{3}$; .62$\frac{1}{2}$; .91$\frac{2}{3}$; .14$\frac{2}{7}$; .71$\frac{3}{7}$ .44$\frac{4}{9}$; .63$\frac{7}{11}$; .46$\frac{2}{13}$; .47$\frac{7}{19}$; .48$\frac{4}{31}$;

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5. 48$\frac{1}{3}$%
7. 33$\frac{1}{3}$%; 25%; 41$\frac{2}{3}$%; 77$\frac{7}{9}$%; 97$\frac{2}{9}$%; 2$\frac{1}{3}$%
8. 1. 5$\frac{1}{3}$%
2. 1$\frac{8}{7}$
3. 62$\frac{1}{2}$
4. 7$\frac{1}{24}$%
5. 4$\frac{1}{6}$%
6. 5%
7. 12$\frac{1}{2}$
8. 50%
9. 4$\frac{1}{6}$
9. $\frac{1}{20}$; $\frac{2}{25}$; $\frac{1}{12}$; $\frac{1}{3}$; $\frac{3}{28}$; $\frac{4}{25}$; $\frac{1}{8}$; $\frac{9}{40}$; $\frac{27}{40}$; $\frac{7}{8}$
10. .07; .0625; .83$\frac{1}{3}$ .875; .015; .0275; .875; .001; .0001
11. 11.27cu. in
12. 4%

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13. 64.7; 59.2; 58.0; 54.2; 50.0 38; 37.9; 37.4
14. 25%; .12$\frac{1}{2}$; .06$\frac{1}{4}$ .18$\frac{3}{4}$; 18$\frac{3}{4}$; .06$\frac{1}{4}$
15. $9000
16. 36
17. $2000
18. $2100

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1. $4; $7.50
2. .02a; .02b; .02x; .12x
3. .05a; .05b; .05x; .05z; .40z
4. .125a; .125x; 1.25x
6. .0a; .0x; .09x; .12y; .45z
7. .16; .29; om; ox; om; .16x
8. 1.6r; 3.5r; Orn; .OrR; .orC; .02mr
9. p = bxr

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1. $20
2. 25c.
3. 13c.
4. $.80
5. 160
6. $128; $179.20; $224; $40; $80; $560
7. $5192
8. $4400
9. 80$\frac{1}{3}$ ft.

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10. 177$\frac{7}{9}$%
11. .16$\frac{2}{3}$
12. 27$\frac{3}{11}$%
13. 66$\frac{2}{3}$%
14. 11,200
15. $52.80
16. $14.00
17. 20%
19. 32,000
20. 597
21. 2000
22. $9.775
23. $750
24. $250
25. $29.75
26. 1.16\frac{2}{3}$

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27. $5,000
28. 15% spilled; $4.5
29. $925
30. 20%
1. 29$\frac{1}{31}$; 48$\frac{2}{31}$; 22$\frac{8}{31}$; 29$\frac{1}{31}$
2. Feb. 35$\frac{5}{7}$; 39$\frac{2}{7}$; 25; 32$\frac{1}{7}$
Mar. 38$\frac{22}{31}$; 45$\frac{5}{31}$; 16$\frac{4}{31}$; 35$\frac{15}{31}$
Apr. 33$\frac{1}{3}$; 43$\frac{1}{3}$; 23$\frac{1}{3}$; 43$\frac{1}{3}$
3. 21$\frac{29}{31}$; 24$\frac{17}{21}$; 29$\frac{11}{11}$; 24$\frac{1}{4}$
4. Cloudy, 28$\frac{15}{83}$; 20$\frac{10}{83}$; 26$\frac{23}{83}$; 24$\frac{18}{83}$
P. Cloudy, 26$\frac{1}{13}$; 26$\frac{1}{13}$; 19$\frac{3}{13}$; 26$\frac{1}{13}$
Rainy or S. 21$\frac{3}{7}$; 21$\frac{3}{7}$; 25$\frac{4}{21}$; 30$\frac{20}{21}$
5. Jan. 71$\frac{3}{7}$%
Feb. 68$\frac{1}{7}$%
March 81$\frac{3}{7}$%
Apr. 100%
May 77$\frac{1}{7}$%
June 64$\frac{2}{7}$%
July 80%
Aug. 67$\frac{1}{7}$%
Sept. 74$\frac{2}{7}$%
Oct. 82$\frac{1}{7}$%
Nov. 74$\frac{2}{7}$%
Dec. 82$\frac{1}{7}$%
6. Jan. 82%
Feb. 66%
Mar. 90%
Apr. 83%
May 80%
June 70%
July 73%
Aug. 67%
Sept. 76%
Oct 84.9 or 85%
Nov 87.7or 88%

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7. Mt. T. 88.9%
Blk. Isl. 94.9%
Chicago 100%
Cleve. 112.9%
N. Y. 114%
Buffalo 116%
Boston 147%
Phila. 152%
St. L. 171.7%
New Or. 195%
Louis. 206%
Wash. 228%
Roseb. 476.4%
8. Mt. T. 34.6%
Blk. Isl. 3165%
Chicago 100%
Cleve. 108%
N. Y. 262%
Buff. 1.07%
Boston 658.4%
Phila. 703%
St. L. 145%
N. Or. 1613.7%
Louis. 156.7%
Wash. 734.8%
Roseb. 158.8%
9. 39.5%; 2.4%
10. 23; 1.6
11. 120 sq. ft.

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12. Jan. def. 46.8%
Feb. ex. 43.6%
Mar. def. 33.7%
Apr. ex. 38 %
1. 60.2%
2. St. Louis, 9 hrs. 29 min., and 51.72 sec.
St. Paul, 8hrs., 49 min., and 14.16 sec.
3. St. Louis, 14hrs 51 min., and 52.14 sec.
St. Paul, 15hrs., 31 min., and 2.52 sec.

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	hrs	min	sec
5. F.	9	52	55.2
M.	11	11	25.62
A.	12	36	31.32
M.	13	57	46.44
J.	14	55	58.08
J.	14	57	54.9
A.	14	26	52.26
S.	13	13	51.24
O.	11	49	18.48
N.	12	7	25.50
D.	9	23	16.44

3. 5.5%; 39%
4. 27.6%; 5.2%

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7. 21%
9. L. Pur 142.8%
Florida 2.9%
Texas 17.9%
Mex pur 21.4%
Tex. pur 3.2%
Gads. pur 1.4%
Alaska 19.02%
Hawaii .18%
P. Rico .09%
Phil Is. 3.07%
Guam .005%
Is. of P. .02%
10. 73.4%; 26.5%

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11. N. A. 43.1%
S. A. 66.4%
Asia 44.7%
Africa 83.5%
Europe 38.5%
Aus. 77.1%
12. N. A. $72\frac{2}{9}$
S. A. $336\frac{4}{11}$
Asia $79\frac{19}{39}$
Africa $43\frac{19}{27}$
Europe 350
Austral. 350
13. 48.7%; 72.5%; 69%; 98.5%
15. 7.7+; 2.2; 1.-7+; 1.4+
16. 47.6; $33\frac{1}{3}$; 47.6; 47.6

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17. 92.8%; 104%
19. 71.9%
20. 96.5%; 86.8%
1. $4.00
2. $62.50
3. $1.50
4.1. $100
2. $382.50
3. $3.00
4. $14.25
5. $3
6. $112.50
7. $75
8. $240

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5. $46.25; $64.75; $231.25
6. $2706.12
7. $1119.52
8. $13,684
9. $53.6085
10. $17.523
11. $3886.40

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2. $60
3. $2992.50 note $2975
4. $5.301

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5.1. $30.375
2. $128.25
3. $1054.62
4. 41.2965
5. 233.284
6. $686
7. $17.859
6. 24%

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1.1. $42\frac{6}{7}$
2. $41\frac{27}{53}$
3. $28\frac{4}{7}$
4. 46
5. $26\frac{4}{5}$

2.1. $\frac{ro}{ao}$; 2. $\frac{on}{m\ s}$; 3. $\frac{hso}{io}$; 4. $\frac{hha}{zs}$; 5. $\frac{hio}{has}$; 6. $\frac{moo}{rao}$ 7. $\frac{nrs}{mos}$

3.1. $\frac{ms}{rs}$; 2. $\frac{zs}{ns}$; 3. $\frac{mas}{rho}$; 4. $\frac{has}{es}$; 5. $\frac{rne}{ais}$; 6. $\frac{zus}{umo}$

4.1. oo; 2. ee; 3. aah; 4. ren; 5. zzs; 6. hhrr
5.1. mo; 2. io; 3. hao; 4. hzr; 5. rio; 6. mso
6.1.k k; 2.ss; 3. lab; 4. aoh; 5. rre; 6. bba lab; 4. aoh; bbaa;
1. ck; 2. ok; 3. blk; 4. bra; 5.aok; 6.cck
8.1. uu $\frac{a}{r}$; 2. zs; 3. hns; 4. rrr $\frac{h}{r}$; 5. z or $\frac{h}{r}$

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1. $31.50; $31.50; $63; $157.50; $110.25; $149.-625; $31.50t
2. $100; $99.20; $297.60; $578.-$66\frac{2}{3}$; $99.20x
3. $128

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5. $62.30
6. $332.4375
7.1. $34.125
Amt. $734.125
2. $33.90
Amt. $433.90
3. $31.43
Amt. $242.43
4. $20.504
Amt. $170.504
5. $19.565
Amt. $299.565
6. $19.40
Amt. $379.40
7. $20.75
Amt. $280.75
8. 68.44\frac{4}{9}$
Amt. 568.44\frac{4}{9}$
9. $39.25
Amt. $339.25
10. $142.50
Amt. $767.50
8. $100.
9. $150.
10. $70.44
11. $240.
12. $850.

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15.1. P = $200
A = $219.50
2. P = $640.22
A = $809.24
3. P = $720.00
I = $278.50
4. P = $135
I = $63.42
5. P = $1200.98
A = $1323.48
6 P = $3424.864
A = $3600.86
7. P = $2888.88
A = 2921.387
16. $3\frac{19}{22}\%$
17. $11\frac{2}{221}\%$
18. 12%
19. $$9\frac{9}{31}$
21. (1) 5.6+; (2) 10.1; (3) 11.9; (4) 10.4
22. 4; yrs.; 3 mo. 15 da.

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23. 5 yrs., 9 mo.
24. 1 yr., 6 mo., 15+ days
25. 12 yrs & 6mo.
26. $16\frac{2}{3}$yrs.; $14\frac{2}{7}$yrs
27. 5yrs, 1mo., 16+ da.

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30.1.6yrs, 2mo, $8\frac{1}{3}$ or 9 da.
2.1yr, 1mo, 15 da.
3.3yrs, 10mo., 27da.
4.3yrs, 4mo, 18 da.
5.3yrs, 3mo, 6 da.
31.1.I,$11.78; A = $71.78
2.I, $41.927; A = $216.927
3.T, 2yr., 3mo; I = $126
4.T, 3yrs, 7mo, 15da; A = $617.50
5.T, 1yr, 7mo, 26da; I = 83.85
6.P, $1500; A = 2009.25
7.I, 87.16\frac{2}{3}$
8.T, 3yr, 7mo, 6da; A = $937.-986
9.R, 8.9%

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1.1. $37.50
2. $75.00
3. $243.75
4. $137.50
5. $968.75
6. $37.50x
7. 37.50t
2.1. $62.50; 2. $206.25; 3.$525 4.$250; 5.$518.-75; 6.$37.50t.; 7.$12.50rt.; 8. $12.50 ru; 9. $12.50 ru.

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4.1. Int., $42.04; Amt., $292.04
2.Int, $162.365; Amt, $797.365
3.Int., $537.53; Amt., $2937.53
4.Int., $330.46; Amt., $4195.46
5.Int., $771.83; Amt., $4411.83
1. $109.09
2. $132; $22.91

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3. $65.79
4. $61.47
5. $4.547; $86.40

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6. 1. D, $5.99; Pro, $59.41
2. D, $31.21; Pro., $284
3. D, $46.70; Pro., $462.36
4. D, $123.12; Pro., $730.88
5. D, $229.77; Pro., $810.42
6. D, $376.48; Pro., $1167.54
7. D, $350.31; Pro., $1862.19
8. D, $193.99; Pro., $3920.137
9. D, $157.438; Pro., $3920.137
10. D, $1101.876 Pro., $875587

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2. $460.676
3. $1920.669

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1. 4
2. 9′5″
3. 1. 85″
2. 61½″
3. 87″
4. 102″
5. 137″
6. 65″
7. 57′5½″
8. 27′11½″
4. 102″ to 1″
5. 63¾″
6. 68ft. ¾in.
7. 140¼inches
8. 191.2 inches
9. 127½inches; 89¼inches; 140¼ inches
10. 25½ x 25½
11. 3″; 49″; 27″

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12. 18″; 113⅓
13. R19″; C119$\frac{3}{7}$
14. 264; 169$\frac{5}{7}$
15. 60mi an hr
16. 2⅓ times
17. 181535 lbs.
18. 121,333⅓lbs.
19. 1,080,868⅔ lbs., or 540.434⅓tons
20. 2522.026$\frac{1}{9}$lbs.
21. 2972.387$\frac{1}{3}$ lbs.

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22. 1.3422.749$\frac{21}{31}$lbs.
2.3873.111$\frac{9}{16}$lbs.
3.4323.473$\frac{1}{3}$ lbs.
4.4773.835$\frac{5}{36}$lbs.
5.5224.196$\frac{17}{18}$lbs.
6.5674.558$\frac{3}{4}$ lbs.
7.6124.920$\frac{5}{9}$ lbs.
8.6575.282$\frac{13}{36}$lbs.
9.7025.644$\frac{1}{6}$ lbs.
10. 7476.005$\frac{35}{36}$
11. 7926.367$\frac{7}{9}$
12. 8376.729$\frac{7}{12}$

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1. 1. 216lbs.
2. 320lbs.
3. 662lbs.
4. 817lbs.
5. 1223½lbs.
2. 1. 85lbs.
2. 203lbs.
3. 270lbs.
4. 480lbs.
5. 572½lbs.
3. 1. 97.50lbs.
2. 155lbs.
3. 117lbs.
4. 425lbs.
5. 212.5lbs.
5. 1. 1040lbs.
2. 1280lbs.
3. 2650lbs.
4. 3880lbs.
5. 4500lbs.

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9. 1. 20; 2. 20; 3. 3; 4. 40; 5. 7000 6. 40.31+; 7. 60; 8. 12
10. 1. 1785.71lbs;
2. 4821.42lbs.;
3. 2010lbs.;
4. 5910.71lbs.;
5. 2449.29lbs.;
6. 39410.7lbs.

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11. 1. L=35.71F
2. L=66⅔F
3. L=30.30F
4. L=52.63F
5. L=83.33⅓F
6. L=52.63F
7. L=125F
8. L=100F
12. 1. 9998.80lbs.;
2. 5177.95lbs.;
3. 2806.806lbs.;
4. 6277.818lbs.;
4. 6277.818lbs;
5. 4999.40lbs.;
6. 24,997lbs.
13. 1. 1999.80lbs.;
2. 10,998.90lbs;
3. 3999.60lbs.;
4. 15,998.40lbs;
5. 799.92lbs.;
6. 1599.84lbs.
14. 1. 10,000lbs.;
2. 50,000lbs.;
3. 60,000lbs.;
4. 40,000lbs.;
5. 70,000lbs.;
6. 7240lbs.
15. (1) 200; (2) 300; (3) 149.69; (4) 90; (5) 180; (6) 54; (7) 1620; (8) 32.4; (9) 1100

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17. 1. 28,000lbs.;
2. 24,000lbs.;
3. 27,066⅔lbs.;
4. 26,400;
5. 28,586⅔;
6. 32,400;
7. 23,280;
8. 45,680;
9. 36,360
19. 1. 105,000lbs.;
2. 114,000lbs.;
3. 99,000lbs.;
4. 122,400lbs.;
5. 141,000lbs.;
6. 148,650
20. 1. 4432
2. 2326.80
3. 4.986
4. 9.0025
5. .4986
6. 2.20215

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18. 6682½ sq. ft.
19. 741 sq. ft.
20. 723 sq. ft.
21. 172$\frac{52}{121}$ sq. rds
22. 168$\frac{24}{121}$ sq. rds.; 163$\frac{17}{121}$
23. 4941 sq. ft.
24. 1$\frac{9}{40}$A
25. 2$\frac{7}{8}$A; 4$\frac{3}{8}$A
26. 8$\frac{3}{8}$A
27. 8A; 9.7¾ sq.rds.

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1. 1. 50°W;
2. 90°W; 30°N;
3. 30°E; 30°N;
4. 90°W, 35°N; 5°E; 5°N
2. 1. +75°, +40°;
2. +75°, +40°;
3. +70°, +45°;
4. +90°, +45°;
5. +105°, +40;
6. +30°, +90;
7. 6°, +52°
8. −5°, +50°
9. +15°, +52°
10. +25°, +80°
11. +68°, −55°
12. −5°, +35°
3. 65°; 72°; 30°
4. 45°; 70°; 50°
5. 35°; 50°
6. 35°; 22°; 37°
7. 35°; 15°; 30°
8. 30°; 60°; 45°

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9. 45°
10. 15°
11. 15°; 35°; 65°; near 45°
12. 30°; near 50°; 15°
13. 15°; near 35°; 15°; 30°

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14. about same; same; 10°; 7°
15. 16.48; 35.42
16.

	dif. in lat.	dif. in long.
1	2.28	262.01
2	70.78	50.80
3	21.53	91.09
4	5.05	63.04
5	93.26	36.52
6	5.67	200.52
7	42.00	7.89
8	20.83	18.58
9	37.80	8.20

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2. 7 o'c; 12 o'c
3. 60°,45′

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4. 7hr., 3min.
5. 1st. is east, 45°, 30′
6. 1st. from E., 2nd. from W.
7. 20°15′; 92°
10. 64°33′; 54″
11. 166°23′24″; 11hrs. 5min., 33.6sec.
12. 3°46′46″; 15min., 7⅛sec.
13. 17°5′; 1hr., 8 min., 20sec.
14. 2°,15′,33.3sec.; 9min., 2.22sec.

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17. 1. 73°,44′59.85″
2. 3°,2′,8.25″
3. 71°,7′,45.75″
4. 77°,3′,56.7″
18. Allegheny, 25 min., 2.94sec.
Berkeley, 2hrs., 34min., 7.53sec.
Ann Arbor, 10 hrs., 26min., 10.93sec.
Chicago, 1hr., 5min., 55.79sec.
Chicago, 5hrs., 59min., 47.81sec
Madison, 5hrs., 44min., 53.73 sec.
Cape G. Hope, 3hrs., 37min., 20.98sec.
Paris, 4hrs., 41 min., 54.77sec.
19. 6° 15′ 44.1″
38° 31′ 52.95″
156° 32′ 43.95″
16° 28′ 56.85″
89° 56′ 57.15″
86° 13′ 25.95″
54° 20′ 14.70″
70° 28′ 41.55″

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1. 9 A.M; 8A.M.; 8 A.M.; 8 A.M.; 7 A.M.; 6 A.M.; 6 A.M.

2. New Or., 9 hr., 10 min., 45 sec., P. M.
Phila., 10 hr., 10 min., 45 sec., P. M.
Buffalo, either of the above
Washington, 10 hr., 10 min., 45 sec., P. M.
Austin, 9 hr., 10 min., 45 sec., P. M.
Boise City, 8 hr, 10 min., 45 sec., P. M.
Los Angeles, 7 hr., 10 min., 45 sec., P. M.
El Paso, 7, 8, 9 hrs, 10 min., 45 sec., P. M.
Salt Lake City, 7 hrs., 10 min., 45 sec., P. M.

3.

	P.M.
New Or.	2:15
Phila.	3:15
Buffalo	2:15
	3:15
Wash.	3:15
Austin	2:15
Boise City	1:15
Los Ang.	12:15
El Paso	2:15
	1:15
	12:15
S. L. City	12:15

Dodge City, 3.-25 A. M.
Dodge City, 3:25 A. M.

	A. M.
New Or.	4:25
Phila.	5:25
Buffalo	4:25
	5:25
Wash.	5:25
Austin	4:25
Boise City	3:25
Los Ang.	2:25
El Paso	3:25
	2:25
	1:25
S. L. City	2:25

5. 12 o'c. noon; 1 P. M., 4 P. M. 9 P. M.; 11 P. M.

6. 180°

7. 1 hr.

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8. 12 o'c. noon; Sat. noon

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9. 4 A. M. Wed.; 5 A. M. Wed.; 3 A. M. Wed.; 2 A. M. Wed.

10. 1. 8.25 P. M.
2. 8.25 P. M.
3. 9.25 P. M.
4. 10.25 P. M.
5. 6.25 P. M.
6. 8.25 P. M.
7. 12.25 P. M.
8. 11.25 P. M.

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2. $1.60 per sq. ft.; $14.40 per sq. yd.

4. 17625 sq. ft.; 16,800; 8000 26,250; 5,200; 25,312.5

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8. 90,000 cu. ft.; 68,060; 75,000; 90,000; 126,000

10. 1. 13,333$\frac{1}{3}$ sq. yds.
2. 13,333$\frac{1}{3}$ sq. yds.
3. 10,000 sq. yds.
4. 11,544$\frac{4}{9}$ sq. yds.
5. 10,736$\frac{1}{9}$ sq. yds.

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12. 800; 889; 1,333$\frac{1}{3}$; 1143

13. 4″ to weather, $38.72
5″ to weather, $30.98
Cost of tin $8.15

14. 31,445

15. $113,202

16. $220.11

17. 18,191$\frac{1}{4}$

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19. $445.536

20. $202.84

21. 1728; $47.52

23. 26,880; $168

24. 53,089$\frac{1}{4}$; $742.-25

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2. 80; 80; 80; 40; 38; 39; 40

5. S. Perry, Lot 51 No. A., 81
J. Hay, Lot 50, No. A., 82
J. Hay, E. $\frac{1}{2}$ N. W. $\frac{1}{4}$, Sec. 7, No A. 80
H. Ochilltree, lot 49, No. A., 41
H. Ochill. S. E. $\frac{1}{4}$, N. W. $\frac{1}{4}$, Sec. 6, No. A. 40
H. Ochill., E. $\frac{1}{2}$, S. W. $\frac{1}{4}$, Sec. 6, No. A., 80
J. Church, lot 48, No. A. 38
J. Church, lot 47, No. A. 41

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5. 32 cu. in.

6. little more than 24 cu. in.

7. 1. 75.3984
2. 269.392
3. 667.590
4. 2150.4252
5. 296.8812
6. $\frac{4\ ar^2}{.7854} = 3.1416\ ar^2$

9. 761.73$\frac{3}{7}$ gal.

10. 9.75 min.

11 14.72625 lbs.

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12. 211.58

13. 552.699

14. 4

15. $\frac{1}{4}$, 360°; 45°

18. 1. 16.5
2. 188$\frac{4}{7}$
3. 27.2485$\frac{5}{7}$
4. 41.88528274

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19. 21.2142$\frac{6}{7}$

20. 169.71$\frac{3}{7}$

21. 148.5

22. 1.80796130$\frac{6}{7}$

23. 27.2397321$\frac{3}{7}$

34. 5903.9357142$\frac{6}{7}$

26. 7.07$\frac{1}{7}$; 28.28$\frac{4}{7}$

28. 1257$\frac{1}{7}$; 314$\frac{2}{7}$

29. 197040561$\frac{1}{7}$ sq. mi.

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30. $S = 4\pi R^2$

31. 120 cu. in.

32. $V = \frac{B \times a}{3}$

33. 64 cu. in.

Page 316

34. 91,636,272 cu. ft.

35. 7[illegible]

36. 2,234,261,760 cu. ft.

37. 12,330,981,000 cu. ft.

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39. 33.51+

40. 33.51+; 2.807; 4.50+; 547.-736291$\frac{2}{3}$

41. Circum. mi.
Moon, 6,788$\frac{4}{7}$
Merc., 9,522$\frac{6}{7}$
Venus, 24,300
Earth, 24,885$\frac{5}{7}$
Mars, 13,294$\frac{2}{7}$
Jup., 271,857$\frac{1}{7}$
Sat., 227,428$\frac{4}{7}$
Uran., 100,257$\frac{1}{7}$
Nept., 109,371$\frac{3}{7}$
Sun, 2,722,971$\frac{3}{7}$

Surface
Moon, 14,463,-314$\frac{2}{3}$
Merc., 28,854,-257$\frac{1}{7}$
Venus, 186,340,000
Earth, 197,040,561$\frac{1}{7}$
Mars, 56,234,-828$\frac{4}{7}$
Jupiter, 23,515,642,857$\frac{1}{7}$
Saturn, 16,748,-285,714$\frac{2}{7}$
Uranus, 3,198,-202,857$\frac{1}{7}$
Neptune, 3,806,125,714$\frac{2}{7}$
Sun, 2,359,182,-445,714$\frac{2}{7}$

Volume mi.
Moon, 5,278,-793,142$\frac{6}{7}$
Merc. 14,571,-399,854$\frac{1}{7}$
Venus, 239,136,333,333$\frac{1}{3}$
Earth, 260,027,860,521$\frac{11}{21}$
Mars, 39,645,-554,142$\frac{6}{7}$
Jup., 339,017,-184,523,809$\frac{11}{21}$
Sat., 203,770,-809,523,809$\frac{11}{21}$
Uranus, 17,003,778,523,809$\frac{11}{21}$
Nept., 22,075,-529,142,857$\frac{1}{7}$
Sun, 340,665,-945,161,142,-857$\frac{1}{7}$

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1. 5
2. 8
3. 1. 15
   2. 16
   3. 20
   4. $36\frac{7}{16}$
   5. 28
   6. 36

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11. 1. 22
    2. 27
    3. 34
    4. 25
    5. 36
    6. 75
    7. 87
    8. 97
    9. 98
    10. 332
14. 1. 43.6
    2. 68.7
    3. 71.1
    4. 7.82
    5. 8.71
    6. 8.94
    7. .74
    8. .677
    9. 1.103

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15. 1. 2.236+
    2. 2.645+
    3. 3.872+
    4. .921+
    5. .5
    6. 11.180+
    7. 41.024+
    8. 82.914+
    9. 85.603+
    10. 1.359+
    11. 5.175+
    12. 8.055+
16. 1. $\frac{1}{4}$
    2. $\frac{4}{9}$
    3. $\frac{16}{25}$
    4. $\frac{144}{169}$
    5. $\frac{64}{225}$
    6. $\frac{441}{1024}$
    7. $\frac{a^2}{b^2}$
    8. $\frac{x^2}{y^2}$
    9. 85.603+
    10. 1.359+
    11. 5.175+
    12. 8.055+
18. 1. $\frac{1}{2}$
    2. $\frac{2}{3}$
    3. $\frac{4}{5}$
    4. $\frac{12}{13}$
    5. $\frac{8}{11}$
    6. $\frac{7}{13}$
    7. $\frac{9}{17}$
    8. $\frac{20}{27}$
    9. $\frac{a}{b}$
19. 42.8+″
20. 70.037+″

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21. 90′
22. $4321.1972\frac{1}{2}$
23. $8.777
24. 11248.72 sq.ft.

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25. 80′
26. $133\frac{1}{3}$yds.
27. 660 ft.
28. 173.2 sq. ft.
29. 3366.36 sq. ft.
30. 804.60 sq. ft.
1. 125 cu. in.; 216 cu. in.; 343; 1331; 5832; 9-261; 13,824

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2. 2 and 3; 3 and 4; 4 and 5; 7 and 8; 8 and 9; 9 and 10; 38 and 39; 78 and 79
3. 15.625; 300.-763; 1953.125; $\frac{1}{8}$; $\frac{8}{27}$; $\frac{64}{125}$; $\frac{343}{1728}$; $37,037\frac{1}{27}$; $6591\frac{17}{64}$
7. $\frac{1}{2}$; $\frac{3}{4}$; $\frac{5}{7}$; $\frac{9}{20}$; $\frac{7}{10}$; $\frac{9}{56}$

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1. 62°
2. 518 ft.
3. yes
4. 45 ft.
5. $67\frac{1}{2}$ ft.
6. $23\frac{1}{7}$

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7. 128 rods
9. $\frac{1}{4}$″
10. 3′
11. 2200 mi.

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13. $704,545\frac{5}{11}$ miles
14. 852,500 miles
15. 1700 ft.

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16. 1504 ft.
17. 54 ft.
18. 846.4

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20. $4\frac{1}{8}$″; 4.52″; 3.78″; 5.27″; 5.35″; 8.32″
21. 280′; 450′; 512′ 295′; 625′; 448′
22. 3.39″; 3.6″; 3.-425″; 3.69″; 3.-49″; 3.125″; 3.-395″
23. 762′; 824′; 572′ 550′; 1172′; 12-36′; 1196′

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25. 988 sq. ft.
26. 607.3 sq. ft.,; 774; 917.2; 749 1672.5; 756.3
1. $11.90
2. $6250
3. 14,000
4. $2,800
5. $16,000

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6. $1\frac{63}{200}$%
7. $2\frac{3}{11}$%
8. 2.6%
9. $700
10. $4500
11. 2.4%
12. 3%
13. $72640
14. $1800
15. $1\frac{1}{4}$%
16. $18.6624
17. 1500; $84
18. 4%
19. $497
20. 75%; $67.50

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1. $24087.50
2. $33,874.35\frac{1}{3}$
3. $7390.512
4. $684,042.55
5. $$3312133\frac{1}{3}$
6. 18 mills on a dollar
7. $3001.471
8. 15
9. $158.40
10. $21.443
11. $84

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12. $100
13. .013;$13,639.11
14. $12767.48
15. 19
16. $21,354.16
1. $578.85; $572.76

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2. $143.66
3. $145.19
4. $1051.84;
5. $1040.77
6. $285.226

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1. 2.1%; $420
2. $26350; $26550 $26,200; $26400
3. $35,000 $35,050; $34,-950; $34,950
4. Gain, $500
5. Neither loses nor gains
6. $1562.50
7. $8\frac{3}{4}$
8. 9.7%

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9. 1.85%; 1.83%
10. 1. 2.76%; 2.74%
    2. 2.76%; 2.74%
    3. 3.55%; 3.53%
    4. 2.80%; 2.85%
11. 3.69%

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12. 4.56%; 4.55%
13. Wab. R. R., 5s. .42%; .41%
14. Strgt. loan, .06%
16. 14.92%;1122% $8850

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2. $60; $7.68
8. $112.61+
9. $277.46

Page 345

10. $225.57
11. $631.24
12. $614.43
2. x+y
3. x+y
4. 80c.−45c.=35c.
5. m−s dollars
6. 45c.
7. xy dollars

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8. 5A°
9. 7 rods
10. $\frac{x}{y}$ inches
11. $\frac{a}{b}$ lots
12. 30c.; 40c.
13. 8x; $3x^2$
15. 28°
16. 12+x; 12−y
17. 2°, 6°, 12°
18. $1.50 hat; $3.00 coat
19. x−$30
20. $\frac{x-30}{2}$

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21. x−y
22. 3a
23. x−y
24. a+b
25. 3+a miles per hr.
26. 3−a
27. 5a cents
28. ax c.
29. $40
30. $\frac{15}{x}$
31. $\frac{m}{l}$ rds.
32. x=30

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33. $10x+8yc$

34. $\frac{m+c}{3}$ c.

35. $5\left(\frac{m+c}{3}\right)$

$5\frac{(m+c)}{3}-(m+c)$ c. $=(m+2c)$ c

36. 72 xy. sq. in.

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37. $\frac{x}{3}$

38. $bx+by$ or $6(x+y)$

1. 75 lbs

2. $p=75$ lbs.

3. $F=300$ lbs.

4. $F=7956$ lbs.

5. $F=3978$ lbs.

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3. 2

4. 10; 7

5. b; $(b+c)$ lb.; $(p+35)$ lbs.

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1. 1. 5lbs.
2. 3 ft.
3. $2\frac{2}{3}$ mi.
4. 4c.
5. \$4
6. 21
7. 5
8. 24
9. 3

2. 6; 21; $1\frac{1}{2}$

3. 15; 2; 19

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4. 5; 15; 25; 25

5. 4;12; 28; 16; 2

6. 9; 27; 81; 3

7. 3

8 $21\frac{1}{2}$

9. 4

10. 2:2

11. 7;7

12. 10; 5; $3\frac{1}{3}$; $2\frac{1}{2}$; 1; $\frac{1}{2}$

14. 1. 13
2. 3
3. 80
4. 1600
5. 9
6. 89
7. 9
8. 1600
9. 39
10. 89
11. 712
12. 15

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15. 1. $x<y$ correct
2. $x<y$
3. $x<y$
4. $y>-x$ correct
5. $x+y=13$ cor.
6. $x-y=5$ cor.
7. $(x+y)^2=1296$
8. $(x-y)^2=25$ cor.
9. $x^2-y^2<25$ cor.
10. $x^2+y^2=97$
11. $(x+y)^2=x^2+2xy+y^2$
12. $(x-y)^2=x^2-2xy+y^2$
13. $(x+y)^2=x^2+2xy+y^2$ cor
14. $(x-y)^2=x^2-2xy+y^2$ cor
15. $(x-y)^2=x^2-2xy+y^2$
16. $(x+y)(x-y)=x^2-y^2$
17. $(x+y)(x-y)=x^2-y^2$
18. $(x-y)\ (x-y)=x^2-y^2$
19. $(x-y)\ (x+y)=x^2-y^2$
20. $x(x+y)=x^2+xy$ cor.

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5. 1. 2
2. 2
3. 3
4. 5
5. 4
6. 2
7. $6\frac{11}{18}$

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4. 4 no. ash trees
12 no. oak trees
16 no. hickory trees

5. 15 5c. stamps
60 2 c. stamps
30 10 c. stamps

6. 20 acres

7. 6; 24; 3

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8. 9; 18; 21

9. 56,650 sq. mi.

10. 16 c.

11. $126\frac{2}{3}$ mi

12. 285 mi.

13. 58 mi.

14. 248 mi.

15. 369 mi.

16. Son, \$15; Father, \$60

17. Joseph 6; William 18

18. 12; 20

19. 30; 45

20. 40

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21. 440 rods

22. \$2.50

23. 40

24. \$37.50

25. 350 rods

26. 8; cir. $=50\frac{2}{7}$

27. $R=\sqrt{\frac{7}{22}}$; $C=6\frac{2}{7}\sqrt{\frac{7}{22}}$

1. 3

2. 8

3. 7

4. 2

5. 24

6. 36

7. 20

8. 12

9. 3

10. 10

11. 3

12. 2

13. $2\frac{1}{2}$

14. 8

15. 15

16. 12

17. 8

18. 9

19. 3

20. 3

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21. Hemp cable 1″ diam.
W. .577 sq. in.
S. .109 sq. in.
S. .654 sq. in.
Hemp cable $2\frac{1}{2}$″ diam.
W. 3.60625 sq. in.
L. .68125 sq. in.
S. 4.0875 sq. in.

22. Tarred hemp rope, 1″ diam.
W. 1.036 sq. in.
L. .247 sq. in.
S. 1.480 sq. in.
Tarred hemp $2\frac{1}{2}$″ diam.
W. 6.475 sq.in.
L. 1.54375 sq. in.
S. 9.25 sq. in.
Manila rope 1″ diam.
W. .765 sq. in.
L. .329 sq. in.
S. 1.877 sq. in.
Manila rope $2\frac{1}{2}$″ diam.
W. 4.78125 sq. in.
L. 2.05625 sq. in.
S. 11.73125 sq. in.

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23. Iron wire $1\frac{1}{4}$″ diam.
W. 6.0109375 sq. in.
L. 4.471875 sq. in.
S. 26.58125 sq. in.
Iron wire, $2\frac{1}{8}$″ diam.
W. 17.371609-375 sq. in.
L. 12.92371875 sq. in.
S. 76.8198125 sq. in.

24. Steel wire, $1\frac{1}{4}$″ diam.
W. 6.165625 sq. in.
L. 6.9390625 sq. in.
S. 43.171875 sq. in.
Steel wire, $2\frac{1}{8}$″ diam.
W. 17.81865625 sq. in.
L. 20.053890625 sq. in.
S. 124.76671875 sq. in.

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2. 12; 6

3. $5\frac{1}{4}$; $2\frac{3}{4}$

4. $12\frac{1}{2}$; $3\frac{1}{2}$

5. 26 girls;14 boys

6. 125; 50

7. 24; 16

8. $15\frac{1}{2}$; $7\frac{1}{2}$

9. $12\frac{1}{2}$; $7\frac{1}{2}$

11. 7; 21

12. 371.25; 41.25

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13. 3 tin; 12 copper

14. $22\frac{1}{2}$tons copper $2\frac{1}{2}$ tons tin.

15. $203\frac{1}{8}$; $28\frac{7}{8}$cu. in.

16. 7; 2

17. 8; 5

18. 3; 7

19. 15 in.; 9 in.

20. 24; 8

21. 16; 12

22. 24; 40

23. $\frac{3}{5}$

25. 2; 12

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26. $2\frac{1}{2}$; $3\frac{1}{2}$

27. -10; -35

28. 2; 3

29. $x=12$; $y=5$

30. $x=14$ $y=3$

31. $x=10$ $y=5$

32. $x=7\frac{1}{2}$, $y=5$

33. $x=16$, $y=14$

34. $x=1\frac{4}{7}$, $y=-1\frac{3}{7}$

35. $a=15$, $b=-21\frac{1}{2}$

36. $c=6\frac{2}{7}$, $d=2\frac{1}{7}$

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1. 212°F; 100°C.; 80°R.

2. 32°F.; 0°C; 0°R.

3. 80°R.; 100°C.; 180°F.

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1. Ice, 32°F.; 0°R.
Benzol, 39.9°F.; 3.52°R.
Tallow, 109.4° F.; 34.4° R.
Parraffin, 114.-8°F.; 36.8° R.
Wax, 143.6° F; 49.6° R.
Sulphur, 239° F.; 92°R.
Tin, 446°F.; 184°R.
Bismuth, 482° F.; 200°R.
Cadmium, 608° F.; 256°R.
Lead, 618.8° F.; 260.8°R.

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Zinc, 773.6°F.; 329.6°R.
Antimony, 829-.6°F.; 345.6°R.
Silver, 1832°F.; 800°R.
Copper, 2012° F.; 880°R.
Gold, 2192°F.; 960°R.
Cast-Iron, 2192° F.; 960°R.
Cast-Steel, 2507°F.; 1100° R.
Wrought-Iron, 2912°F.; 1280° R.
Platinum, 3227° F.; 1420°R.
Iridium, 3542°F.; 1560° R.

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2. Ether, 35°C.; 28°R.
Carbon Dis., 46°C.; 36.8° R.
Sulph. Acid, −10°C.; −8.° R.
Chloroform, 61°C.; 48.8°R.
Alcohol, 78°C.; 62.4°R.

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Water, 100°C.; 80.°R.
Mercury, 357° C.; 285.6°R.
Zinc, 1040°C.; 832°R.

4. 98.96° Fahr. 29.76° Ream.

6. Ether vapor, 384.8° Fahr.
Carbonic acid, 87.8° Fahr.
Etheline, 48.2° Fahr.
Oxygen, 180.4° Fahr.
Nitrogen, 229° Fahr.
Hydrogen, 281.2° Fahr.

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2. $114.5625

3. $338.125

4. 5.5; 2.40625 sq. in.

5. 254.57

6. 48.43″

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2. $156.45

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2. 1. 884
2 396
3. 1591
4. 2436
5. 3575

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6. 8036
7. 4899
8. 8019
9. 8075

3. 1. 1296
2. 2209
3. 4096
4. 3364
5. 361
6. 9025
7. 15625
8. 21904

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6. 1. 114.43
2. 20159.81
3. 8.56
4. 4780.30

7. 1. 248.729
2. 496.885
3. 1977.279
4. 4035.960

2. 1. 568.75
2. 73.27
3. 42.51
4. 2.39

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2. 1. 80.20
2. 196.54
3. 93.62

4. 1. 34
2. 212
3. 13.2
4. 222
5. 234
6. 538